Optimal production size with partially inspected batches

Arik Sadeh[1*] and *Cristina* Feniser[2]

[1]HIT, Holon Institute of Technology, Technology Management Faculty, 52 Golomb St. Holon, 58102, Israel

[2]Technical University of Cluj Napoca, Department of Management and Economic Engineering, Cluj Napoca, Romania

Abstract. While it is desirable to manufacture and ship perfect products, in many cases it is possible to ship products even though they do not stand up to all aspects of inspections. In this study a single machine is considered, that produces m products per day and there are n parameters that each product should pass. The number of passes (b) is a decision vector. A mathematical programming approach is used in order to model and optimize the number of products to be shipped under the constraints of failures. The results can be used for predetermined manufacturing, i.e. to set a series of tests less than m for each batch of production. In addition, aspects of calculation and computation are considered since the problem involves integer variables. Numerical examples and simulation runs of the suggested model are provided along with sensitivity runs.

1. Introduction

The practice of inspecting products before their shipment is a common procedure. While it is desirable to pass all inspections, there are many cases in which it is acceptable to ship products that may not pass all inspections. On the other hand, it could be of interest to avoid a certain number of inspections in order to ease operations and to reduce costs. The risk associated with this should then be taken into consideration.

A product or a batch of products either passes or fails a given inspection. There are cases in which not reaching full quality is applicable, yet a reasonable threshold for a maximum of failures is essential. One way to handle such a case is by using binary planning and zero-one programming.

This study intends to explore some aspects of partially inspected batches of products. It is of interest to provide a mechanism to help decision makers to handle the optimal set of rules. In the next section a literature review is provided, followed by a formal description of the problem. Then a numerical example is given along with an application section and a summary.

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[*] Corresponding author: sadeh@hit.ac.il

1.1 Literature review

The problem of partially inspected batches can be treated as a binary optimization problem. Padberg [1] and Nemhauser and Wholsey [2] discussed thoroughly the usage and advantage of zero-one problems. Bilitzki and Sadeh [3] addressed several special cases using zero-one programming.

These types of problems are also addressed in the literature on treating missing data. Missing data values appear in surveys in many disciplines, such as natural sciences, engineering, social sciences and clinical research. The naïve approach is to discard a batch when there is a missing inspection, as in the case of missing values.

A counterpart to eliminating all observations with any missing value is to fill in the missing values with empirical indices such as the sample average, the sample median, etc. Raaijmakers [4] introduced the relative mean substitution as a procedure for replacing missing data based on sample means. Other categories of procedures are based on the correlations between variables with and variables without missing data, e.g. multiple regression imputation (Raymond [5]). Another example of the second type of procedure is given by Mason [6]. Three tests for examining and treating missing values as a result of attrition in clinical studies are described in that study.

Rubin [7] introduced the method of multiple imputation (MI) following the work by Dempster, Laird and Rubin [8], which provided the EM algorithm for efficient estimation when there are missing values in the data set. A comprehensive description of the MI method is provided in Schafer and Olson [9] along with practical tools to use that method.

This study intends to provide a useful method to deal with partially inspected batches of manufactured products. It takes into consideration the predetermined number of inspections needed for a complete inspection along with a predetermined number of inspections to be executed.

The suggested model is described first along with an illustration; then a few examples based on simulated data are used in order to show the usefulness of the mechanism. The paper concludes with a summary.

2. The model

2.1 General formulation

Consider a machine that produces n products per day and each product j (j=1,2…n) has to pass m inspections (i=1,2…,m).
Let's mark

$$A_{ij} = \begin{cases} 1, \text{if product j fails to pass inspection i} \\ 0, \text{otherwise} \end{cases} \tag{1}$$

That is, a failure to pass an inspection gives the value 1. Each column in matrix A is associated with inspection i, while each row in matrix A is associated with product j.

The decision is whether or not to consider manufactured product j as a suitable product for shipment. Let's define the decision variables x_j:

$$x_j = \begin{cases} 0, & \text{if product } j \text{ to be discarded from the shipment set} \\ 1, & \text{otherwise} \end{cases} \tag{2}$$

For a given inspection i, d is the percent (proportion) of allowed failures. Consider the criterion for maximizing the number of manufactured products to be shipped:

$$\max \sum_{j=1}^{m} x_j$$

$$s.t. \quad \frac{\sum_{j=1}^{m} x_j A_{ij}}{\sum_{j=1}^{m} x_j} \le d_i \;, \; i = 1,2,,,,n \tag{3}$$

$$x_j = \begin{cases} 0 \\ 1 \end{cases}$$

The constraint equations can be rewritten as

$$\sum_{j=1}^{m} x_j A_{ij} - d \sum_{j=1}^{m} x_j \le 0 \;, \; i = 1,2,,,,n \tag{4}$$

This is an integer programming formulation with binary decision variables and the constraints mean that for each variable i, the ratio between the number of not shipped products and the total number shipped should be below a given threshold d_i. The value of d_i should be predetermined. When d_i is zero, only products that have fully passed inspection i can be considered for shipment. When d_i is 1, any product can be considered for shipment regardless of the result of inspection i.

When a portion of a product is applicable, e.g. j is a batch of many products, then the variables x_j can be bounded between zero and one and linear programming can be used.

Let's consider the parameter b_i as the number of allowed failures of inspection i, rather than the parameter d_i. Without loss of generality, the problem becomes:

$$\max \sum_{j=1}^{m} x_i$$

$$s.t. \quad \sum_{j=1}^{m} x_j A_{ij} \le b_i \;, \; i = 1,2,,,,n \tag{5}$$

$$x_j = 0,1$$

2.2 Special common cases

Let's consider a case with p inspections where q inspections of p can be ignored (q<p). Now define matrix $B^{q,p}$ with p rows and $\binom{p}{p-q}$ columns. Each column in matrix $B^{p,q}$ has (p-q)

values of "1" and p values of "0". Note that each row of matrix $B^{p,q}$ has $\binom{p-1}{q-1}$ values of "1".

It would be easy to define index t (t=0,1,2...$\binom{p-1}{q-1}$-1).

Now define the revised optimization problem to be:

$$\max \sum_{j=1}^{\binom{p}{q}} x_j$$
$$\text{s.t.} \quad B^{pq}x^{pq} \le t \tag{6}$$
$$x_j = 0, 1$$

where $x^{p,q}$ is a vector of size $\binom{p-1}{p-q}$ and **t** is a vector of t and t can be any number between 0 and $\binom{p-1}{p-q}$.

When p is large, this is a big problem to be solved using binary programming. A way to ease the computation is by using linear relaxation, then using the solution to find the binary solution.

When the product $\left(t \cdot \frac{p}{q}\right)$ is an integer, the solution is not necessarily an integer, but the objective value is an integer. This could help to find the optimal solution of the binary problem. In practice, it is advised to choose parameters l, k and r such that this product is an integer.

These findings lead to the following suggested procedure.

Step 0:   Define values for parameter p and parameter r (q<p).

Step 1:   Compute the range values of t between 0 and $\binom{p-1}{p-q}$, choose t.

Step 2:   Compute the term $\left(t \cdot \frac{p}{q}\right)$. If it is an integer, go to Step 3, if not, go to Step 1 and manipulate t.

Step 3:   Build the matrix B^{pq}. It is a matrix of zeros and ones, with k rows and $\binom{p}{q}$ columns. Note that in each column there are q ones, and in each row of B^{pq} there are $\binom{p-1}{q-r}$ ones.

Step 4:   Solve the linear relaxation of Equation (6) and keep the objective value.

Step 5:   Add a constraint that the objective value is bounded by the objective value of Step 4. Solve Equation (6) with this additional constraint.

The procedures described above are illustrated using numerical examples in the next section.

3. Numerical example

Consider a production line with p=9 inspections. If a batch passes any five of them, it is defined as a good batch. That is, q=5. The parameter t can be between 0 and 70, but for t=0,5,10,15…,70, the linear relaxation of Equation (6) will be an integer. For this

illustration, the value t=20 is chosen. The term $\left(t \cdot \frac{p}{q}\right)$ is 36. Let's build the matrix $B^{9,5}$ and optimize Equation (6) without linear relaxation. The solution is a set of 36 variables (36 columns of the matrix $B^{9,5}$). Each column has 5 ones, and each row has 20 ones.

This gives the five inspections that each batch should undergo.

4. Conclusions

A mathematical programming is used to develop a model to choose which batches to consider after inspections. The model helps to design the set of inspections needed per batch. Several convenient working rules are determined in the study that can help to ease the computation in the case of a large number of batches and inspections.

The model is very useful for cases with a limited number of batches and many inspections. The contribution of the suggested model is when there are k inspections, and it is for the decision maker to decide both how many inspections to make and how many batches to inspect.

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