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Multiobjective optimization of planar truss girder

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Abstract The continuous increasing trends in the case of the raw material-, energy-, production-, and operation costs have been made necessary, the development of the numerical methods made it possible to spread widely the optimal design methods in the technical practice.The method of optimal design with multiobjective functions is useful when the designer has to rank the different construction variations. We apply this method to optimal design of truss girder when the aim functions are the mass of girder and the deflection of the structure. We change the cross-section of beam of girder as unknown parameter. The restrictions are concerning the design resistance of tensile bars and the stability of compressed bars.

1 Introduction

Thecontinuous increasing trends in case of the raw material-, energy-, production-, and operation costs have been made necessary the development of the numerical methods have been made possible the wide spread of the optimal design methods in the technical practice. Using these methods we can ensure to fullfil not only the different requrements, but we also are able to reduce the cost level of the product. The reduction of the structural mass or volume was the target at the early stage of the optimal design studies. Later the overal cost of the product to be reducedwas the goal.

When specifying the technical optimization problems, product and process optimization is defined. The product optimization can be further specified: topology optimization, form optimization, dimension optimization and material optimization, T. Kulcsár, I. Timár [1].

2 Multiobjective optimization

The *multiobjective optimization* is widely used for solving optimization tasks of engineering problems. The method was developed by Vilfredo Pareto (1848 – 1923). The Pareto optimum is a solution point, where no other main function can be improved without the degradation of another main function. We cannot find too many papers about the theory of the multi-objective optimization by the 60-es, but a lot of papers came out in this topic after that, including also the decision making applications, K. Jármai, M. Iványi [2].

Multiobjective optimum design of structures is very significant for enhancing the quality of structural design. Multiobjective optimization is a vector optimization, each element of which represents the objective functions being optimized. The mathematical expressions are as

$$\begin{aligned} &min\ F(\underline{x}) = \{f_1(\underline{x}),\dots,f_n(\underline{x})\},\\ &0 \le g_j(\underline{x}), \rightarrow j = 1,2,\dots,m \\ &0 = h_j(\underline{x}), \rightarrow j = m+1,\dots,p, \end{aligned} \tag{1}$$

where $\underline{x} = [x_1,\ x_2,\dots x_n]^T$ is a column vector of design variables; $f_i(\underline{x})$ is the i-th objective function; $g_j(\underline{x})$ are inequality constraints and $h_j(\underline{x})$ are equality constraints. Several effective methods for multiobjective optimization are introduced as follows: weighting method, hierarchical optimization method and goal programming, S. Hernadez, M. El-Sayed [3]. We have applied the weighting method to the optimal design. The summation of each individual objective function multiplied by its weighting factor is considered as a new scalar objective function

$$\begin{aligned} &F(\underline{x}) = \sum_{i=1}^{3} w_i f_i(\underline{x}),\\ &\sum_{i=1}^{3} w_i = 1\ and\ w_i \ge 0. \end{aligned} \tag{2}$$

We should substitute a single objective optimization for the multiobjective optimization

$$min\ F(\underline{x}). \tag{3}$$

The weighting factors represent the relative importance of the objective functions from the decision maker's view point. Because there is no practical analytical method to define the weighting factors now they are selected by experience. The weighting method

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has been applied extensively in engineering design because of its simplicity.

3 Optimization of planar truss girder

In the most of the optimization problems of truss girder we have two cases in the optimization tasks:

- the topology of the structure is fixed, so the cross sections of the beams are unknown quantities,
- the overall topology of the structure is variable, so we have to find also the optimal overall geometry, M. P. Bendsoe [4].

At the majority of optimization task of truss girder can be related, that in the bulk of the cases the mass of the construction, his material cost or his volume are considered an objective function, what with the expenses concerning the nodal forming heavy to formulate and onto this literature data do not stand for a provision relevantly.The restriction conditions onto the stresses, the deflections, the eigenfrequencies, and they concerned the stability.The single pole cross-sections, which may record discreet or constant values, appear as unknown.The employment of beams of truss girder is tension or compression. So the tension forces have to be smaller in each beam than given by EUROCODE 3 (EC3). In the compressed beams of truss girder is the bending danger, so the force in a beam have to be smaller than the prescribed of norm (EC3). The design value of the tension force N_{Ed} at each cross-section should satisfy

$$\frac{N_{ed}}{N_{t,Rd}} \leq 1. \tag{4}$$

For sections with holes the design tension resistance $N_{t,Rd}$ should be taken as the smaller of:

a) the design plastic resistance of the gross cross-section

$$N_{pl,Rd} = \frac{Af_y}{\gamma_{M0}}, \tag{5}$$

where A is the cross sectional area of member; f_y is the yield strength; γ_{M0} partial factor for resistance of cross-section whatever the class is,

b) the design ultimate resistance of the net cross section at holes for fasteners

$$N_{u,Rd} = \frac{0,9 A_{net} f_u}{\gamma_{M2}}, \tag{6}$$

where A_{net} is the net area of a cross-section, f_u is the ultimate strength, γ_{M2} is the partial factor for resistance of cross-section in tension to fracture.

The design value of the compression force N_{Ed} at each cross-section should satisfy

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1, \tag{7}$$

where $N_{b,Rd}$ is the design resistance. The design resistance of the cross-section for uniform compression $N_{b,Rd}$ should be determined as follows

$$N_{pl,Rd} = \frac{Af_y}{\gamma_{M0}}, \tag{8}$$

for class 1, 2 and 3 cross-sections,

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}, \tag{9}$$

for class 4 cross-section, where χ is the reduction factor for the relevant buckling mode; A is the cross sectional area of beam; A_{eff} is the effective cross-section area of the compressed member (EC3/1.5); γ_{M1} is the partial factor for resistance of members to instability. For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ should be determined from the relevant buckling curve according to

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \quad but \quad \chi \leq 1, \text{ where}$$

$$\Phi = \frac{1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2}{2}, \tag{9}$$

where α is an imperfection factor (EC3). The non-dimensional slenderness is given by

$$\bar{\lambda} = \sqrt{\frac{\beta\, Af_y}{N_{cr}}} = \frac{\lambda}{\lambda_1}, \tag{10}$$

for class 1, 2, and 3 cross sections,

$$\bar{\lambda} = \sqrt{\frac{\beta_A Af_y}{N_{cr}}} = \frac{\lambda}{\lambda_1}, \tag{11}$$

for class 4 cross-sections, where β is A_{eff}/A, N_{cr} is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties, $\lambda = l_{cr}/i$ is the slenderness ratio, l_{cr} is the buckling length in the buckling plane considered, i is the radius of gyration about the relevant axis, determined using the properties of the gross cross-section and

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93,9\sqrt{\frac{235}{f_y}} = 93.9\varepsilon. \tag{12}$$

In the additional we show the optimization of truss girder ones on the Fig. 1., F_1 and F_2 forces loaded. We regard it as given ones the geometrical form of the structure wich consist square crosssectional beams and this cross-sections are unknown, I. Timár [5]. We solve the problem with the multiobjective optimization. The objective functions are the mass of construction and the displacements of joints "C" and "D".

3.1 Compose of the objective functions

The mass of the planar truss (Fig. 1.) is given by

$$m = f_1(x) = \sum_{i=1}^{11} A_i l_i \rho \tag{13}$$

where A_i is each beam cross-section, l_i is the length of beams and ρ is the density of the material.

The deflections of joints C and D can be calculated by Betti's law

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$$w_c = f_2(x) = \sum_{i=1}^{11} l_i \frac{N_i n_{Ci}}{A_i E} \quad (14)$$

$$w_D = f_2(x) = \sum_{i=1}^{11} l_i \frac{N_i n_{Di}}{A_i E} \quad (15)$$

where $N_{i,}$ are the internal forces in the i-th member due to

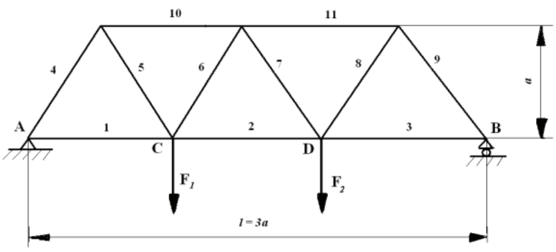

Fig. 1. The structure of truss girder

the applied load (F_1 and F_2), n_{Ci} and n_{Di} are the internal forces in the i-th member due to the load [Q_C=*1*(kN) and Q_D=*1*(kN)], E is the modulus of elasticity. Because of we have 3 main functions, the problem can be solved with the so called multiobjective optimization method. One of these methods is the so called as weighted method. The value of the weighting factors it is necessary to elect it with the respect of how important one we should award the single objective functions to in terms of the problem.

3.2 Compose of the design constrains

To the displacement of joints "C" and "D" we prescribe that should be smaller than the admissible displacements

$$w_{C\,max} \le w_{adm} = C_w l, \quad (16)$$

$$w_{D\,max} \le w_{adm} = C_w l, \quad (17)$$

where C_w is the allowable deflection ratio. l is the length of planar truss.

4 Numerical example

Consider the planar truss shown in Fig. 1 under the action of concentrated forces F_1 and F_2. Data: a = 1,5 (m); C_w= 1/300; E = 210 (GPa); f_y= 235 (MPa);f_u= 360(MPa);F_1 = 105 (kN); F_2 = 63 (kN); l = 4,5 (m); Q_C= 1; β_A = 1; γ_{M1} = 1,1; ρ = 7850 (kg/m^3); σ_m = 200 (MPa); value of χ calculate the software by Eurocode 3.

We solved the task with the multiobjective optimization method. Fig. 2.shows the minimal value of the optimal beams cross-sections depends on the objective functions. On the left side column the mass of truss girder was the objective function in the majority of cases (w_1=0,995, w_2=0,005, w_3=0,000) and the deflection of points "C" and "D" was practically negligible (w_2= 0.005, w_3=0,000). We got the right side higher columns that the aim was the minima of deflection of point "C" (w_1=0,005, w_2=0,995, w_3=0,000). This aim could be achieved with a more stiffness and heavier structure.

The Fig. 3.shows the mass of truss depends on the load (n*(F_1+F_2). On the left side column the mass of truss girder was the objective function in the majority of cases (w_1=0,995, w_2=0,005, w_3=0,000) and the deflection of

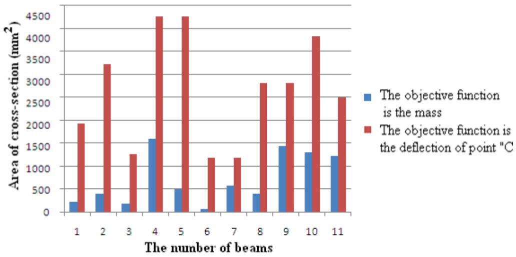

Fig. 2.Cross-section area of bars depends on the objective functions

points "C" and "D" was practically negligible (w_2= 0.005, w_3=0,000). We got the right side higher columns that the aim was the minima of deflection of point "C" (w_1=0,005, w_2=0,995, w_3=0,000), this case we got higher mass (right side columns).

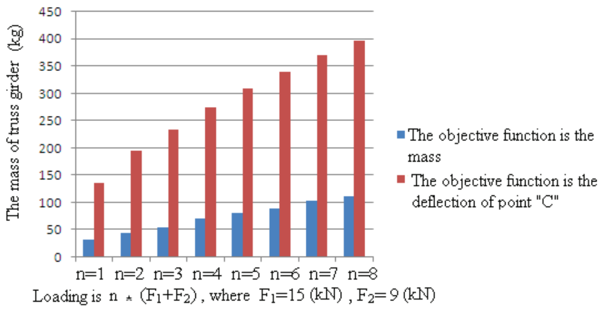

Fig. 3. Mass of truss depends on the load

The Fig. 4.shows the deflection of point "C" depends on the load (n*(F_1+F_2)).On the left side column the mass of truss girder was the objective function in the majority of cases (w_1=0,995, w_2=0,005, w_3=0,000) and the deflection of points "C" and "D" was practically negligible (w_2= 0.005, w_3=0,000). In this case the deflections are higher.

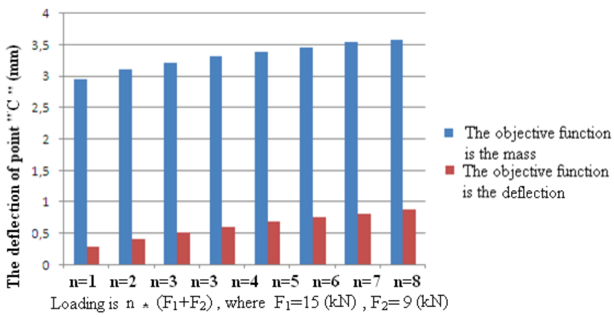

Fig. 4. Deflection of point "C" depends on the load

We got the right side higher columns that the aim was the minima of deflection of point "C" (w_1=0,005, w_2=0,995, w_3=0,000), and the deflections are smaller.

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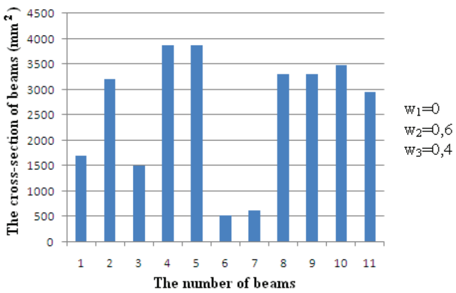

Fig. 5. The calculated cross-sections of beams

The Fig. 5. shows the the beam of planar truss girder cross-sections in this case F_1=105 (kN) and F_2=63 (kN) load. The weighting factors w_1=0,000,w_2=0,6, w_3=0,4, that mean we desire the deflection of junctions “C” and “D”at least to reduce.

Table 1. The cross-sections of beams depends on the loading

Loading	Cross-sections of beams (mm^2) (Number of beams)					
	1	2	3	4	5	6
F_1=15 (kN), F_2=9 (kN).	530	970	450	1160	1160	140
F_1=30 (kN), F_2=18 (kN).	720	1320	620	1650	1650	220
F_1=45 (kN), F_2=27 (kN).	880	1640	770	2020	2020	280
F_1=60 (kN), F_2=36 (kN).	1020	1910	890	2310	2310	320
F_1=75 (kN), F_2=45 (kN).	1140	2140	1000	2600	2600	360
F_1=90 (kN), F_2=54 (kN).	1260	2380	1100	2860	2860	400
F_1=105 (kN), F_2=63 (kN).	1360	2570	1200	3090	3090	420
F_1=120 (kN), F_2=72 (kN).	1440	2740	1280	3300	3300	440
F_1=135 (kN), F_2=81 (kN).	1530	2890	1360	3480	3480	460
F_1=150 (kN), F_2=90 (kN).	1620	3000	1440	3580	3580	480

Table 2. The cross-sections of beams depends on the loading

Loading	Cross-sections of beams (mm^2) (Number of beams)				
	7	8	9	10	11
F_1=15 (kN), F_2=9 (kN).	160	1020	1020	1100	880
F_1=30 (kN), F_2=18 (kN).	260	1380	1380	1460	1280
F_1=45 (kN), F_2=27 (kN).	330	1710	1710	1820	1560
F_1=60 (kN), F_2=36 (kN).	380	2010	2010	2100	1800
F_1=75 (kN), F_2=45 (kN).	420	2240	2240	2370	2000
F_1=90 (kN), F_2=54 (kN).	460	2440	2440	2592	2192
F_1=105 (kN), F_2=63 (kN).	480	2660	2660	2790	2370
F_1=120 (kN), F_2=72 (kN).	500	2840	2840	2970	2510
F_1=135 (kN), F_2=81 (kN).	520	2980	2980	3160	2650
F_1=150 (kN), F_2=90 (kN).	540	3120	3120	3340	2780

5 Conclusion

Mathematical optimization methods are generally applicable in the case of technical and economic problems. The optimization problem in general case is to build up a suitable model: to set up the multiobjective function(s) and to formulate the restrictions as mathematical functions.The truss girder optimizing received results show it, that the objective functions (mass and deflections) we receive different results differing at the time of his weighting factors. The bending of the construction with bigger mass will be smaller, but his expense will be bigger according to the meaning. The one with smaller mass (expense) we receive bigger bending on the other hand in case of a construction. The multiobjective optimising so it is possible to apply it successfully at the time of the solution of decision tasks.

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