Feedback linearized trajectory-tracking control of a mobile robot

Aleksey Kabanov^{1,2,*}

¹Federal Research Centre "Computer Science and Control" of Russian Academy of Sciences, Moscow, Russian Federation

²Sevastopol State University, 299053 Sevastopol, Russian Federation

Abstract. This paper is devoted to the designing of a trajectory-tracking control system for a unicycle-type mobile robot. Synthesis of the trajectory control law is based on the feedback linearization method and a canonical similarity transformation of nonlinear affine system in state-dependent coefficient form. The result of experimental test of the trajectory control system for mobile robot Rover5 is presented.

1 Introduction

The problem of mobile robot (MR) control is relevant over the years because of the wide range of theoretical problems and practical applications associated with it. There is a great number of works in the field of motion control of MR that are published in recent decades. Their reviews can be founded in [1-4].

Trajectory-tracking control problem is a typical control problem for MR. It is mainly concerned with the design of control laws that force a MR to reach and follow a time parameterized reference trajectory [5]. Thus, as a rule, simple non-linear models are operated, which characterize only the kinematic relationship between the MR's motion parameters and parameters that are predetermined by a reference trajectory.

At this point it is possible to mark out two basic approaches to design the trajectory-tracking system. The first approach is based on linear control design methods and tangent linearization of system model about the reference trajectory. At that, a pole placement method or linear optimal control method are usually used to adjust controller parameters [6-8]. Another approach is based on non-linear control design methods, such as feedback linearization (FL), Lyapunov techniques [9-11].

A lot of publications are devoted to the trajectory-tracking control problem for MR with nonholonomic constraints by means of FL (static and dynamic). In [9] applicability of static FL for setpoint regulation problems and for trajectory-tracking problems is considered. In [11] the trajectory-tracking control problem based on dynamic FL is considered. It is shown that dynamic FL is an efficient design tool leading to a solution simultaneously valid for both trajectory tracking and setpoint regulation problems. The main drawback of such solution is aligned with the raise of the system order and the order of the dynamic feedback.

In this paper, we propose another approach for static FL control design for trajectory-tracking, based on representing the original nonlinear system into a state-

dependent coefficient form and applying the canonical similarity transformation, that allow getting the system to canonical form. This similarity transformation allow accomplishing linearization of a system without determining of a virtual system output.

The remaining sections of the paper proceed as follows: Section 2 describes the trajectory-tracking control problem statement; Section 3 is devoted to the problem of designing of trajectory control system for MR based on FL method; Section 4 describes the structure of the trajectory control system, results of experimental tests for MR Rover5; some conclusions are shown in Section 5.

2 Formulation of a Trajectory-Tracking Control problem for MR

The simplest model of a nonholonomic MR is the unicycle. Let us consider the kinematic model of a unicycle-type MR motion in a horizontal plane (Fig.1a) [1-4]:

$$\begin{aligned}\dot{X} &= V \cos \varphi = \frac{1}{2} \cdot \cos \varphi (V_1 + V_2), \\ \dot{Y} &= V \sin \varphi = \frac{1}{2} \cdot \sin \varphi (V_1 + V_2), \\ \dot{\varphi} &= \omega = \frac{1}{2d} \cdot (V_2 - V_1),\end{aligned}\quad (1)$$

where X, Y are coordinates of MR posture in the earth coordinate system; V_1, V_2, V are linear velocities of left and right wheels and velocity of MR; ω is a MR angular velocity; φ is an angle between the vector V and the axis OX .

We suppose that there is a given trajectory of MR

$$p_{ref}(t) = (X_{ref}(t), Y_{ref}(t), \varphi_{ref}(t)).$$

* Corresponding author: kabanovaleksey@gmail.com

MR trajectory-tracking errors (Fig. 1b) is defined as follows [1-4]

Fig. 1. a The MR on a plane. **b** Statement of the trajectory-tracking control problem for MR.

$$\begin{aligned} e_x &= X_{ref} - X, \\ e_y &= Y_{ref} - Y, \\ e_\varphi &= \varphi_{ref} - \varphi, \\ e_1 &= e_x \cos \varphi + e_y \sin \varphi, \\ e_2 &= e_y \cos \varphi - e_x \sin \varphi, \\ e_3 &= e_\varphi. \end{aligned} \quad (2)$$

Differentiating (2), we obtain the dynamic model of MR trajectory following errors:

$$\begin{aligned} \dot{e}_1 &= \omega_{ref} \cdot e_2 + u_1, \quad \dot{e}_2 = -\omega_{ref} \cdot e_1 + V_{ref} \cdot \sin e_3, \\ \dot{e}_3 &= u_2, \quad u_1 = V_{ref} \cdot \cos e_3 - V, \quad u_2 = \omega_{ref} - \omega. \end{aligned} \quad (3)$$

The problem is to find control inputs u_1 and u_2 , such that system (3) be an asymptotically stable, i.e. $e_i \rightarrow 0, i=1,2,3$ for $t \rightarrow \infty$.

3 Control law design

3.1. Feedback Linearization of MIMO nonlinear system

Nonlinear MIMO system in state-dependent coefficient form is considered:

$$\dot{x}(t) = A(x)x + B(x)u, \quad (4)$$

where $x \in R^n$ is a state vector; $u \in R^r$ is a control vector; matrix $A(x) - (n \times n)$ and $B(x) - (n \times r)$ are continuous differentiable matrices and have bounded derivatives. Let us suppose that the system (4) is controllable.

The feedback linearization problem of the system (4) consists in searching a nonsingular similarity transformation $z = T(x)x$ and a control input $u = u_{FL}$ by means of which the system (4) can be transformed into a linear canonical form.

The procedure of constructing the similarity transformation is given in detail in [13]. Here is an example of the system conversion for the case $u \in R^2$. Applying the similarity transformation $z = T(x)x$ to the system (4) we get

$$\begin{aligned} \dot{z} &= A_C(z)z + B_C(z)u, \\ A_C &= TAT^{-1} + \dot{T}T^{-1}, \quad B_C = TB, \\ A_C &= \begin{pmatrix} J_{r_0} & E_{r_0 r_1} \\ E_{r_1 r_0} & J_{r_1} \end{pmatrix}, \quad B_C = \begin{pmatrix} e_{p_1 p_1} & B_{12} \\ 0 & e_{p_2 p_2} \end{pmatrix}, \\ J_k &= \begin{pmatrix} 0 & I \\ a_{k1} & a_{k2} \dots a_{kk} \end{pmatrix}, \quad E_{kl} = \begin{pmatrix} 0 \\ a_{k1} \dots a_{kl} \end{pmatrix}, \\ B_{12} &= \begin{pmatrix} 0 \\ b_{12} \end{pmatrix}, \quad I = \text{diag}(1, \dots, 1), \\ J_k &= (k \times k), \quad E_{kl} = (k \times l), \quad B_{12} = (r_0 \times 1), \\ I &= (k-1, k-1), \quad k = r_0, r_1; \quad l = r_0, r_1; \quad m = 2. \end{aligned} \quad (5)$$

Matrix B_C can be presented in terms of

$$\begin{aligned} B_C &= \bar{B}C, \quad C = (2 \times 2), \\ \bar{B} &= \begin{pmatrix} e_{p_1} & 0 \\ 0 & e_{p_2} \end{pmatrix}, \quad C = \begin{pmatrix} 1 & b_{12} \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

Since C is nonsingular, there is a new equivalent input $\bar{u} = Cu$. Feedback linearizing control for system (4) can be defined in the next form

$$\begin{aligned} u &= u_{FL} + v = C^{-1}(K_{FL}z + v), \\ K_{FL} &= - \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \end{pmatrix}. \end{aligned}$$

Instead of elements k_{ij} , it is necessary to take corresponding elements of J_k, E_{kl} .

3.2 Design of Trajectory-Tracking Control Law

Trajectory-tracking control loop is designed on the basis of FL method (4), (5) for tracking error dynamic system (3), which can be written in a state-dependent coefficient form:

$$\begin{aligned}\dot{e} &= A(e)e + Bu, \\ e &= (e_1 \ e_2 \ e_3)^T, u_K = (u_1 \ u_2)^T, \\ A(e) &= \begin{pmatrix} 0 & \omega_{ref} & 0 \\ -\omega_{ref} & 0 & a_{23} \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \\ a_{23} &= V_{ref} e_3^{-1} \sin e_3.\end{aligned}\quad (6)$$

The similarity transformation for system (6) under condition of controllability of pair of matrices $(A(e), B)$ (holds for all V_{ref} , ω_{ref} , except $V_{ref} = 0, \omega_{ref} = 0$) is

$$\tilde{e} = T(e)e, \quad T(e) = \begin{pmatrix} 0 & -\omega_{ref}^{-1} & 0 \\ 1 & \dot{\omega}_{ref} \omega_{ref}^{-2} & a_{23} \omega_{ref}^{-1} \\ 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

Under transformation (7), the system (6) takes the form

$$\begin{aligned}\dot{\tilde{e}} &= \tilde{A}(e)\tilde{e} + \tilde{B}\tilde{u}, \quad \tilde{u} = C^{-1}u, \\ \tilde{A}(e) &= \begin{pmatrix} 0 & 1 & 0 \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ 0 & 0 & 0 \end{pmatrix}, \tilde{B} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & \tilde{b}_{22} \\ 0 & 1 \end{pmatrix}, \\ \tilde{a}_{21} &= -\frac{\omega_{ref} \dot{\omega}_{ref} - \dot{\omega}_{ref}^2 + \omega_{ref}^4}{\omega_{ref}^2}, \\ \tilde{a}_{22} &= -\frac{\dot{\omega}_{ref}}{\omega_{ref}}, \tilde{b}_{22} = \frac{V_{ref} \sin e_3}{\omega_{ref} e_3}, \\ \tilde{a}_{23} &= \frac{\sin(e_3)(\dot{\omega}_{ref} V_{ref} - \dot{V}_{ref} \omega_{ref})}{e_3 \omega_{ref}^2}.\end{aligned}$$

The resulting control law is

$$\begin{aligned}u &= C^{-1}(K_{FL} + G_{ST})T(e)e, \\ K_{FL} &= -\begin{pmatrix} \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ 0 & 0 & 0 \end{pmatrix}, \\ G_{ST} &= -\begin{pmatrix} g_{11} & g_{12} & 0 \\ 0 & 0 & g_{23} \end{pmatrix}, g_{11}, g_{12}, g_{23} > 0.\end{aligned}\quad (8)$$

Numerical values of g_{11}, g_{12}, g_{23} may be calculated by a pole placement method. When $V_{ref} = const$, $\omega_{ref} = const \neq 0$ we have a special case:

$$\begin{aligned}T(e) &= \begin{pmatrix} 0 & -\omega_{ref}^{-1} & 0 \\ 1 & 0 & a_{23} \omega_{ref}^{-1} \\ 0 & 0 & 1 \end{pmatrix}, \\ u_2 &= -g_{23}e_3, \quad u_1 = \omega_{ref}e_2 - g_{12}e_1 + \\ &+ \frac{g_{11}e_2 + g_{12}V_{ref} \sin(e_3) - g_{23}V_{ref} \sin(e_3)}{\omega_{ref}}.\end{aligned}\quad (9)$$

The control law (9) is not applicable when $\omega_{ref} = 0$. So on the basis of this law it is impossible to realize a

tracking along a straightforward trajectory. To overcome this disadvantage, it is possible to change the similarity transformation:

$$\tilde{e} = T(e)e, \quad T(e) = \begin{pmatrix} 0 & -1 & 0 \\ \omega_{ref} & 0 & a_{23} \\ 0 & 0 & 1 \end{pmatrix},$$

which leads to transformed system

$$\begin{aligned}\dot{\tilde{e}} &= \tilde{A}(e)\tilde{e} + \tilde{B}C^{-1}\tilde{u}, \\ \tilde{a}_{21} &= -\omega_{ref}^2, \quad \tilde{b}_{22} = -a_{23}, \\ \tilde{A}(e) &= \begin{pmatrix} 0 & 1 & 0 \\ \tilde{a}_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tilde{B} = \begin{pmatrix} 0 & 0 \\ \omega_{ref} & \tilde{b}_{22} \\ 0 & 1 \end{pmatrix}, \\ C^{-1} &= \begin{pmatrix} \omega_{ref} & -\tilde{b}_{22} \\ 0 & 1 \end{pmatrix}.\end{aligned}$$

In this case, the resulting control law has the form (8) where $\tilde{a}_{22} = \tilde{a}_{23} = 0$. It can be written in a scalar form

$$\begin{aligned}u_1 &= \omega_{ref}(e_2 + g_{11}e_2 + g_{12}V_{ref} \sin(e_3)) - \\ &- g_{12}e_1 \omega_{ref}^2 - g_{23}V_{ref} \sin(e_3), \quad u_2 = -g_{23}e_3.\end{aligned}\quad (10)$$

Finally, for the required values of MR linear and angular velocities, we have:

$$V_{req} = V_{ref} \cos(e_3) - u_1, \quad \omega_{req} = \omega_{ref} - u_2.$$

4 The experimental results

Object of the study is a MR Rover5 by DAGU [8]. Power section of robot includes two electric drives based on the DC Motors. As a control unit Arduino UNO controller is used. To control the motors Rover5 Explorer PCB board is used. It is equipped with two FET H-bridges rated at 4A. Wireless communication interface is implemented through the usb-programmable wireless module Wixel.

Some parameters of the Rover5: track length $l = 0,245$, m; driving wheel's radius $r_w = 0,03$, m; robot's mass $m = 0,83$, kg; distance between tracks $d = 0,19$, m. For measuring the motor's speed MR Rover5 is equipped with two quadrature encoders. Encoder's resolution is 1000 pulses for 3 turns of the driving wheel.

An eight-shaped reference trajectory was taken as a test trajectory. At the initial time the robot is at the point with coordinates (0,25; -0,25), the trajectory starts to move from the point (0,0). The frequency of harmonic signals $X_{ref}(t)$ and $Y_{ref}(t)$ is set so that the trajectory $p_{ref}(t)$ forms an "eight" for 40 seconds.

The experimental results on a real Rover5 robot are shown in Figure 2.

Fig. 2. The experimental results: reference and real trajectory of MR Rover5.

Reference and real Rover5's trajectories are shown on the fig.2. The required V_{req} and real V linear velocities, required ω_{req} and real ω robot's angular velocities are shown on the fig.3.

The results show us that the system has an acceptable quality. Robot enters the reference trajectory for 5 seconds, then deviations are $|e_x| \leq 0.015$, m, $|e_y| \leq 0.01$, m, $|e_\phi| \leq 0.05$, rad, the relative tracking error doesn't exceed 2% after the robot reached reference trajectory.

Fig. 3. The experimental results: reference and real trajectory of MR Rover5.

5 Conclusions

The problem of MR trajectory-tracking control was considered. The design of trajectory control loop is based on FL problem for tracking error dynamic. In general case, the control law is a tracking error feedback with time-varying coefficients. Non-stationary nature of the problem is determined by the time-varying values of reference linear and angular velocities of the robot. In such a case when these velocities are constant, the problem becomes stationary and the feedback's coefficients also become time-invariant.

Experimental verification of developed trajectory-tracking control system was performed on mobile robot Rover5. The results of experiment confirm the acceptable quality of designed control system.

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References

1. B. Siciliano, O. Khatib, *Springer handbook of robotics* (Springer, Berlin Heidelberg, 2016)
2. B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo, *Robotics: modelling, planning and control*. (Springer, London, 2009)
3. K. Kozłowski, *Robot motion and control: recent developments* (Springer, London, 2006)
4. G. Cook, *Mobile robots: navigation, control and remote sensing* (Wiley-IEEE Press, 2011)
5. A.P. Aguiar, J.P. Hespanha, IEEE Trans. on AC, **52**(8) (2007)
6. G. Klančar, D. Matko, S. Blazič, Proc. of the 13th Mediterranean conf. on control and automation, Limassol, Cyprus, June 27-29, (2005)
7. C.D. Charalambous, A. Lambis, X. Li, Proc. of 16th Mediterranean conf. on control and automation. Ajaccio, France, 946-951 (2008)
8. A.A. Kabanov, WSEAS Trans. on Sys. Cont., **9** (2014)
9. B. D'Andréa-Novell, G. Campion, G. Bastin, Int. J. of Robotics Research. **14**(6), 543-559 (1995)
10. W.E. Dixon, D.M. Dawson, E. Zergeroglu, *Nonlinear control of wheeled mobile robots*. Springer, London (2001)
11. G. Oriolo, A.D. Luca, M. Vendittelli, IEEE Trans. on Cont. Sys. Tech., **10**(6) (2002)
12. I.E. Zuber, J. App. Math. and Stoch. Anal. **11**(4) (1998)
13. A.A. Kabanov, Int. Siberian Conf. on Cont. and Com. (SIBCON). Proc. National Research University Higher School of Economics. Russia, Moscow, May 12-14 (2016)