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The evaluation of the correlation between entropy and negentropy in the structure of a technical system

Aleksander Dulesov[1], *Denis* Karandeev[1,*], and *Tatyana* Krasnova[1]

[1]Katanov Khakass State University, 655017 Lenina ave. 92, Abakan, Russia

Abstract. The role of information uncertainty in the structure of a technical system is defined. The application possibility of the R. Hartley's and C.E. Shannon's approaches in determination of entropy and negentropy of structural content of the system are selected and justified. The connection between entropy and negentropy in the case of system states changes is justified. The determination example of the quantitative values of information that characterizes connection association between entropy and negentropy is done.

1 Introduction

According to cybernetics, a technical system must not only maintain its properties (for example, to ensure a high reliability level [1]), but also to evolve. Therefore, information also develops after the system. The change of structural connection of the system in time is reflected through information. And the pre-processing of this information allows to monitor the system condition ("health"). The processed information allows a control subject to develop acceptable solutions that are aimed to maintain the state according the required level. Thus, timely and operational information is required to stabilize the system through reconstruction of broken structural connections between elements. The development and stability of the control system depends on information awareness of the system, connections with the environment, modern instruments of control and experience of system interaction.

In the operation of mechanisms and machines, information value consists in the measure determination of its uncertainty or information entropy in the system. The determination of its value allows to combine information uncertainty when it is important to consider the influence of random factors of different nature. At the same time, information contents can be represented through entropy of both quantitative and qualitative forms [2]. Entropy participates in the whole cycle of the system operation: collection of statistical information about system states → information processing and its analysis → obtaining entropy → searching of removal uncertainty → determining control resources → control and results estimation of removal uncertainty.

It is obvious, optimally using the resources in the operation process, we both increase quality and reliability of system operation [3] and temporarily reduce the uncertainty by introducing it to the known information.

2 The R. Hartley's and C.E. Shannon's connections of measures with the entropy of a technical system

Let's consider the possibilities application of information measure. We have a system structure (for example, as a graph), where the connections (edges) – elements of the system and the nodes (peaks) – concatenations of elements. Parameters are only for graph edges: the states of elements, the probability of events occurrence changing the element state. It is necessary to determine information value in order to estimate the system state and to pass on variations of the uncertainty removal in the system.

R. Hartley offered a combinatorial determination of an information amount using the logarithmic function following the principle of additivity and having connected information concept to implementation of the choice from a set of opportunities [4]. His approach to information retrieval can be regarded as the choice of one system state from a finite preassigned set of N equally probable states of its elements:

$$I = \log_2 N, \qquad (1)$$

where I – the information amount in bits, $N=2^I$ – the number of equally probable states or events. If $N = 2$ (a choice from two opportunities), then $I = 1$ bit.

In spite of the fact that the logarithm base in expression (1) can be various, we have assumed the base is 2. It is explained by the following reason: in this example, only two states (for example, operable and non-operable state, enabled and disabled, "yes" and "no") are considered. Further, these states can be designated as "1" and "0".

The Hartley's binary logarithm (1) allows to determine entropy of the set of equally probable states. It

* Corresponding author: den_dr_house_1991@mail.ru

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is can be removed through the information amount, compensating for any states (for example, unwanted).

As the combinatorial approach to the determination of the information amount allows the various combinations of the system state or its element, then the entropy H can characterize the hypothesis about a possible arrangement of the states. In this case, it can pass on to the states probabilities. Hartley's formula is written so:

$$I = \log_2 N = \log_2 (1/p) = - \log_2 p, \tag{2}$$

and all states of N have the equally probable result: $p = 1/N$, $N = 1/p$.

R. Hartley's approach is based on the combinatorics theory and simple clear intuitive assumptions. When we look at the formula (2), we can tell: all the states have equal probability and we can find information in bits, thereby determining the number of the elements that generating this information. For example, when $N = 8$, then $I = 3$. It means that there are 3 elements, each of which generates 2 equally probable opposite states and in total 8 states is generated.

According to the additivity properties: all the independent elements (sources), which realize couples of the states simultaneously, generate the same information as the system itself. Thus, the uncertainty of the system is equal to the uncertainties amount of its elements. However, if we don't have details about the system state as source of a message, then there is an uncertainty about which of the element states or set of the states reflected on the state of the entire system. If we have information about the states that are generated by the selected element among the possible states, we can say that the existence of this information will reduce the information uncertainty about the system state.

The quantitative information measure meets the condition of the monotone increasing and increase of the choice possibility, i.e. the system states amount. In addition, as there is the additivity property then states total amount: $N = N_1 + N_0$, and the required function with equal probabilities elements states must meet the condition: $f(N_1 \cdot N_0)=f(N_1)+f(N_0)$. Here N_1 and N_0 – the number of the opposite states. These quantitative values indicate that we have divided the states on the qualitative character (for example, the number of the operable N_1 and non-operable N_0 element states).

If we consider the system as analysis then according to (2) we can write the equation by $N1 = N0$:

$$\log_2 N_1 = \log_2 N_0, \tag{3}$$

where private information about the occurrence of the operable states in the left formula part, and non-operable in the right formula part.

The equality (3) testifies about equilibrium of opposite states, whereas the formula (2) testifies about the maximum of the information I. For example, Hartley's model gives an indication of the equilibrium gas in a closed system when for all the molecules the equality of the states probabilities is caused by thermodynamic equilibrium. The equilibrium according to (3) is unsuitable for the majority of technical systems, because there equilibrium "is displaced" in the area that is ensuring the robustness of system structure functioning. For such systems the ratio of the states should not be in favor of the entropy (information uncertainty), because $N_1 >> N_0$. In this case, Hartley's approach is of little use due to the inequality of the probabilities opposite states.

Further, we are going to pass on to C.E. Shannon's approach in the determination of information amount. Shannon has applied the probabilistic approach to the information measurement (the information as the removal uncertainty, which reduces entropy) [5, 6]. In a technical system, the elements states follow each other occurring as events and have their own duration in time. In the operation process, registration of the states allows to determine the probability of the occurrence of events based on accumulated statistical data. The connection between the states probability and information amount, which obtained in case of an event approach, expressed by the Shannon's formula:

$$I = -\sum_{i=1}^{N} p_i \log_2 p_i, \text{ by } \sum_{i=1}^{N} p_i = 1, \tag{4}$$

where I – the information amount; N – the number of possible events; p_i – the probability of i-th event.

In Shannon's formula (4), we think, he reflected the possibility of the information differentiation on a qualitative character [2]:

$$I = -(\sum_{i=1}^{N_1} p_i \log_2 p_i + \sum_{j=1}^{N_0} q_j \log_2 q_j), \tag{5}$$

by $\sum_{i=1}^{N_1} p_i + \sum_{j=1}^{N_0} q_j = 1$,

where p_i and q_j – the probability of the opposite states (for example, p_i – the probability of an operable element state, q_j – the probability of a non-operable element state).

When we consider the states of a technical system based on the results of the processing statistical data, it becomes obvious that $p_i >> q_j$. Therefore, the value of the left term in the formula (5) exceeds the value of the right term. Note that: equality of these terms will be reached when $p_1 = q_1 = \ldots = p_i = q_j = \ldots$, that coincides with the relation (3) and indicates the equilibrium of the opposite states or the presence of chaos (disorder) in the system. However, as noted earlier, this equilibrium (lack of the order) is unacceptable in case of the operation of technical systems. In order to preserve and maintain a high scientific and technical level, it is necessary to involve additional resources in the system, for example, timely maintenance of equipment, scheduled preventive maintenance, etc. Then the number of the type "1" states (that measured by the number of negentropy) will greatly exceed the number of the type "0" states (that measured by the number of entropy). In case of information measurement on a state system, when it is necessary to maintain the order, we must neutralize a part of an entropy (to remove the information uncertainty) through the introduction into it of the negentropy [7]. In our opinion, the connection of these information components confirms the principle of the information mutual exclusion.

3 Detailing of connection the "entropy-negentropy"

Considering the system structure and its states, however, the question of the connection between entropy and negentropy requires a detailed study. In information theory [8], both information components are determined through a "probability of the state ", because the Nature laws and human behavior are subordinated to the probabilistic dominance. The growth of the "0" type states in relation to "1" type and, as a result, to entropy is tells us about the aging process of the object (destruction). In turn, the negentropy as order measure is aimed at removal uncertainty and increasing of the "1" type state. The exchange between information components provides the principle of the mutual exclusion. At the same time, it should be appreciated that exchange between information components is carried out within the full information about the closed system structure. Therefore, the increasing of one of components reduces the opposite component with the same value. These information components are changed to self-law and their absolute values are practically independent of each other.

Further, according to our example of structure states, we are going to consider the correlation between entropy and negentropy. For simplicity, from the standpoint of reliability [9], we are going to consider only the presence of the form "1" - operable states and "0" - non-operable states [10-13].

The regulation sets the rules on timely preventive maintenance of equipment (elements) to maintain a high reliability level in the system operation [14]. All repair and interrepair periods are specified in time based on the operating experience of similar facilities, or empirical way for newly constructed facilities. The process of systematic purposeful actions on the part of the control subject are connected with the determination of negentropy amount, which can be determined by the Shannon's formula:

$$I = -\sum_{i=1}^{N_1} p(A_i^{(j)}) \log p(A_i^{(j)}), \qquad (6)$$

by $\sum_{i=1}^{N_1} p(A_i^{(j)}) = 1,$

where $A_i^{(j)}$ – an event $i = \overline{1, N_1}$ that has probability p_i or the proportion of the time t_i / T, $T = \sum t_i$ in the interval *T,* the reflecting signs (planned repair; interrepair period) of the element *j*. Additionally, we note the following: formula (6) is applicable to a structure element, because it's a planned outage should not significantly affect the operability of the entire system.

The nature impact on the system though isn't considered aggressive, however, introduces some chaos, which indicates the occurrence of the states "0" and the presence of the entropy. The entropy "will replace" negentropy, because the information about the element won't be changed.

Information about the state of the entire system can be determined by the expression:

$$I_\Sigma = I + H, \qquad (7)$$

where *I* and *H* – respectively, negentropy and entropy.

The expression (7) reflects information about the states, which connected with the implementation of specified functions and opposite states that violate these functions. When $I = H$ – utter chaos or a balance between the opposite states. The task of a control subject: to bring the system through an orderly influence in the area where the condition $I >> H$ is satisfied, when the order is considerably exceeds chaos. Moving from chaos to order the negentropy increases, i.e. the anti-entropic process is carried out. L. Brillouin has shown that the quantity of the negentropy is exactly equal to the decreasing in entropy [15]. Thereby, the control subject tries to maintain equilibrium at the required level.

Look at the example. Let the system consists of n=3 of elements, where each can be fitted in one of two opposite states with equal probabilities. All elements are independent. It is necessary to determine the information components about system state.

Full or total information amount about the possible system states can be determined by the expression (4). Since the statement of the problem is associated to existence of the equally probable events, then quantity of the all equally possible states of the system - $N=2^n=2^3=8$ and states variety of forms "1" and "0" for each of 3 elements: 1,1,1; 1,1,0; 1,0,1; 1,0,0; 0,1,1; 0,1,0; 0,0,1; 0,0,0. The occurrence probability each of 8 states is equal to 1/8. Then we will receive required information value according to the expression:

$$I_\Sigma = \sum_{i=1}^{N} I_i = \sum_{i=1}^{N} \frac{1}{N} \log_2 \frac{1}{N} = \sum_{i=1}^{8} \frac{1}{8} \log_2 \frac{1}{8} = 3, \qquad (8)$$

where I_i =3/8 – the number of the private information about the state of the *i* system.

The following follows from a statement of the problem (8): the maximum of information amount (in bits) is determined and we have $I = H = 1.5$ (or utter chaos) for the variety of 8 system states. This ratio doesn't suit us. For example, we are able to provide a high order in the system due to the control actions. Then the elements states will be mostly of only one type, for example: 1,1,1; 1,1,0; 1,0,1; 1,1,0; 0,1,1; 0,1,1; 0,0,1; 0,0,0. These 8 system states are placed in random order and have the condition: one state of type "0", which inherent in the element, doesn't result in state "0" of the entire system. There are two qualitatively different types in this combination of the system states: 6 states correspond to the qualitative type "1" and 2 states correspond to the qualitative type "0". Let's determine the information components about system states:

$$I = \frac{N_1}{N}\log_2\frac{1}{N} = \frac{6}{8}\log_2\frac{1}{8} = 2.25; \quad (9)$$

$$H = \frac{N_0}{N}\log_2\frac{1}{N} = \frac{2}{8}\log_2\frac{1}{8} = 0.75. \quad (10)$$

In the expressions (9) and (10), the probability of generating of the private information amount about the system state of the type "1" is equal 6/8 and type "0" is equal to 2/8. The property (7) is confirmed according to this example.

4 Conclusion

The negentropy characterizing with systematic actions of the control subject removes uncertainty (entropy) and it is the quantitative information measure. The removing of the uncertainty is expressed through the changes of conditions that imposed on the system and its entropy. The preserving of system structure stability is a fight against entropy, which reflects aging process in technical systems.

In technical systems, the information attachment from the control subject allows to accumulate the ability to probability differentiation of the negative reactions on the external environment influence through the equation (4). At the same time, the system structure is changed due to the using of material resources, i.e. the entire complex of signs. The necessaty to increase the negentropy – is the necessaty to compensate the entropy as much as the entropy can be decreased itself. As a result, the control subject tends to that the probability of one type "has won", for example a type "1". However, the technical system can't reach absolutely deterministic state, because the Nature looks out for the existence of the system and elements lifecycle to make them more perfect and replace.

We can determine the entropy and negentropy amounts based on C.E. Shannon's formulas for discrete systems due to the processing of statistical data about the system states and executing the differentiation of the probability about the changing control conditions. The problem solution of the removal uncertainty will allow to plan actions to reduce entropy based on the negentropy principle of L. Brillouin's information about the structural system changes.

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