Direction-of-arrival and Range Estimation of Wideband Sources in the Near Field

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Abstract. In this letter, we address the problem of wideband sources localization in the near field of the sensor array. It is shown that the spatial sampling rate can be considered as a function of temporal frequency so that wideband sources can be represented in the spatial frequency domain. A computationally efficient localization method is then developed based on the Toeplitz matrices constructed from the frequency-dependent cross-correlation between sensors. This allows us to resolve near field sources with quite a few numbers of sensors. Indeed, the sources can be located with fewer sensors. Finally, the simulation results are presented to validate the performance of the proposed method.

1 Introduction

The localization of radiating sources using sensors arrays is a problem of considerable importance, occurring in a variety of fields ranging from radar, sonar, wireless communication, and radio astronomy to seismology [1-4]. Therefore, this problem has received considerable attention in the literature, resulting in a plethora of estimation algorithms [5-7]. However, the majority of such algorithms have been restricted to DOA estimate of narrowband sources in the far-field. Recently, there has been growing interest in developing sources location estimators in the near-field, in which case, both the angle and the range are required to be estimated simultaneously. Huang and Barkat [8] presented two techniques, i.e., two-dimension (2-D) MUSIC method and the maximum likelihood estimator (MLE), to determine the range and the angle of sources, which can be considered to be the modified versions of far-field cases. In [9-12], a series of ESPRIT-like methods are proposed, where higher order statistics is involved. Lately, some computationally simple second-order statistics-based methods have been proposed in the literature, such as the cyclic-based method [13], the weighted linear prediction method [14], the generalized ESPRIT method [15-16], the CRB-Based method [17-18], the NF-DSPE and the JADSSE method [19] etc. Recently, mixed near-field and far-field sources localization are proposed [20-21]. Significantly, most of the aforementioned methods rely on the narrowband assumption, therefore, to deal with wideband signals, e.g., acoustic or seismic, in [22-23] the authors provided some schemes to solve the problem of joint range and DOA estimation for multiple wideband near field sources. However, these methods have been designed under the

assumptions that the number of the sources can be estimated is less than or equal to the number of antenna elements. This is a central problem in some environments and situations in which the number of sensors is limited by physical or economical considerations.

Therefore, the objective of our letter is to propose a novel scheme to simultaneously estimating the DOA and sensor location using fewer sensors, possibly less than the number of wideband sources in the near-field. Like the conventional methods for wideband sources [24-26], the received data is transformed to the frequency domain firstly, and each frequency bin can be represented by the narrowband model. Secondly, we make use of the fact that the relative distance between different elements (i.e. the physical size measured in wavelengths) is a function of the temporal frequency. Therefore, the Toeplitz matrices including the angle and the range information can be obtained by the frequency-dependent cross-correlation between sensors [27]. Then, we can employ the traditional subspace-based methods to estimate the angle and the range of wideband near field sources. Finally, the simulation results show that our proposed technique can improve greatly the detection and estimation capability for wideband near field sources.

2 Problem Formulation and Proposed Method

Consider K near field, wideband, and uncorrelated radiating sources observed by a passive nonuniform antenna array of five sensors indicated in Figure 1,

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Figure 1. Near-field, wideband sources observed by a five nonuniform linear array which is symmetric about the original element.

where the array center is the origin and the phase reference point. Since we consider wideband sources in the near field, the array signal model in the frequency domain can be given by

$$x_p(f) = \sum_{k=1}^K s_k(f) e^{j\tau_{pk}(f)} + N_p(f) \quad p = 0, \pm 1, \pm 2 \quad (1)$$

where $s_k(f)$, $N_p(f)$, and $\tau_{pk}(f)$ denote the k th source signal spectrum, the additive noise, and the delay associated with the k th source signal propagation time between the original sensor at the p th sensor in the f frequency band, respectively. The noise $N_p(f)$ is a zero-mean spatially white process uncorrelated with the signal sources, and $\tau_{pk}(f) \approx \omega_{pk}(f) + \phi_{pk}(f)$ [14] and ω_{pk} and ϕ_{pk} are given by

$$\omega_{pk}(f) = \begin{cases} -p2\pi f d_1 \sin(\theta_k)/c & p = \pm 1 \\ -p\pi f d_2 \sin(\theta_k)/c & p = \pm 2 \\ 0 & p = 0 \end{cases} \quad (2)$$

$$\phi_{pk}(f) = \begin{cases} \pi f d_1^2 \cos^2(\theta_k)/c r_k(f) & p = \pm 1 \\ \pi f d_2^2 \cos^2(\theta_k)/c r_k(f) & p = \pm 2 \\ 0 & p = 0 \end{cases} \quad (3)$$

where c is the speed of the signal propagation in the considered medium, and $r_k(f) = r_k c / f$ is a frequency dependent value.

In addition, the wideband sources are assumed to be the same power spectrum over a given frequency band $[f_{\min}, f_{\max}]$ provided with symmetric characteristic with

respect to the central frequency $f_0 = \frac{f_{\min} + f_{\max}}{2}$, i.e.,

$s_k(f_0 + n\Delta f) = s_k(f_0 - n\Delta f)$ $k = 1, \dots, K$, $n = 1, \dots, N$, without loss of generality, where we can make the following assumptions that each wideband source can be decomposed into $2N+1$ frequency bins with inter-frequency bin difference $\Delta f = \frac{f_{\max} - f_{\min}}{2N}$.

Next, making use of the assumptions mentioned above, we can define the cross-correlation between $x_1(f_0 + n\Delta f)$ and $x_{-1}(f_0 + n\Delta f)$ or $x_2(f_0 + n\Delta f)$ and $x_{-2}(f_0 + n\Delta f)$ as

$$\begin{aligned} & r_p(f_0 + n\Delta f) \square E \{ x_p(f_0 + n\Delta f) x_{-p}^*(f_0 + n\Delta f) \} \\ &= E \left\{ \left(\sum_{k=1}^K s_k(f_0 + n\Delta f) e^{j(\omega_{pk}(f_0 + n\Delta f) + \phi_{pk}(f_0 + n\Delta f))} \right) \left(\sum_{k=1}^K s_k(f_0 + n\Delta f) e^{j(-\omega_{pk}(f_0 + n\Delta f) + \phi_{pk}(f_0 + n\Delta f))} \right)^* \right\} \\ &+ E \{ N_p(f_0 + n\Delta f) N_{-p}^*(f_0 + n\Delta f) \} \\ &= \sum_{k=1}^K P_k(f_0 + n\Delta f) e^{j2\omega_{pk}(f_0 + n\Delta f)} \quad p = 1, 2 \quad n = -N, \dots, -1, 0, 1, \dots, N \end{aligned} \quad (4)$$

where $E\{\bullet\}$ and $(\bullet)^*$ denote the statistical expectation and the complex conjugate, respectively. Additionally, $P_k(f_0 + n\Delta f) = E\{s_k(f_0 + n\Delta f)s_k^*(f_0 + n\Delta f)\}$ is the power spectral of the k th source at frequency $f_0 + n\Delta f$. To simplify the descriptions, we will consider the scenario as in [28] where the power spectral is constant in different frequency in the subsequent section.

Thus, the cross-correlation in (4) can be rewritten by

$$r_p(n) \square r_p(f_0 + n\Delta f) = \sum_{k=1}^K P_{kf_0} e^{j2\omega_{pk}(n\Delta f)} \square \sum_{k=1}^K P_{kf_0} e^{j\alpha_{pk}} \quad (5)$$

where $P_{kf_0} \square P_k e^{j2\omega_{pk}(f_0)}$, $\alpha_{pk} \square 2\omega_{pk}(n\Delta f)$, and $p = 1, 2$ $n = -N, \dots, -1, 0, 1, \dots, N$, then we can construct the following Toeplitz matrix $\mathbf{R}_{p,1}$ [27] and by the cross-correlations $\{r_p(n)\}_{n=-N}^N$,

$$\begin{aligned} \mathbf{R}_{p,1} &= \begin{bmatrix} r_p(0) & r_p(1) & \dots & r_p(N) \\ r_p(-1) & r_p(0) & \dots & r_p(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ r_p(-N) & r_p(-N+1) & \dots & r_p(0) \end{bmatrix} \\ &= \mathbf{A}_{p,1} \mathbf{D}(\mathbf{P}) \mathbf{A}_{p,1}^H \end{aligned} \quad (6)$$

where $(\bullet)^H$ denotes the complex conjugate, and $\mathbf{A}_{p,1} = [\mathbf{a}_{p,1}(\theta_1), \mathbf{a}_{p,1}(\theta_2), \dots, \mathbf{a}_{p,1}(\theta_K)]$ with

$$\mathbf{a}_{p,1}(\theta_k) = [1, \exp(-j\alpha_{pk}), \dots, \exp(-jN\alpha_{pk})]^T \quad \text{and}$$

$$\mathbf{D}(\mathbf{P}) \square \text{diag}(\mathbf{P}) = \text{diag}\{P_{1f_0}, P_{2f_0}, \dots, P_{Kf_0}\}.$$

Since $\mathbf{A}_{p,1}$ is a $(N+1) \times K$ Vandermonde matrix [29] with full column rank when $\theta_i \neq \theta_j$ ($i \neq j$), the columns are linearly uncorrelated matrix. From the expression of P_{kf_0} $k=1, 2, \dots, K$ in (5), we can see that $P_{kf_0} \neq 0$ and $\mathbf{D}(\mathbf{P})$ has a rank of K . Therefore, the rank of $\mathbf{R}_{p,1}$ equals to the number of wideband sources and the signal subspace or noise subspace can be obtained by singular value decomposition (SVD). Finally, the traditional narrowband subspace-based DOA techniques [30] are applicable to $\mathbf{R}_{p,1}$ in (6) to obtain $\{\alpha_{pk}\}_{k=1}^K$, and the DOAs are estimated by the simple operation.

Then we also define the cross-correlation between $x_1(f_0 + n\Delta f)$ and $x_{-1}(f_0 - n\Delta f)$ or $x_2(f_0 + n\Delta f)$ and $x_{-2}(f_0 - n\Delta f)$ as

$$\begin{aligned}
 & r_p(n\Delta f) \square E\{x_p(f_0 + n\Delta f)x_p^*(f_0 - n\Delta f)\} \\
 & = E\left\{\left(\sum_{k=1}^K s_k(f_0 + n\Delta f)e^{j(\omega_{\theta_k}(f_0 + n\Delta f) + \theta_{\theta_k}(f_0 + n\Delta f))}\right)\left(\sum_{k=1}^K s_k(f_0 - n\Delta f)e^{j(-\omega_{\theta_k}(f_0 - n\Delta f) + \theta_{\theta_k}(f_0 - n\Delta f))}\right)^*\right\} \\
 & \quad + E\{N_p(f_0 + n\Delta f)N_p^*(f_0 - n\Delta f)\} \\
 & = \sum_{k=1}^K P_k(n\Delta f)e^{j(\omega_{\theta_k}(f_0 + n\Delta f) + \omega_{\theta_k}(f_0 - n\Delta f))}e^{j(\theta_{\theta_k}(f_0 + n\Delta f) - \theta_{\theta_k}(f_0 - n\Delta f))} \\
 & = \sum_{k=1}^K P_k(n\Delta f)e^{j2\omega_{\theta_k}(f_0)}e^{j\pi d_p^2 \cos^2(\theta_k)4f_0 n\Delta f / c^2 r_k} \quad p=1,2 \quad n=-N, \dots, -1, 0, 1, \dots, N
 \end{aligned} \quad (7)$$

where $P_k(n\Delta f) \square E\{s_k(f_0 + n\Delta f)s_k^*(f_0 - n\Delta f)\}$
 $= P_k(f_0 + n\Delta f) = P_k(f_0 - n\Delta f) = P_k$, then Eq.(6) can be rewritten as

$$r_{p\Delta f}(n) \square r_p(n\Delta f) = \sum_{k=1}^K P_{k\Delta f} e^{j\pi d_p^2 \cos^2(\theta_k)4f_0 n\Delta f / c^2 r_k} \square \sum_{k=1}^K P_{k\Delta f} e^{jn\beta_{pk}} \quad (8)$$

where $p=1,2$, $n=-N, \dots, -1, 0, 1, \dots, N$,

$$P_{k\Delta f} \square P_k e^{j2\omega_{pk}(f_0)}, \quad \beta_{pk} \square \pi d_p^2 \cos^2(\theta_k)4f_0 \Delta f / c^2 r_k.$$

In the similar way to (6), we can construct the following Toeplitz matrix $\mathbf{R}_{p,2}$,

$$\begin{aligned}
 \mathbf{R}_{p,2} &= \begin{bmatrix} r_{p\Delta f}(0) & r_{p\Delta f}(1) & \dots & r_{p\Delta f}(N) \\ r_{p\Delta f}(-1) & r_{p\Delta f}(0) & \dots & r_{p\Delta f}(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ r_{p\Delta f}(-N) & r_{p\Delta f}(-N+1) & \dots & r_{p\Delta f}(0) \end{bmatrix} \\
 &= \mathbf{A}_{p,2} \mathbf{D}(\mathbf{P}) \mathbf{A}_{p,2}^H
 \end{aligned} \quad (9)$$

where $\mathbf{A}_{p,2} = [\mathbf{a}_{p,2}(\theta_1, r_1), \mathbf{a}_{p,2}(\theta_2, r_2), \dots, \mathbf{a}_{p,2}(\theta_K, r_K)]$ with

$$\mathbf{a}_{p,2}(\theta_k, r_k) = [1, \exp(-j\beta_{pk}), \dots, \exp(-jN\beta_{pk})]^T \text{ and }$$

$\mathbf{D}(\mathbf{P}) \square \text{diag}(\mathbf{P}) = \text{diag}\{P_{1\Delta f}, P_{2\Delta f}, \dots, P_{K\Delta f}\}$. Similarly, we can also obtain $\{\beta_{pk}\}_{k=1}^K$ and then the ranges of wideband sources can be estimated by

$$r_k = 4\pi d_p^2 \cos^2(\theta_k) f_0 \Delta f / c^2 \beta_{pk} \quad k=1, \dots, K \quad (10)$$

Remark: It is worth noting that for unique estimation of the DOA and the range, we need to make the following assumption

$$d_p \leq c/2\Delta f \quad \text{and} \quad d_p \leq c\sqrt{r_{\min}/f_0 \Delta f}/2 \quad (11)$$

where $r_{\min}$ denote the minimum value in the set $\{r_k\}_{k=1}^K$. Without loss of generality, we can employ $\mathbf{R}_{2,1}$ and $\mathbf{R}_{1,2}$ to estimate the DOA and the range respectively due to a tradeoff between the estimation performance and the phase ambiguity in the following simulations.

3 Simulations

We now evaluate the performance of the proposed method in estimating the DOAs and the range of

wideband near-field sources. The spectral range of the wideband sources is taken to be the normalized frequency interval $[0.8, 1.2]$ with respect to the central frequency f_0 , that is a bandwidth of 40% of f_0 , where we decompose the wideband sources into 21 frequency bins with $\Delta f = 0.02$. The element spacing $d_1 = c\sqrt{r_{\min}/f_0 \Delta f}/2$ and $d_2 = c/2\Delta f$ is considered.

Firstly, we examine the estimation performance of the proposed method. The impinging source signals are two wideband sources signals located at $[\theta_1, r_1] = [-10^\circ, 4]$ and $[\theta_2, r_2] = [5^\circ, 2]$, respectively. The performance is measured by the normalized root mean square error (RNMSE) for the range and the root mean square error (RMSE) for the DOA defined in [12]. The results shown below are based on 1000 independent trials. Figure 2 and Figure 3. show the RMSE and RNMSE of the estimated source angles and ranges with respect to the SNR from -3dB to 15dB. It is observed that the RMSE of DOA estimation is very small even at low SNR; for example, the RMSE is only 0.2 degree for SNR = -2 dB. In addition, we also see that the smaller the range is, the better estimation performance is.

Figure 2. RMSE of the angle estimation versus SNR for two wideband near field sources.

Figure 3. RNMSE of the range estimation versus SNR for two wideband near field sources.

Next, we examine the estimation capability for multiple sources of the proposed method shown Figure 4(a) and (b) show that the DOA of five and six near-field uncorrelated wideband sources are placed at $[-20^\circ - 10^\circ \ 0^\circ \ 10^\circ \ 20^\circ]$ and $[-30^\circ - 20^\circ - 10^\circ \ 0^\circ \ 10^\circ \ 20^\circ]$. It can be seen from Figure 4 that we can estimate much more wideband near field sources than the number of sensors while it is impossible for the conventional methods to deal with more sources than sensors.

Figure 4. DOA estimation of uncorrelated wideband sources by the proposed algorithm, (a) five sources $[-20^\circ -10^\circ 0^\circ 10^\circ 20^\circ]$ at SNR=5dB, (b) six sources $[-30^\circ -20^\circ -10^\circ 0^\circ 10^\circ 20^\circ]$ at SNR=10dB.

4 Conclusions

We have presented a novel joint DOA and range estimation algorithm for wideband near field sources to deal with the scenarios in which there are more sources than sensors. This method takes advantage of the characteristics of wideband sources and obtains cross-correlation between sensors at each frequency bin to form Toeplitz matrixes that contains all DOA and range information. As a result, DOA and range estimation can be achieved using the conventional narrowband signal subspace techniques. Simulation results have demonstrated the effectiveness of the proposed method.

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