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Numerical-analytical study on effect of three-phase poroelastic medium model parameters on dynamic displacements and porous pressures response

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Abstract. The problem of loading on a one-dimensional partially saturated poroelastic rod is considered. A mathematical model of a partially saturated porous medium based on the Bio model was used. Solid skeleton displacements and pore pressures of fillers are chosen as basic functions defining wave process. A mathematical model of the boundary-value problem of the dynamic theory of poroelasticity is considrered in Laplace domain. The solution in time domain is obtained using the stepping method of numerical inversion of the Laplace transform. Dynamic displacement responses and pore pressures are presented for different values of the parameters of the poroelastic material model.

1 Introduction

Porous materials are widely used in nature and engineering. An example of such materials may be a water saturated soils, rocks, biological tissues, foam metals, etc. The mechanics of porous media is important in many domains of science, such as geotechnics, geomechanics, engineering geology, biomechanics, engineering and materials science.

Research of wave processes in saturated porous media was started by Frenkel [1] and Biot [2, 3]. The proposed approach was later developed by other authors for the case of partially saturated porous media containing several fluid fillers. A model describing the dynamic behavior of a three-phase porous medium is presented in Beskos [4], Beskos et al. [5, 6] and Vgenopoulou et al. [7] researches. Garg and Nayfeh [8], Santos [9, 10], Tuncay and Corapcioglu [11, 12], Wei and Muraleetharan [13], Lo [14], Lu [15] studies on the theory of porous media with two fillers belong conceptually to the general approach with different details in implementation. A general system of equations for a multiphase porous medium was proposed by Nikolaevskiy [16].

Applying a model of a fluid saturated porous material (even with simplifications) results in significantly more complicated computational scheme of the boundary value problem in comparison with elastic or viscoelastic formulations when considering wave processes. The dynamic of the filler fundamentally changes the form of the wave patterns, which can be predicted with the help of advanced computational methods.

Nevertheless, analytical solutions are important since allow to identify mechanisms of interaction between the solid and fluid phases, conduct a posteriori evaluation of numerical results, and investigate transient processes in a porous medium subjected to dynamic loading.

Review of available analytical solutions of wave propagation problems in saturated porous media can be found in Schanz article [17]. Li and Schanz [18] presented an analytical solution for a one-dimensional partially saturated poroelastic rod.

2 Problem formulation

A set of fully coupled governing differential equations of a porous medium saturated by two compressible fluids (water and air) subjected to dynamic loadings is considered. In this formulation the solid skeleton displacements u_i, water pressure p^w and air pressure p^a are presumed to be independent variables [18]. The final differential equations in Laplace domain yield:

$$G\bar{u}_{i,jj} + (K + \frac{G}{3})\bar{u}_{j,ij} - (\rho - \beta S_w \rho_w - \gamma S_a \rho_a)s^2 \bar{u}_i - \\ -(\alpha - \beta)S_w \bar{p}^w_{,i} + (\alpha - \gamma)S_a \bar{p}^a_{,i} = -\bar{F}_i, \tag{1}$$

$$-(\alpha - \beta)S_w s\bar{u}_{i,i} - (\zeta - S_{aa}S_w + S_u)s\bar{p}^a + \\ + \frac{\beta S_w}{\rho_w s}\bar{p}^w_{,ii} - (\zeta S_{ww}S_w + \frac{\varphi}{K_w}S_w - S_u)s\bar{p}^w = -\bar{I}^w, \tag{2}$$

$$-(\alpha - \gamma)S_a s\bar{u}_{i,i} - (\zeta S_{ww} + S_u)s\bar{p}^w + \frac{\gamma S_a}{\rho_a s}\bar{p}^a_{,ii} - \\ -(\zeta S_{aa} + \frac{\phi}{K_a}S_a - S_u)s\bar{p}^a = -\bar{I}^a,\ x \in \Omega, \tag{3}$$

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where K_w and K_a are bulk moduli of the fluid, $\overline{F}_i, \overline{I}^w, \overline{I}^a$ are bulk body forces. The porosity φ is defined as

$$\varphi = \frac{V_{void}}{V} \tag{4}$$

where V_{void} is volume of interconnected pores in the specimen, V is total volume of the material. The bulk density is denoted by

$$\rho = (1-\varphi)\rho_s + \varphi S_w \rho_w + \varphi S_a \rho_a \,, \tag{5}$$

where ρ_s is the density of the solid, ρ_w is water density, ρ_a is air density. The saturation degrees are defined as the ratios of the volume occupied by the fluid V_w or V_a to the void volume, i.e. it holds

$$S_w = \frac{V_w}{V_{void}},\ S_a = \frac{V_a}{V_{void}},\ S_w + S_a = 1. \tag{6}$$

The following abbreviations

$$\zeta = \frac{\alpha - \varphi}{K_s}, \tag{7}$$

$$S_{ww} = S_w - \vartheta(S_w - S_{rw}), \tag{8}$$

$$S_{aa} = S_a + \vartheta(S_w - S_{rw}), \tag{9}$$

$$S_u = -\frac{\vartheta(S_{ra} - S_{rw})}{p^d}\left(\frac{S_w - S_{rw}}{S_{ra} - S_{rw}}\right)^{\frac{\vartheta+1}{\vartheta}}, \tag{10}$$

are introduced, where S_{rw} is the residual water saturation and S_{ra} is air entry saturation. The symbol p^d is non-wetting fluid entry pressure, ϑ skeleton grain size distribution coefficient while the value of ϑ lies between 0.2 and 3, normally. The symbols β and γ are Laplace parameter dependent variables and expressed as

$$\beta = \frac{\kappa_w \phi \rho_w s}{\varphi S_w + \kappa_w \rho_w s},\ \gamma = \frac{\kappa_a \varphi \rho_a s}{\phi S_a + \kappa_a \rho_a s} \tag{11}$$

where κ_w and κ_a the phase permeability of the water and the air are given by $\kappa_w = K_{rw} k / \eta_w$ and $\kappa_a = K_{ra} k / \eta_a$ respectively. K_{rw} and K_{ra} denotes the relative fluid phase permeability, k denotes the intrinsic fluid permeability, η_w and η_a are viscosity of the fluid. To evaluate relative phase permeability following equations are used

$$K_{rw} = S_e^{(2+3\vartheta)/\vartheta},\ K_{ra} = (1-S_e)^2\left[1 - S_e^{(2+\vartheta)/\vartheta}\right]. \tag{12}$$

S_e denotes the effective water saturation degree given by

$$S_e = \begin{cases} 0 & S_w \le S_{rw} \\ \dfrac{S_w - S_{rw}}{S_{ra} - S_{rw}} & S_{rw} < S_w < S_{ra} \\ 1 & S_w \ge S_{ra} \end{cases} \tag{13}$$

For the extreme case $S_w = 0$, the equations turn out to be simple a elasticity problem, for the extreme case $S_w = 1$ – saturated poroelacticity problem.

3 Analytical solution

Consider the problem of loading as a Heaviside function in time on a one-dimensional partially saturated poroelastic rod (1-d column) of length l. The scheme of the problem is presented in fig. 1.

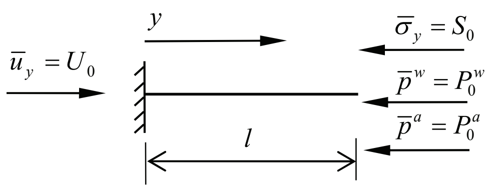

Fig.1 A 1-D column under dynamic loads.

Differential equations of one-dimensional propagation of poroelastic waves in Laplace domain has the following form:

$$\begin{aligned}&(K + \frac{G}{3})\overline{u}_{y,yy} - (\rho - \beta S_w \rho_w - \gamma S_a \rho_a) s^2 \overline{u}_y - \\ &-(\alpha-\beta) S_w \overline{p}^w_{,y} + (\alpha - \gamma) S_a \overline{p}^a_{,y} = 0,\end{aligned} \tag{14}$$

$$\begin{aligned}&-(\alpha-\beta) S_w s \overline{u}_{y,y} - (\zeta S_{aa} S_w + S_u) s \overline{p}^a + \\ &+ \frac{\beta S_w}{\rho_w s}\overline{p}^w_{,yy} - (\zeta S_{ww} S_w + \frac{\varphi}{K_w} S_w - S_u) s \overline{p}^w = 0,\end{aligned} \tag{15}$$

$$\begin{aligned}&-(\alpha-\gamma) S_a s \overline{u}_{y,y} - (\zeta S_{ww} + S_u) s \overline{p}^w + \\ &+ \frac{\gamma S_a}{\rho_a s}\overline{p}^a_{,yy} - (\zeta S_{aa} + \frac{\phi}{K_a} S_a - S_u) s \overline{p}^a = 0.\end{aligned} \tag{16}$$

Boundary conditions in Laplace domain in addition to equations (14)-(16):

$$\overline{u}_y\big|_{y=0} = 0,\ \overline{\sigma}_y\big|_{y=l} = 0\,, \tag{17}$$

$$\overline{p}^w\big|_{y=l} = P_0,\ \overline{q}^w\big|_{y=l} = 0\,, \tag{18}$$

$$\overline{p}^a\big|_{y=l} = 0,\ \overline{q}^a\big|_{y=l} = 0. \tag{19}$$

Exponential substitution

$$\overline{u}(y) = U e^{\lambda s y},\ \overline{p}^w = U^w e^{\lambda s y},\ \overline{p}^a = U^a e^{\lambda s y} \tag{20}$$

allow to have the following analytical solution:

$$\bar{u}_y = \frac{P_0}{Ms}\sum_{i=1}^{3}\left(t_i\frac{e^{\lambda_i s(y-l)} - e^{-\lambda_i s(y+l)}}{1+e^{-2\lambda_i sl}}\right) \tag{21}$$

$$\bar{p}^w = \frac{P_0}{M}\sum_{i=1}^{3}\left(a_i t_i\frac{e^{\lambda_i s(y-l)} + e^{-\lambda_i s(y+l)}}{1+e^{-2\lambda_i sl}}\right) \tag{22}$$

$$\bar{p}^a = \frac{P_0}{M}\sum_{i=1}^{3}\left(b_i t_i\frac{e^{\lambda_i s(y-l)} + e^{-\lambda_i s(y+l)}}{1+e^{-2\lambda_i sl}}\right) \tag{23}$$

Coefficients in equations (21)-(23) may be found in paper [18].

4 Method for numerically inverting Laplace transform

The stepping method of numerical inversion of the Laplace transform used to obtain the solution in time domain. The method, based on the operational calculus theorem, allows to calculate the required original function using quadrature formula [19]:

$$y(0) = 0, \quad y(n\Delta t) = \sum_{k=1}^{n}\omega_k(\Delta t), \quad n = 1,...N . \tag{24}$$

Weights ω_k are determined using the Laplace image of the inverse function $\bar{f}(s)$ and the linear multistep method

$$\omega_n(\Delta t) = \frac{R^{-n}}{L}\sum_{l=0}^{L-1}\bar{f}(s)se^{-in\phi}, \tag{25}$$

$$s = \frac{\vartheta(z)}{\Delta t}, \quad z = Re^{i\phi}, \quad \phi = 2\pi\tfrac{l}{L}, \tag{26}$$

$$\vartheta(z) = 3/2 - 2z + z^2/2, \tag{27}$$

where N is number of time steps, L is number of nodes for numerically integrating on argument ϕ.

5 Numerical results

The following values of the boundary value problem parameters used for calculations: $l = 10m$, $P_0 = 1N/m^2$. Water-saturated sand was used as a porous material of the rod with following parameters:

$K = 1.02\cdot10^9\,N/m^2$,
$G = 1.44\cdot10^9\,N/m^2$,
$\phi = 0.23$,
$\rho_s = 2650kg/m^3$,
$\rho_w = 997kg/m^3$,
$\rho_a = 1.10kg/m^3$,
$K_s = 3.55\cdot10^{10}\,N/m^2$,
$K_w = 2.25\cdot10^9\,N/m^2$,
$K_a = 1.10\cdot10^5\,N/m^2$,
$\kappa = 2.5\cdot10^{-12}\,m^2$,
$\eta_w = 1.0\cdot10^{-3}\,Ns/m^2$,
$\eta_a = 1.8\cdot10^{-5}\,Ns/m^2$,
$S_{rw} = 0$, $S_{ra} = 1$, $\theta = 1.5$.

The solution in time domain found with values $\Delta t = 5\cdot10^{-5}$, $N = L = 2000$, $R = 0.997$.

Displacement and pore water pressure are presented on fig. 2-4.

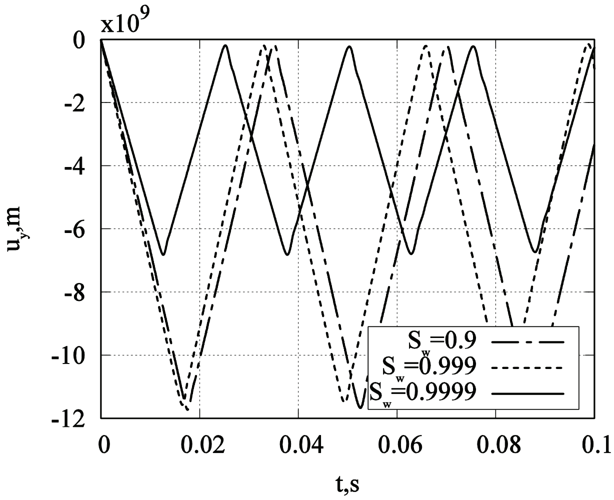

Fig. 2. Displacement versus time for different water saturation

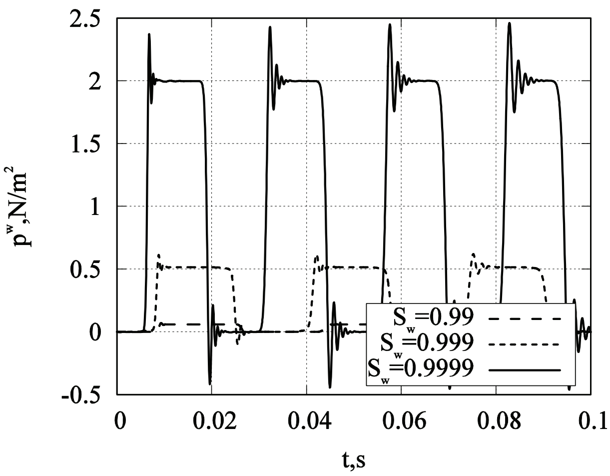

Fig. 3. Pore water pressure versus time for different water saturation

With an increase of the saturation degree from 0.99 to 0.9999, the amplitude of displacements decreases and velocity of the longitudinal wave increases (fig. 2). An increase in the amplitude is observed on the dynamic responses of pore pressures (fig. 3-4). The results of calculations are in qualitative agreement with the results presented in papers [18, 20]. Although these results were obtained for the case of loading $\bar{\sigma}_y = 1N/m^2$, similar effect of the saturation degree observed.

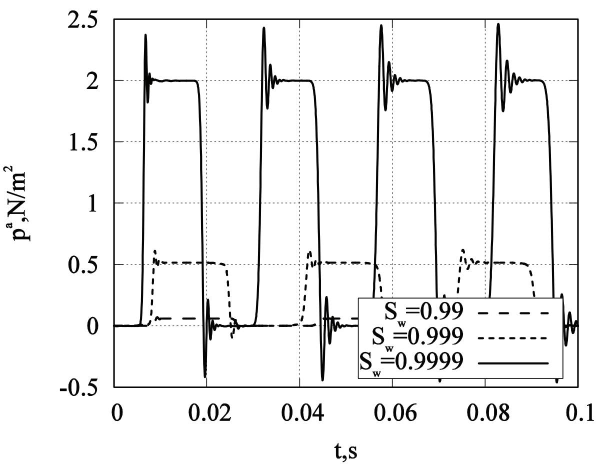

Fig. 4. Pore air pressure versus time for different water saturation

6 Conclusions

The analytical solution for the problem of one-dimensional partially saturated poroelastic rod dynamic loading is considered. The results of calculations obtained using the stepping method of numerical inversion of the Laplace transform are presented. The effect of the saturation degree on the dynamic responses of displacement and pore pressures is demonstrated. A qualitative coincidence of the results with the results of other authors observed.

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