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# Analyzing The Behavior of Classical Functionally Graded Coated Beam

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**Abstract.** The governing equation of a classical rectangular coated beam made of two homogeneous layers at top ceramic coated layer and bottom metal layer and also single Functionally Graded Material (FGM) as a sub coated layer subjected to uniform distributed mechanical load are derived by using principle of virtual displacements and based on Euler-Bernoulli deformation beam theory (EBT). In FGM layer the material properties are presumed differ as an exponential function form in thickness coordinate. Hence, the aim of this paper is analyzed the static behavior of clamped-clamped thin coated beam under mechanical load.

## NOMENCLATURE

a = Length of beam
$E_c$ = Modulus elasticity of ceramic
$E_m$ = Modulus elasticity of metal
b = Width of beam
$h$ = Total thickness of beam
$M_x$ = Total moment
$u$ = Displacement in x direction
$v$ = Displacement in y direction
$w$ = Displacement in z direction
$u_0, w_0$ = Displacement mid-plane
$I$ = Second moment of area
$\delta W$ = Total virtual work
$\delta W_I$ = Internal virtual work
$\delta W_E$ = External virtual work
$D_{xx}$ = Flexural rigidity of beam
$\varepsilon_{xx}$ = Strain
$R$ = Matrix
$\sigma_{xx}$ = Stress
$m$ =Series member
$\gamma$ = Row number
$F_\gamma$ = Force column
$A_i$ = Fourier coefficient column

## INTRODUCTION

Generally, the surface of structures or elements is weaker than the inside of them. Therefore, the coating has been used for decades to increase the resistance of the surface of structures, reduce the stress concentration on the surface and stop cracks which have been generated. In the conventional surface coating method, one layer as a coated will be covered the surface of the body and the thickness range is from a micrometer to several millimeters. The coating thickness usually is depended on the type of material which is used as a substrate layer [1]. For example, in Thermal

Barrier Coating (TBC) one-layer ceramic is bonded to the substrate layer. The disadvantage of this type of coating is that due to the use of two completely different materials (ceramic-metal), the concentration of stress between the two layers' increases, and thus the two layers are separated [2]. To improve the weakness, Functionally Graded Materials (FGMs) have lately been suggested to modify the conventional coating [3]. Due to the characteristic of FGM composites, the properties of materials in the composite change very gradually between two different materials. This property makes the separation between the composite layers not present. Mainly, under a high-temperature atmosphere like a nozzle of the shuttle, nuclear fusion reactor, internal combustion engine and so on at the interface of two layers due to different thermal expansion, the mismatch will be happened [4]. Coating structures with FGM are called FGC. FGC beams under mechanical, thermal and thermos-mechanical loads are studied by several researchers [5-11]

The present article, a static analysis of FGC beam is studied by using a series displacement as a linear combination of known function which satisfies the boundary conditions and unknown parameters. The governing equation of FGC beam based on classical beam displacement theory and principle of virtual is derived. The material properties of FG layer through the thickness is considered an exponential function. In order to demonstrate the behavioral difference of a conventional composite with a modern composite which is consisting of an FGM layer. Two composite models are considered. Lastly, the effect of the rectangular clamped-clamped beam under distributed load is investigated.

## EXTRACTING GOVERNING EQUATIONS

The displacement field components in Timoshenko's beam are considered in the form below [12]:

$$\begin{aligned} u(x,y,z) &= u_0\left(x,y\right) - z\frac{dw_0\left(x\right)}{dx} \\ v\left(x,y,z\right) &= 0 \\ w\left(x,y,z\right) &= w_0\left(x\right) \end{aligned} \tag{1}$$

Thus, the axial strain component in mid-plane direction of beam is

$$\varepsilon_{xx} = \frac{du}{dx} = \frac{du_0}{dx} - z\frac{d^2 w_0}{dx^2} \tag{2}$$

Based on EBT strains are small in the case of the following terms are negligible compared to equation (2)

$$\frac{du}{dz}, \left(\frac{du}{dx}\right)^2, \varepsilon_{zz}, \varepsilon_{xy} \tag{3}$$

## FGM material properties

The continuous composition of FGM can be described by using several mathematical models to show the material properties of the composite. The material properties of FGM are inhomogeneous microscopically, therefore the material properties of FGMs depend on position. One of the mathematical models of material properties was used in a lot of research in order to study fracture mechanics, crack propagation, vibration, and bending, is exponential function [4, 5, 13-20].

Position dependent material properties with exponential function of FGM are

$$E_{fe}\left(z\right) = E_c e^{\left(\frac{1}{h}\ln\left(\frac{E_m}{E_c}\right)\left(z+\frac{h}{2}\right)\right)} \tag{4}$$

## Governing equation of beam

To derive the governing equations of the beam, we use the principle of virtual work defined as [21]:

$$\delta W = \delta W_I + \delta W_E = 0 \tag{5}$$

Where $\delta W_I$ and $\delta W_E$ are virtual work cause of internal forces and virtual work cause of external forces respectively. The boundary conditions of clamped-clamped beam are

$$w(0) = w(a) = \frac{dw}{dx}(0) = \frac{dw}{dx}(a) = 0 \tag{6}$$

Also, the variation form of the beam should satisfy the equation (6):

$$\begin{gathered} \delta w(0) = \delta w(a) = 0 \\ \frac{d\delta w}{dx}(0) = \frac{d\delta w}{dx}(a) = 0 \end{gathered} \tag{7}$$

The stress-strain relation in the mid-plane direction is

$$\sigma_{xx} = E(z)\varepsilon_{xx} \tag{8}$$

Substitution equation (2) into equation (8), the resulting moment per unit length of the beam is

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) b\left[ z\frac{du_0}{dx} - z^2\frac{d^2 w}{dx^2} \right] dz \tag{9}$$

The total internal virtual work done is

$$\delta W_I = \int_0^a \int_A \sigma_{xx}\delta\varepsilon_{xx} dA dx \tag{10}$$

The virtual work done by external distributed load by using the virtual displacement is

$$\delta W_E = -\int_0^a bP_0 .\delta w dx \tag{11}$$

By using equations (10) and (11) into equation (5) and with consideration the boundary conditions (equations (6) and (7)), it can be obtained the governing equation of beam, taking into account Euler-Bernoulli theory, according to the following:

$$\int_0^a D_{xx}\frac{d^2 w}{dx^2}\frac{d^2\delta w}{dx^2} dx - \int_0^a bP_0\delta w dx = 0 \tag{12}$$

Where $D_{xx} = IE_{fe}(z)$

## SOLUTION TO RECTANGULAR COATED BEAM

An approximate solution, the deflection of the beam will be obtained based on boundary conditions (equation (7)) and geometry which is occupying the space defined by

$$0 \le x \le a \qquad -\frac{h}{2} \le z \le \frac{h}{2} \tag{13}$$

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Where $a$ and $h$ are the length of the beam and the total thickness of beam included the three layers respectively.

To find the answer to equation (12), based on boundary condition and geometry of beam, an approximate solution is obtained as:

$$w = \sum_{i=1}^{m} A_i \sin^2\left(\frac{i\pi x}{a}\right) \tag{14}$$

By using the solution equation (14) and variation form of solution in equation (12) we get

$$R_{(\gamma)(i)} A_i - F_\gamma = 0 \tag{15}$$

## NUMERICAL CALCULATIONS

In order to analyze the behavior of coated beam, two cases are considered. In case one, the coated beam has two layers (coating ceramic layer with (4 mm) thickness on top and substrate metal layer with (6 mm) thickness on bottom). Case two, the coated beam has three different layers. The homogeneous ceramic layer with (2 mm) thickness as a coated, FGM layer with (2 mm) thickness as an under coated (bonded) of coated layer and substrate as a homogeneous metal layer with (6 mm) thickness which are located from top to bottom of the model in thickness direction respectively. In the whole layers of the coated beam, the Poisson's ratio was presumed to be constant $(\nu = 0.3)$ [22, 23]. In the exponential function which defined in Equation (4), the ratio of Young's modulus $\left( E_m / E_c \right)$ was assumed constant and equal to $\left( 1/10 \right)$, ( $E_m = 10$ GPa and $E_c = 100$ GPa).

### Static analysis

To illustrate the numerical approach, the dimensions of the coated beam in both cases were taken as a = 50 cm, b = 5cm and h = 1cm. The coordinate axes were located in the middle of the volume of the coated beam. The uniformly distributed load is assumed to be $P_0 = 22kPa$ on top of the beam with clamped-clamped boundary conditions (Equation (6)). With consideration $i = \gamma = 2$ in equation (15), the series of the solution will be achieved with two members.

In Figure (1), the deflection of the coated beam is shown for two cases. The behavior of beam in both case slightly was almost identical, yet, in FGM beam (case two) it was apparent that the deflection at the middle of the beam was observed to be less than 2mm bigger in contrast with the conventional coated beam (case one). Figure (2) shows the variation of the stress $(\sigma_{xx})$ throughout the thickness of the coated beam for both cases. In case one, it clearly shows that the stress singularity at the interface between the ceramic layer and substrate layer. Because of this phenomenon, de-bonding will appear at the interface between two layers. For FGM beam graph or case two in Figure (2), because of FGM layer at the interface between the ceramic coated layer and substrate layer, it was observed that the stress gradually changed. Thus, stress concentration and singularity will be decreased. It means, in case two, there is the possibility to create separation at interface two layers will be much less compared to case one.

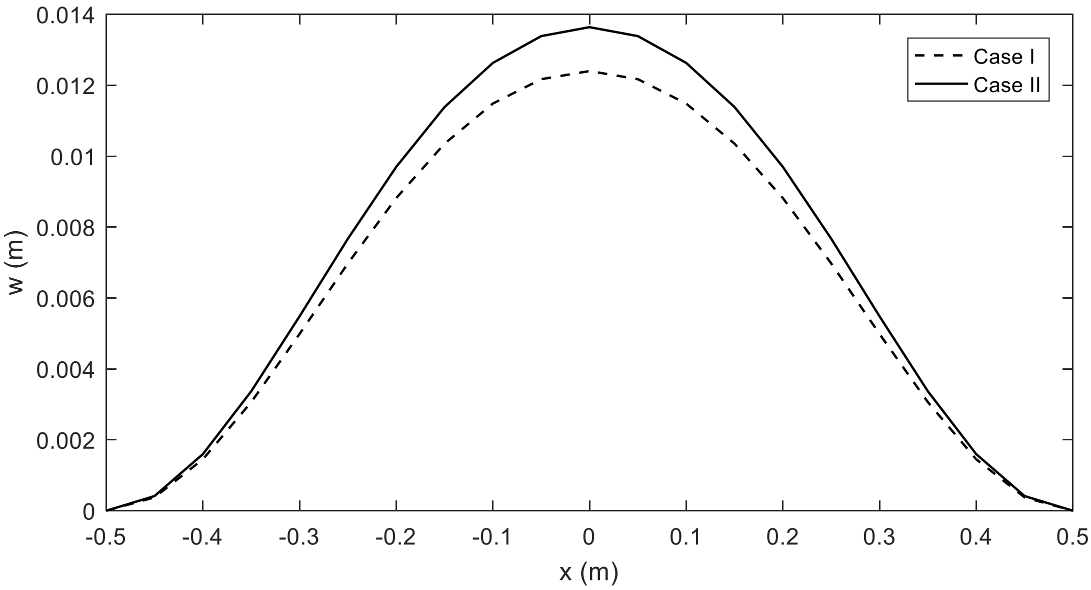


Fig. 1. The deflection $(w)$ of coated beam in case I and case II along x axis $\left(-\frac{a}{2} \leq x \leq \frac{a}{2}\right)$

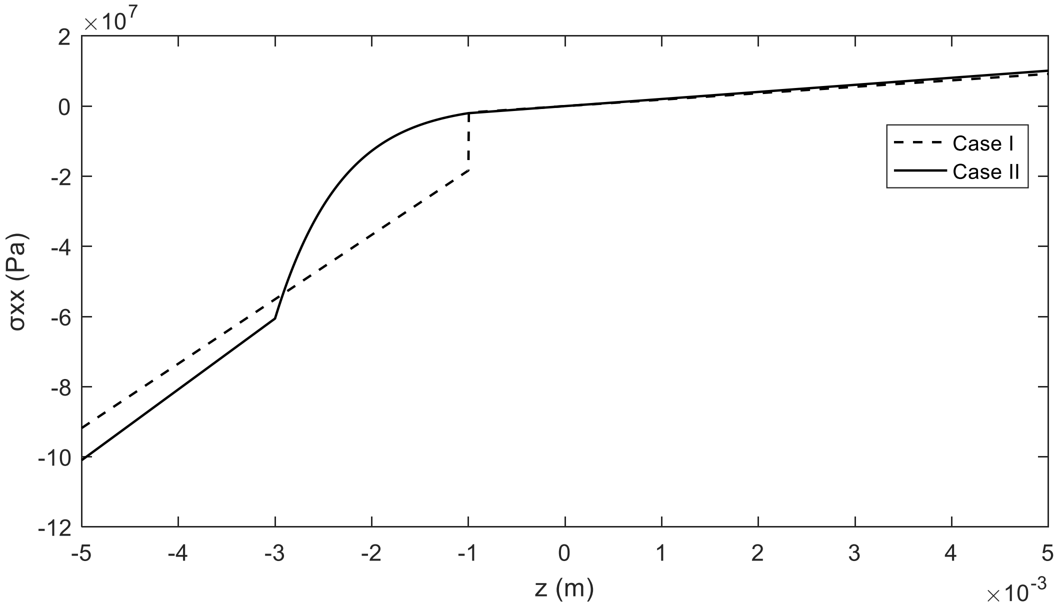


Fig. 2. The stress $(\sigma_{xx})$ throughout the thickness of the coated beam in case I and case II

## CONCLUSION

The static analysis of conventional coated and FGC under coated rectangular beam under the transverse distributed load was performed. The exponential function in demonstrating the material's properties behavior was considered. Henceforth, the governing equation was then derived based on the Euler-Bernoulli theory and exponential function for FGM layer by using the virtual work principle with only the transverse displacement and stress throughout the thickness of the beam being considered. In conclusion, the obtained approximation function for the displacement or behavior of structure under mechanical load was found. It should be noted that this approximation function is supported by the clamped-clamped boundary conditions. Clearly, from this study, the superiority of FGC composite against conventional coated composite has been demonstrated.

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