Modelling of cutting block cut surface at faceted surfaces machining using planetary gear set

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Abstract. The article resolved the relevant problem to assign process parameters for polygonal turning machining ensuring the component has its pre-set characteristics. To accomplish this, the authors modelled the machining process and described the cut surface formed by the cutter block edge during machining.

Today cylindrical components with multi-faceted surfaces have found application in various areas of the economy, in particular: machine construction, instrument engineering, robotics, agriculture, mining and even the medical industry [1].

The conducted analysis of the production methods for faceted surfaces helped to choose the planetary motion method of the turning machine cutting edges. This method works as follows: the planetary gear set assigns a complex curve trajectory (trochoid) to the cutting tool point Fig. 1 [1].

To determine the planetary gear set structural parameters, the authors resolved the problem of the rectangle side approximation error calculation with the prolonged trochoid section [1]. However, to assign the machining process parameters the component has its pre-set characteristics, it is necessary to model the machining process, namely, to make a cut surface model.

Fig. 1. Examples of details with polyhedral surfaces

Fig. 2. Planetary Gear Set

Generally, the cut surface is described by the formula

\[ Q((O\phi(t), \theta(t)z(t), S) = \left\{ A(\phi(t)), A(R-r)A(-\theta(t))r(S) \right\} (1) \]

Where

\[ \phi(t) = \frac{\pi n}{30}t, \theta(t) = \frac{R}{r} \frac{\pi n}{30}t, z(t) = \frac{S_{min}}{60}, t, S_{min} \] – feed per minute, \( N \) – number of the cutting block rotations in relation to the workpiece.

Let us build a model for cutting block edges. Fig. 3 presents a square indexable insert.

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Let us determine the generating points coordinates

Point 1 coordinates:

\[ x_1 = 0.5d \cdot \sin \eta_1; \quad y_1 = 0.5d \cdot \sin \eta_1, \]

Point 2 coordinates:

\[ x_2 = \frac{\sqrt{2}}{2}d \cdot \cos \eta_1; \quad y_2 = -r \cdot \sin \eta_1, \]

Point 3 coordinates:

\[ x_3 = x_2; \quad y_3 = -y_2, \]

Point 4 coordinates:

\[ x_4 = x_3; \quad y_4 = -y_3, \]

Point 5 coordinates:

\[ x_5 = \frac{\sqrt{2}}{2}d - r \cdot \sqrt{2}; \quad y_5 = 0, \]

\[ r_{II} = [x(S), y(S), z(S), 1]^T \]

where

\[ x(S) = \begin{cases} x_1; S \leq 0 \\ (x_2 - x_1) \cdot (x(S) - y_1)/(y_2 - y_1) + x_1; S \leq y_1 - y_1 \\ x_1 + \sqrt{2}/2 \cdot (y_2 - y_1); y_2 - y_1 < S \leq y_1 - y_1 \\ (x_2 - x_1) \cdot (x(S) - y_1)/(y_2 - y_1) + x_1; y_1 - y_1 < S \leq y_1 - y_1 \\ x_1; S < y_1, \end{cases} \]

\[ y(S) = \begin{cases} y_1; S \leq 0 \\ y_1 + S; 0 < S \leq y_1 - y_1 \\ y_1 + S > y_1 - y_1, \end{cases} \]

\[ N_x(S) = \begin{bmatrix} f_1 \cdot \gamma_1 \cdot \sin \gamma_2 \cdot \sin \gamma_3 & f_1 \cdot \gamma_1 \cdot \sin \gamma_2 \cdot \cos \gamma_3 & f_1 \cdot \gamma_1 \cdot \cos \gamma_3 \end{bmatrix} \]

\[ N_y(S) = \begin{bmatrix} f_2 \cdot \gamma_1 \cdot \cos \gamma_2 \cdot \sin \gamma_3 & f_2 \cdot \gamma_1 \cdot \cos \gamma_2 \cdot \cos \gamma_3 & f_2 \cdot \gamma_1 \cdot \sin \gamma_3 \end{bmatrix} \]

Further, let us set the indexable insert parameters; the transition to the coordinates system 1 is the first action

\[ A_1^{-1} = A(-d)A\left(\frac{\pi}{2}\right)^2A\left(-\frac{\pi}{2}\right) \]

Rotation for the angle \( \lambda \) is the second action

\[ A_2^{-1} = A(-\lambda) \]

After that, let us set the insert for the angle

\[ \gamma_1 = \gamma_{\text{in}} \]

Move to the rotation center

\[ A_5^{-1} = A(-m)A(-r) \]

If \( \phi \neq \phi_2 \), set the angle \( \phi \)

\[ A_4^{-1} = A(\phi) \]

\[ A_6^{-1} = A(-0.5d + m - r) \]

\[ A_7^{-1} = A\left(\frac{\pi}{2}\right)^2A\left(-\frac{\pi}{2}\right) \]

The setting matrix calculation in the coordinates system

Reference data:

\[ \mathbf{j}_{21} \cdot \text{vector} \mathbf{j}_2 = [0,1,0,0]^T \text{ presented in the coordinates system } x_1, y_1, z_1 \]

\[ \mathbf{k}_{21} \cdot \text{vector} \mathbf{k}_2 = [0,0,1,0]^T \text{ presented in the coordinates system } x_1, y_1, z_1 \]

\[ \mathbf{r}_{21} \cdot \text{vector defining the position of the coordinates } x_2, y_2, z_2 \text{ origin within the system } x_1, y_1, z_1 \]

The \( A_{21} \) matrix elements are determined by the system

\[ \begin{align*}
\begin{bmatrix}
\mathbf{j}_{21} \\
\mathbf{k}_{21} \\
\mathbf{r}_{21}
\end{bmatrix} &= A_{21} \cdot \begin{bmatrix}
\mathbf{i}_2 \\
\mathbf{i}_2 \\
\mathbf{i}_2
\end{bmatrix} \\
\mathbf{r}_{21} &= A_{21} \cdot \mathbf{e}^i
\end{align*} \]

where \( i_2 = (1,0,0,0)^T \)

\[ A_{21} = \begin{bmatrix}
\mathbf{r}_{21} \cdot \mathbf{k}_{21} & \mathbf{j}_{21} & \mathbf{i}_2 & \mathbf{r}_{21} \\
\mathbf{r}_{21} \cdot \mathbf{j}_{21} & \mathbf{i}_2 & \mathbf{i}_2 & \mathbf{r}_{21} \\
\mathbf{r}_{21} & \mathbf{k}_{21} & \mathbf{i}_2 & \mathbf{r}_{21} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1}
\end{bmatrix} \]

The method for calculation of the residual ridges and roughness parameters

1. Define the point on the cutting edge which contacts with the component surface (shape-generating point) by solving the following equation

\[ \mathbf{J} - \mathbf{Q}(S,t) = 0 \]

In relation to the S edge parameter at \( t=0 \)
2. Calculate the $A_w$ matrix elements for setting the sectional plane coordinates system under the set parameters

$$[0,0,-1,0]^T, [-1,0,0,0]^T, Q(S,t)$$

(13)

Determine the coordinates for the A point

$$r_a = A_w \cdot Q(S,t)$$

(14)

Determine the coordinates for the B point

$$r_a = A_w \cdot Q(S,t + T_{oa})$$

(15)

Determine the coordinates for the C point by solving the equation system

\[
\begin{align*}
\bar{i}_z \cdot A_v \cdot Q(S,t_1) &= \bar{i}_z \cdot A_v \cdot Q(S_2,t_2) \\
\bar{j}_z \cdot A_v \cdot Q(S,t_1) &= \bar{j}_z \cdot A_v \cdot Q(S_2,t_2) \\
\bar{k}_z \cdot A_v \cdot Q(S,t_1) &= 0 \\
\bar{k}_z \cdot A_v \cdot Q(S_2,t_2) &= 0 \\
0 \leq S_1 \leq S_{a4} \\
0 \leq S_2 \leq S_{w4} \\
t - \frac{T_{oa}}{4} \leq t_1 \leq t + \frac{T_{oa}}{4} \\
t + T_{oa} - \frac{T_{oa}}{4} \leq t_1 \leq t + T_{oa} + \frac{T_{oa}}{4}
\end{align*}
\]

(16)

In relation to the $S_1, S_2, t_1, t_2$, parameter

Hence $r_a = A_w \cdot Q(S,t)$

Assessment for the layers being cut

1) Define the normal vector towards the cut surface at the cut-in moment $t$

$$\vec{N}_{\varphi} (S,t) = \left[ \bar{Q}_w \cdot (S,t) \times \bar{Q}_w \cdot (S,t) \right]_{\text{sign}} \left\{ \bar{Q}_w \cdot (S,t) \times \bar{Q}_w \cdot (S,t) \right\}^4 A(\varphi(t))^z \frac{A(R - r)^z A(Q(t)) - N_z(S)}{N_z(S)}$$

Let the $A_j$ transition matrix be defined in the system of $x_j, y_j, z_j$ coordinates with the origin in the $\bar{Q}_j(S,t)$ point and the $y_j$ axis along the $\bar{Q}_j(S,t)$ vector and the $z_j$ axis along the $\vec{N}_{\varphi} (S,t)$ vector

Define the cross point of the $z_j$ axis with the cut surface of the $i$ cutter shaped during the previous cutting block rotation

\[
\begin{align*}
\bar{i}_z \cdot A_j \cdot Q(S', t + \Delta t) &= 0 \\
\bar{j}_z \cdot A_j \cdot Q(S', t + \Delta t) &= 0 \\
0 \leq S \leq S_j \\
t - \frac{T_{oa}}{N_z} < \Delta t < t - \frac{T_{oa}}{N_z}
\end{align*}
\]

(17)

The system is solved in relation to the $S'$ and $\Delta t$ parameters

Define the thickness $Q_i(S,t)$

$$Q_i(S,t) = F_i(S,t) \bar{k}_z \cdot A_j \cdot Q(S', t + \Delta t)$$

(18)

Let the thickness value of the layer being cut limited by the workpiece material be defined as follows:

Solve the equation system in relation to the $U$ and $V$ parameters

\[
\begin{align*}
\bar{i}_i \cdot A_j \cdot PR(U,V) &= 0 \\
\bar{j}_i \cdot A_j \cdot PR(U,V) &= 0 \\
0 \leq U &\leq 2\pi \\
0 \leq V &\leq H_w
\end{align*}
\]

where $H_w$ - workpiece thickness.

2) Calculate the thickness $a_i(S,t)$

$$a_i(S,t) = F_i(S,t) \bar{k}_z \cdot A_j \cdot QPR(U,V)$$

(19)

The thickness for the layer being cut is calculated as

$$a(S,t) = \min(a_i(S,t), a_j(S,t))$$

(20)

Basing upon the model built for the cut surface, it is possible to calculate assessment parameters for the multi-faceted surfaces machining with further determining of the rational values for the polygonal turning parameters.

References


6. M. Razumov, *Form error calculation during polygonal sharpening of polyhedrons with even number of sides*, Metallurgical and Mining Industry 7 (1), pp. 66-69


14. V.V. Kuts, *The profile distortion value calculation for shaped to be machined surface at the CAD/CAM - development for assembled shaped mills*, Avtomatizatsiya i Sovremennye Tekhnologii, Issue 11, Pages 5-7 (2004)


16. Y.A. Maksimenko, V.V. Kuts, *Analysis of changes in error processing r8-profile shaft with teeth resharpening cutters with radial constructive feed*, Spravochnik. Inzhenernyi zhurnal, pp.008-012 (2014)


21. A.I. Timchenko, *Tekhnologiya izgotovleniya detaley profil’nykh bezshponochnykh soedineniy [Manufacturing technology of keyless connection of profile pieces]* (Moscow, VNITTEMP, 1988)


