Modelling the Reduced Coefficient of Volumetric Rigidity in High-Pressure Rubber-Cord Shell Hoses

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Abstract. In recent years, methods for calculating the hydraulic power drive have become increasingly widespread, taking into account the violation of the continuity equation during its operation at unsteady modes due to the use of the theory of volumetric rigidity in modeling. In this article approach to calculation of the reduced coefficient of volume rigidity of high-pressure hoses is considered.

1 Introduction

Hydraulic systems are increasingly used in modern machines and mechanisms. Typically, the working fluid in such systems operates under high pressure. To transport this liquid, metal pipelines and high-pressure hoses (HPH) are used. The working fluid can be various kinds of mineral and synthetic oils, liquid fuels, water-oil emulsions or water.

One of the most important characteristics of a hydraulic drive is the quality of its operation at non-steady-state conditions, when the equation of continuity of flow is violated because of the compressibility of the working fluid and the deformation of the hydraulic system [1-3]. In this case, in modelling, a certain dummy flow of fluid is introduced into the equation of continuity, which compensates for these effects.

However, in recent years, another method of modelling hydraulic power drives has been increasingly used, to take account of the violation of the continuity equation during its operation at unsteady modes in modelling - the theory of volumetric rigidity [4,5].

Thus, instead of the reduced modulus of elasticity of the system, the reduced coefficient of its volume rigidity is used [6,7]. Several formalized dependencies have been obtained to determine the reduced coefficient of volume hardness of various hydraulic devices [8-15]. However, for rubber-cord shell high-pressure hoses, such dependencies have not yet been obtained, since their elastic properties still do not lend themselves to a rigorous analytical description. Here, we address this omission.

2 Theory

The volumetric rigidity of a hydraulic drive is the property of the hydraulic system to resist changing its internal volume when the pressure changes. It is characterized by “reduced coefficient of volumetric rigidity” of a system.

The given volume rigidity will allow us to judge pliability of system in general and each her element separately similar to a concept of rigidity of solid bodies which indicates a change of the linear sizes of elastic elements at impact on them of external forces [1].

Liquid compresses when forcing in system with increased pressure that imposes the influence on property of the system in general. Therefore, the “volume rigidity” characterized by a coefficient of volumetric rigidity may serve as an integral criterion of dynamic properties of hydraulic system considering that change of volume of hydraulic system results from the increase liquid pressure boost that at the same time compresses [6].

\[ C_{np} = - \frac{dp}{dV}, \] (1)

where \( C_{np} \) - the reduced coefficient of volumetric rigidity of the considered site of the hydraulic actuator; \( dp \) – pressure increment of the considered site; \( dV \) – interior content increment of the considered site corresponding to an increment of pressure upon \( dp \).

We will define coefficient of volumetric rigidity of plant and equipment, hydraulic actuator as pressure increment caused by additional injection of a unit of volume, power liquid given to power liquid condition in hydraulic reservoir and environmental condition.

In addition to metal pipelines, high-pressure hoses (HPH) are also in current use. They are characterised by being composed of a rubber-cord shell and are used in the machine-building industry, in agriculture, in various types of mining and mining machines, in road-building machinery and in the metallurgical industry and other industries. They are also used in vehicles as power steering hoses and brake hoses. HPHs are thus essential to a wide range of different applications. The technique and experimental investigations made for a narrow class of high pressure hoses [16-18], but a general mathematical formula, suitable for calculating the reduced volumetric coefficient of stiffness of any type and size do not exist.

The aim of this work is to find the single formula for...
calculating the reduced volumetric stiffness of any high pressure hoses.

3 Analysis

According to GOST 6286-73 [19] the high-pressure hoses depending on the strength of the wire used, must be made in three groups: A, B and B.

Group A is manufactured with the use of a wire with a breaking force of not less than 15 kg.

Group B is manufactured using a wire with a breaking force of not less than 17.5 kg.

Group Z should are made of a lattin wire and a breaking force of at least 20 kg.

The high-pressure hoses of each group, depending on the design, can be of two types:

Type I- with one metal braiding (fig. 1)

Fig. 1. The high-pressure hoses type I construction with metallic braidings. Z group: 1 — hose tube; 2 — metal braiding; 3 — squeegee; 4 — outer rubber layer. A, B, C group: 1 — hose tube; 2 — textile braiding; 3 — metal braiding; 4 — provisional rubber layer; 5 — outer rubber layer.

Type II- with two metal braiding (fig. 2)

Fig. 2. The high-pressure hoses type II construction with metallic braidings. Z group: 1 — hose tube; 2 — metal braiding; 3 — squeegee; 4 — outer rubber layer. A, B, C group: 1 — hose tube; 2 — textile braiding; 3 — metal braiding; 4 — provisional rubber layer; 5 — outer rubber layer.

In the HPH, the first layer (the inner rubber hose chamber), which is firmly connected to the frame, receives a radially directed hydrostatic pressure \( p_\text{i} \). On the outer surface of the chamber, of radius \( r_\text{i} \), there is a pressure \( p_\text{z} \). The last pressure is assumed equal to 0, thus excluding the influence of the third layer—the external rubber lining. From the assumptions made, a calculation formula can be obtained for a single-loop HPH:

\[
p_1 = 0.184 \cdot K_b \cdot N \cdot n \cdot \frac{1}{(r + r_\text{i})^2}
\]

where \( K_b \) — braid durability to a break-up of N/thread, \( N \) — quantity of the streams of a wire put by the spool of the braiding machine, \( n \) — number of wires in the spool.

The second layer is the frame consisting of a row concentric or spiralate located laying which elements have some possibility of a shift, unlike the first layer. The special layer and structural design represent in homogeneous and anisotropic material.

We will accept the following assumptions for simplification of calculations:
1. Forces arising in a hose wall are perceived generally by a frame as the most rigid part of a hose;
2. The frame is considered as thin-walled cylinder closed from edges the outer rubber cover isn’t taken into consideration;
3. The outer rubber cover is not taken into consideration.

According to Sukharev and Lepetov [20] the significant difference in the loadings perceived by the first and last braidings is revealed. They represented the distribution of loads in the frame of a hose having homogeneous braidings in the form:

\[
p = p_1 + p_2 + p_3 + \cdots + p_i,
\]

where \( p_1, p_2, p_3, \ldots, p_i \) are the fractions of the hydraulic pressure \( p \) perceived by the corresponding metal braidings.

The equation of joint deformations takes the form:

\[
p \cdot r_\text{i}^2 = p_\text{i} \cdot (r_\text{i}^2 + 2r_\text{z}^2),
\]

where \( r_\text{i} \) and \( r_\text{z} \) are the average calculated diameters of the overlap of the first and second braidings (in centimeters), and \( p_\text{i} \) is the intermediate pressure between the first and second braidings (10^5 Pa).

It follows from (2) that

\[
p_1 = p \cdot \left(1 - \frac{r_\text{i}^2}{r_\text{i}^2 + 2r_\text{z}^2}\right)
\]

or

\[
p_2 = p \cdot \frac{r_\text{i}^2}{r_\text{i}^2 + 2r_\text{z}^2}
\]

From here:

\[
p_1 = p_\text{i} \cdot \left(1 - \frac{r_\text{i}^2}{r_\text{i}^2 + 2r_\text{z}^2}\right)
\]

\[
p_2 = p_\text{i} \cdot \frac{r_\text{i}^2}{r_\text{i}^2 + 2r_\text{z}^2}
\]

Expressing \( p_2 \) in terms of \( p_1 \), we obtain the calculated formula for finding \( p_2 \):

\[
p_2 = \frac{r_\text{i}^2}{r_\text{i}^2 + 2r_\text{z}^2}
\]
Using formula (2), we thus obtain a formula for calculating the pressure $p_2$:

$$p_2 = 0.184 \cdot k_b \cdot N \cdot n \cdot \frac{1}{(r + r_1)^2} \cdot \frac{C_1}{C_2}$$

We will receive the generalized formula for HPH containing $i$ of braidings, having entered coefficient

$$\theta_i = \frac{0.735}{2^{i+1}}$$

$$p_i = \theta_i \cdot k_b \cdot N \cdot n \cdot \frac{1}{(r + r_1)^2} \cdot \frac{C_1}{C_2} \quad (4)$$

We will find pressure increment, having differentiated (4)

$$dp_i = -\theta_i \cdot k_b \cdot N \cdot n \cdot \frac{dr}{(r + r_1)^2} \cdot \frac{C_1}{C_2} \quad (5)$$

The internal volume of pressure will change due to the deformation which has resulted because of change of pressure in the pipeline [3]:

$$V_{\text{trp}} = \pi \cdot r^2 \cdot l,$$

where $d$ and $l$ are the current values of the diameter and length of the pipe section in question, respectively.

The increment in the volume of the pipe is determined by the formula

$$dV_{\text{trp}} = d(\pi \cdot r^2 \cdot l) = 2 \cdot \pi \cdot r \cdot dr + \pi \cdot r^2 \cdot dl \quad (6)$$

where $dr$ and $dl$ — increment of the radius of a pipe and its length corresponding to pressure increment $dp$.

We will find a formula for calculation of the reduced factor of volume rigidity of i-braided HPH having substituted the found increments (5) and (6) in a formula (1)

$$C_{\text{np}} = \frac{\theta_i \cdot k_b \cdot N \cdot n \cdot C_1}{\pi \cdot 1 \cdot r \cdot (r + r_1)^3 \cdot C_2}$$

5 Conclusions

The analytical formula of calculation of the reduced factor of volume rigidity of HPH is received. It is required to carry out a series of tests for her confirmation and specification.

References

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