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The elastic-plastic contact of a single asperity of a rough surface

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Abstract. A penetration of spherical asperity into the elastic-plastic hardening half-space is described. The elastic-plastic material properties correspond to Hollomon's power law. In this case the empirical Meyer law relating a spherical indentation load with an indentation diameter *d* is used. Initially, the Meyer law is not related to the mechanical characteristics of the test material. The study used the relations between the strain hardening exponent *n* and the Meyer law constant obtained by S.I. Bulychev. The effects relating to elastic punching and plastic displacement of material are taked into account. It is shown that there is no need to define Meyer law constants. Expressions relating the value of the relative load magnitude to the relative indenter penetration magnitude are presented. The scope of application of the proposed equations is defined. A comparison of the obtained results with the experimental data and published data of the finite element analysis is given.

1 Introduction

Widespread use in tribology finds a discrete model of roughness, in which the asperities are presented as a set of bodies of regular geometric shapes, for which solutions of contact problems are available [1, 2]. In this case, the asperity model in the form of a spherical segment is considered to be optimal.

Problems of spherical asperity elastic-plastic penetration are not sufficiently studied and the solutions suggested require clarification and improvement [3]. One of important problems to be taken into account is material hardening. The authors' approach to solving this problem is given in [4-6 et al.]. The method consists in the use of the kinetic indentation load-displacement diagram and the similarity method of deformation characteristics. In this case, the notion "plastic hardness" is used as characteristics of material resistance to contact plastic strain. Plastic hardness has the form

$$HD = K_h(\varepsilon_y, n) \cdot \sigma_y,$$

where σ_y is the yield strength, $K_h(\varepsilon_y, n)$ is a parameter defined by the double indentation technique [4] using the results of the finite element analysis [7], ε_y and n are the characteristics of Hollomon's elastic-plastic material.

In a number of works [8, 9] the empirical Meyer law linking the spherical indentation load and an indenter diameter was used to allow for material hardening in solving the tribomechanics problems. In [9] an influence of some physical and mechanical properties of real materials on the features of contact elastic-plastic deformation formation is emphasized. However, the limitation of the method is that it does not explicitly take into account elastic-plastic characteristics of the hardened material.

The aim of this work is to use the above mentioned hardened material characteristics in the Meyer law application.

2 Problem solution

In describing elastic-plastic characteristics of the hardened material Hollovon's power law is widely used. According to this law the relation between the true stress *S* and the strain ε under uniaxial tension or compression is described by equations

$$S = \begin{cases} \varepsilon E, & \varepsilon \le \varepsilon_y; \\ K\varepsilon^n, & \varepsilon \ge \varepsilon_y; \end{cases} \quad (1)$$

where E is an elastic modulus, n is a strain-hardening exponent.

The constant K is determined from the equality condition for σ at ε_y. Then the second expression in Eq. (1) can be written as

$$\frac{S}{\sigma_y} = \left(\frac{E\varepsilon}{\sigma_y}\right)^n = \left(\frac{\varepsilon}{\varepsilon_y}\right)^n, \ \varepsilon \ge \varepsilon_y. \quad (2)$$

where $\sigma_y \approx S_y$, σ_y - yield strength, $\varepsilon_y = \sigma_y / E$.

Taking into accord that the limiting uniform strain $\varepsilon_u = n$, the strain hardening exponent can be defined from following Eq. as [10]

$$n \ln n - n(1 + \ln \varepsilon_y) - \ln \frac{\sigma_u}{\sigma_y} = 0, \quad (3)$$

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where σ_u is the tensile strength.

E. Meyer was the first to describe a material behavior in the elastic-plastic domain. He related the load P to the indentation diameter d as

$$P = Ad^m. \tag{4}$$

The empirical Meyer law is often written as

$$\frac{4P}{\pi d^2} = A^*\left(\frac{d}{D}\right)^{m-2}. \tag{5}$$

where m, A, A^* are constants. A^* has a dimension of strength.

The equation in the left part is a mean contact area pressure referred to as the Meyer hardness

$$\frac{4P}{\pi d^2} = \frac{P}{\pi a^2} = p_m = HM, \tag{6}$$

where a is the radius of the contact area.

Using the maximum Meyer hardness concept we have

$$HM_{\max} = HM\big|_{d=D},$$

from equations (5) and (6) it follows that

$$p_m = HM_{\max}\left(\frac{d}{D}\right)^{m-2} = HM_{\max}\left(\frac{a}{R}\right)^{m-2}. \tag{7}$$

The maximum Meyer hardness relates to the Brinell hardness as [11]

$$HM_{\max} = 2HB/k_m, \tag{8}$$

$$k_m = m^{\frac{m}{2}}(m-1)^{1-m}(m-2)^{\frac{m-2}{2}}. \tag{9}$$

From [12] it follows that

$$\sigma_u = k_\sigma \cdot HB, \tag{10}$$

where $k_\sigma = 0.333$ for carbon and pearlitic steel, for other materials the values of k_σ are given in [13, 14]. As it shown in [14], in practice well justifies the dependence

$$k_\sigma = 0.333 \cdot (\delta_u/\psi)^{0.25}$$

where δ_u is the uniform elongation, ψ is the deformation in a print of a spherical indenter.

According to the data obtained by S.I. Bulychev [15], the limiting uniform strain ε_u corresponding to σ_u is equal to

$$\varepsilon_p = n = 0.961(m-2). \tag{11}$$

From here we have

$$m = 2 + 1.041n. \tag{12}$$

From Eq. (10) with regard to Eqs. (2) and (11) it follows that

$$HB = \frac{\sigma_y}{k_\sigma}\left(\frac{n}{e}\right)^n \cdot \varepsilon_y^{-n}. \tag{13}$$

where e is base of natural logarithm.

For strain (load) $P = \pi a^2 p_m$ with regard to Eqs. (7), (8) and (13), we have

$$P = 2\pi \frac{\sigma_y a^2}{k_\sigma \cdot k_m}\left(\frac{n}{e}\right)^n \varepsilon_y^{-n}\left(\frac{a}{R}\right)^{m-2},$$

$$\frac{P}{E^* R^2} = \frac{2\pi}{k_\sigma \cdot k_m}\left(\frac{n}{e}\right)^n \varepsilon_y^{1-n}\left(\frac{a}{R}\right)^m. \tag{14}$$

With regard to (12) we have

$$\frac{P}{E^* R^2} = \frac{2\pi}{k_\sigma \cdot k_n}\left(\frac{n}{e}\right)^n \varepsilon_y^{1-n}\left(\frac{a}{R}\right)^{2+1.041n}. \tag{15}$$

where E^* is reduced elastic modulus,

$$k_n = \frac{(2+1.041n)^{1+0.5205n}}{(1+1.041n)^{1+1.041n}}(1.041n)^{0.5205n}.$$

As $a = \sqrt{2Rh_c} = \sqrt{2Rc^2h}$, where h_c is the depth at which the sphere contact with a half-space takes place, h is the depth of indentation from the initial surface level, we have

$$\frac{P}{E^* R^2} = \frac{2\pi}{k_\sigma \cdot k_n}\left(\frac{n}{e}\right)^n \varepsilon_y^{1-n}\left(\frac{2c^2h}{R}\right)^{1+0.5025n}. \tag{16}$$

The parameter $c^2 = h_c/h$ can be defined based on the data obtained in [16]

$$c^2 = c^2(n) = 1.4e^{-0.97n}, \tag{17}$$

Equation (17) is in good agreement with the results of FE simulations [17].

A lower limit is $\bar{a}_y = a_y/R$ corresponding to ε_y. According to [15] under spherical indentation, the strain is defined by the relation

$$\varepsilon = \alpha\left(\frac{d}{D}\right)^\beta = \alpha\left(\frac{a}{R}\right)^\beta, \tag{18}$$

where

$$\alpha = 0.15 + 1.85(m-2)^{1.4},\ \beta = 1.18\left(1 + (m-2)^{1.6}\right).$$

Or with regard to Eq. (12) it follows that

$$\alpha = 0.15 + 1.85(1.041n)^{1.4},\ \beta = 1.18\left(1 + (1.041n)^{1.6}\right).$$

Then we have

$$\bar{a}_y = \left(\frac{\varepsilon_y}{\alpha}\right)^{\frac{1}{\beta}}, \tag{19}$$

and the corresponding load is defined from Eq. (15).

An upper limit is defined from the condition $h_c = c^2 h^* = R$.

With regard to Eq. (12) we have

$$\frac{h^*}{R} = 1.4e^{-0.97n}. \tag{20}$$

The corresponding load is defined from Eq. (16).

3 Comparison with experimental data and published data of the finite element analysis

For estimation of the obtained results, we will compare it with the experimental study given in [18]. In this paper for calculating parameters of elastic-plastic contact, the authors used the generalized deformation curve and method of variable elasticity parameters. The experimental study were to determine the contact radius curvatur during indentation hardened steel ball (HRC 63…64) with a radius of curvature 2.5 mm into material samples specimens in the table. Mechanical properties of the material of specimens defined by results of tension.

To the analytical dependences for each material from the Eq. (3), the values of hardening exponent n were determined. The values of ε_y were determined by the formula

$$\varepsilon_y = \frac{\sigma_y}{E^*} = \sigma_y \cdot \left(\frac{1-\nu_i^2}{E_i} + \frac{1-\nu^2}{E}\right), \tag{21}$$

where ν_i, E_i are Poisson's ratio and the elastic modulus of the indenter.

Table 1. Mechanical properties of materials [18]

No	Materials	σ_y, MPa	σ_u, MPa	E, GPa	ν
1	Armko-iron	256	410	210	0.3
2	Steel 45	480	725	204	0.26
3	Steel 30ХГСА	667	942	215	0.3
4	Steel 30ХГСА	1207	1344	215	0.3
5	Copper M2	69	196	132	0.35
6	Duraluminium	265	392	72	0.3
7	Titanium BT6	687	883	118	0.32

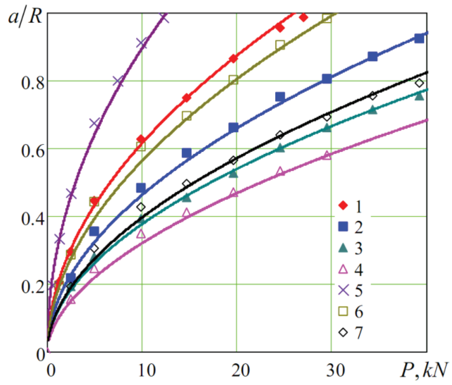

Fig. 1. Dependence of the relative radius of contact of the load P.

The fig. 1 show the experimental data [18] are presented by points, the coordinates were have been "digitized" in the processing of the results. The lines denote the corresponding dependence, calculated by the Eq. (15).

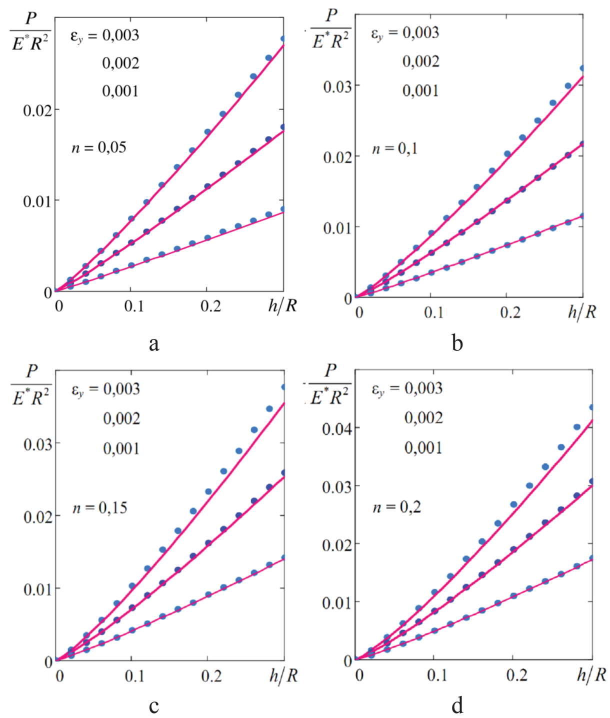

Fig. 2. Dependences $\bar{P} - \bar{h}$: for ε_y = 0,001; 0,002; 0,003; *a*) n = 0,05, *b*) n = 0,1, *c*) n = 0,15, *d*) n = 0,2; solid lines - according to expression (18); dashed - according to the expression (16).

Let us also compare the results with the data of the finite element analysis [19].

$$\frac{P}{E^* R^2} = e^{-B(\varepsilon_y,n)} \left(\frac{h}{R}\right)^{A(\varepsilon_y,n)}, \tag{22}$$

where

$$A(\varepsilon_y, n) = \frac{\left[\begin{array}{l} 0{,}93 + 290{,}65\varepsilon_y - 23408{,}85\varepsilon_y^2 + \\ + 546404.43\varepsilon_y^3 + 3{,}11n + 0{,}77n^2 \end{array}\right]}{\left[\begin{array}{l} 1 + 194.49\varepsilon_y - 17101{,}85\varepsilon_y^2 + \\ + 407006{,}05\varepsilon_y^3 + 2{,}1684n \end{array}\right]},$$

$$B(\varepsilon_y, n) = \frac{\begin{bmatrix} 5{,}30 + 81{,}12\varepsilon_y - 21284{,}04\varepsilon_y^2 + \\ + 381357.70\varepsilon_y^3 - 8{,}34n + 1{,}81n^2 \end{bmatrix}}{\begin{bmatrix} 1 + 437{,}16\varepsilon_y - 9993{,}45\varepsilon_y^2 + \\ + 0{,}34{,}05n - 3{,}43n^2 \end{bmatrix}}.$$

The fig. 2 show the dependences calculated by Eq. (16) and Eq. (22). As it follows from Fig. 1 and Fig. 2, there is a good agreement between the analytical dependences, the experimental data and the results of the finite element analysis.

Thus, the proposed approach suggests an alternative to a more complex method for describing elastoplastic penetration of a sphere on the basis of the kinetic indentation diagram [5], which was used in solving problems of elastoplastic contacting of rough surfaces.

4 Conclusions

1. Using the empirical Meyer law initially not linked to the physical and mechanical properties of materials, we obtained Eqs. (15) and (16), which allows *defining* a contact area radius and spherical indentation depth into an elastic-plastic hardening half-space.
2. Characteristics of elastic-plastic material such as $\varepsilon_y = S_y / E^*$ and strain hardening exponent n are taken into account. It is shown that there is no need of determine the Meyer law constants m and A^*.
3. The scope of the obtained equations application is defined.
4. A comparison of the results obtained by the authors with the published results of spherical indentation into elastic-plastic half-space obtained by the experimental data allows recommending the given relations for the solution to problems of surface plastic deformation problems.

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