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The continuous-time H∞ model matching problem with integral control in lmi optimization

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Abstract. In this paper, the continuous-time H_∞ model matching problem with integral control by using 2 DOF static output feedback is presented. The controller synthesis theorem and the controller design algorithm is elaborated. Simulation study illustrates the effectiveness and usability of the presented controller design.

1 Introduction

The model matching problem has attracted a lot of attention in the control theory [13-14]. If $G_m(s)$ and $G(s)$ are the model and the system matrices, respectively, the continuous-time H_∞ model matching problem (MMP) is introduced to derive a controller transfer matrix $R(s)$ that minimizes the H_∞ norm of $G_m(s)$-$G(s)R(s)$. The model transfer matrix $G_m(s)$ has the desired performance specifications defined by its poles and zeros. Moreover, $G_m(s)$ and $G(s)R(s)$ are stable and proper transfer matrices, that is $G_m(s)$ and $G(s)R(s) \in RH_\infty$. The closed-loop performance $G(s)R(s)$ is considered to be approximated by the desired performance $G_m(s)$ such that,

$$\gamma_{opt} = \inf_{R(s)\in RH_\infty} \left\| G_m(s) - G(s)R(s) \right\|_\infty . \quad (1)$$

H_∞ MMP is elaborated in [5, 8, 9]. In these studies, the dynamic precompansator $R(s)$ is obtained and then it is implemented by dynamic state feedback, [13]. Formerly, continuous-time H_∞ MMP with one degree of freedom (1 DOF) static state feedback is derived in [1], the discrete-time H_∞ MMP with 1 DOF static output feedback and the continuous-time H_∞ MMP with 2 DOF static output feedback is presented in [2-3], respectively. On the other hand, the integral control structure subject to the existence of state feedback is firstly used in [4].

In this paper, the continuous-time H_∞ MMP with integral control is proposed by using a 2 DOF static output feedback. Both the solution of the continuous-time static H_∞ optimal control problem (OCP) and continuous-time H_∞ MMP is revisited toward the solution of our presented problem, whereas continuous-time H_∞ MMP can be completely solved by the LMI-based numerical optimization.

This paper is organized as follows: In Section 2, a special formulation for the continuous-time H_∞ MMP by a 2 DOF static output feedback with integral control in linear matrix inequalities (LMIs) is elaborated. In Section 3, the main result is given by a theorem that provides two existence conditions of the solution. In Section 4, we construct the 2 DOF static output feedback with integral control by using this theorem. A numerical system example and some conclusions are finally given in Section 5 and 6, respectively.

Notations

R	The set of real numbers.
C	The set of complex numbers.
R^{nxm}	The set of nxm real matrices.
$Re(\alpha)$	The real part of $\alpha \in C$.
L_∞	The functions bounded on $Re(s)=0$ including at ∞.
H_∞	The set of L_∞ functions analytic in $Re(s)>0$.
I_n	An identy matrix of nxn dimension.
0_n	A zero matrix of nxn dimension.
0_{nxm}	A zero matrix of nxm dimension.
KerM	The kernel space the linear operator M.
ImM	The image space of the linear operator M.
N^T	The tranpose of the matrix N.
$P>0$	P positive definite matrix.
dim(U)	The dimension of the linear space U.
$\lambda_{max}(A)$	The largest eigenvalue of the matrix A.
$\sigma_{max}(A)$	The largest singular value of the matrix A defined as $$\sigma_{\max}(A) = \sqrt{\lambda_{\max}(A^T A)}.$$
$\|G(s)\|_\infty$	The norm of the transfer matrix G(s) defined as $$\| G(s) \|_\infty = \sup_{\omega\in R} \sigma_{\max}\left[G(j\omega)\right].$$

In order to present a synthesis theorem on the LMI-based characterization of the continuous-time H_∞ model

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matching problem with integral control, the following lemmas are given. The first lemma is **The Bounded Real Lemma** and it is used to turn the continuous-time H_∞ optimal control problem into an linear matrix inequality (LMI):

Lemma 1.1 Consider a continuous-time transfer matrix T(s) of (not necessarily minimal) realization $T(s)=D+C(sI-A)^{-1}B$. The following statements are equivalent:

i) $\|T(s)=D+C(sI-A)^{-1}B\|_\infty<\gamma$ and the matrix A is Schur ($Re(\lambda_i(A))<0$, i=1,…,n).

ii) There is a solution X>0 to the LMI:

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0. \tag{2}$$

Proof: See [7]. ■

Lemma 1.2 Suppose P, Q and H are matrices and that H is symmetric. The matrices N_P and N_Q are full rank matrices satifying ImN_P=KerP and ImNQ=KerQ. Then there is a matrix J such that,

$$H+P^TJ^TQ+Q^TJP<0 \tag{3}$$

if and only is the inequalities

$$N_P{}^THN_P<0 \qquad \text{and} \qquad N_Q{}^THN_Q<0 \tag{4}$$

are both satisfied.

Proof: See [10]. ■

Lemma 1.3 The block matrix

$$\begin{bmatrix} \mathrm{P} & \mathrm{M} \\ \mathrm{M}^\mathrm{T} & \mathrm{N} \end{bmatrix} < 0 \tag{5}$$

if and only if

$$N<0 \qquad \text{and} \qquad P-MN^{-1}M^T<0 \tag{6}$$

In the sequel, $P-MN^{-1}M^T$ is referred to as the **Schur complement** of N.

Proof: See [6]. ■

2 The continuous-time H∞ mmp by 2 dof static output feedback with integral control in lmi optimization

Toward the solution of the continous-time H_∞ MMP via LMI approach, the problem should be reformulated as standard continuous-time H_∞ OCP. The state-space representation of the system G(s) and the model system $G_m(s)$ is given:

$$G(s): \quad \begin{cases} \dfrac{dx(t)}{dt} = Ax(t)+Bv(t) & (7) \\ y_S(t) = Cx(t) & (8) \end{cases}$$

$$G_m(s): \quad \begin{cases} \dfrac{dq(t)}{dt} = Fq(t)+Gw(t) & (9) \\ y_m(t) = Hq(t)+Jw(t) & (10) \end{cases}$$

where $x(t)\in R^n$ is the system state, $q(t)\in R^{n_m}$ is the model state, v(t), w(t), $y_s(t)$ and $y_m(t)\in R^m$. We take that the given system is strictly proper in order to simplify the solution of the problem. The integral control is modelled by a serie integrator:

$$\frac{d\hat{x}(t)}{dt} = w(t) - y_S(t) \tag{11}$$

$$v(t) = \hat{x}(t) + u(t) \tag{12}$$

where $\hat{x}(t)$ and $u(t)\in R^m$. The control input u(t) is generated by a two degrees of freedom static output feedback controller:

$$u(t)=Ly_s(t)+Mw(t)=\begin{bmatrix} L & M \end{bmatrix} \begin{bmatrix} y_S(t) \\ w(t) \end{bmatrix}. \tag{13}$$

The block diagram of a continuous-time H_∞ MMP by a static output feedback with integral control is illustrated in Figure 1. In this formulation, the steady-output value $y_s(t)$ will follow a step function input with zero error. We will use a 2 DOF feedback control structure which is defined in the control theory, [12]:

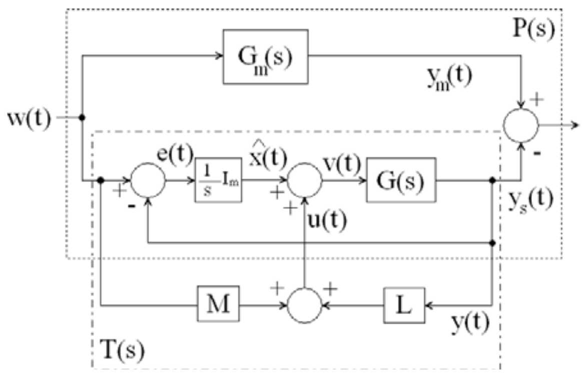

Fig. 1. The block diagram of model matching system with 2 DOF static output feedback in the integral control.

The generalized plant P(s) shown in Figure 1 can be modelled as,

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{bmatrix}\begin{bmatrix} x(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I \\ G \end{bmatrix} w(t) + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(t) \tag{14}$$

$$z(t) = \begin{bmatrix} -C & 0 & H \end{bmatrix}\begin{bmatrix} x(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} + Jw(t) \tag{15}$$

$$y(t) = \begin{bmatrix} y_S(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w(t). \tag{16}$$

Matrices are defined as follows :

$$\underline{A} = \begin{bmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ I \\ G \end{bmatrix} \qquad B_2 = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

$$C_1 = \begin{bmatrix} -C & 0 & H \end{bmatrix} \qquad C_2 = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$D_1 = J \qquad D_2 = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{19}$$

The above formulation concludes that the continuous-time H_∞ model matching problem plus integral control with the two degrees of freedom static output feedback is equivalent to the continuous-time H_∞ optimal control problem. This equivalency is drawn in Figure 2:

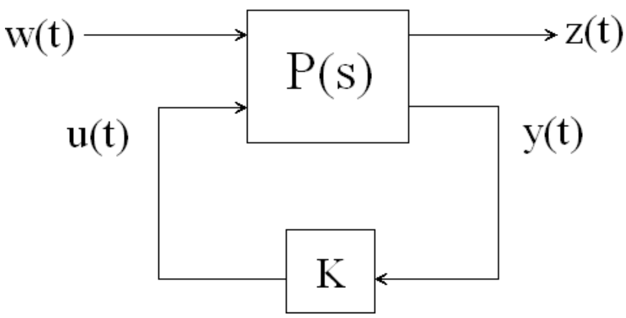

Fig. 2. The block diagram of the general form of H_∞ OCP with a static controller.

The closed-loop transfer matrix from w(t) to z(t) is derived by

$$T_{zw}(z) = D_{cl} + C_{cl}(zI - A_{cl})^{-1} B_{cl} \tag{20}$$

where

$$A_{cl} = \underline{A} + B_2 K C_2 \tag{21}$$

$$B_{cl} = B_1 + B_2 K D_2 \tag{22}$$

$$C_{cl} = C_1 \tag{23}$$

$$D_{cl} = D_1. \tag{24}$$

A synthesis theorem on the LMI-based solution of the problem is presented in the following section.

3 Main result

We can now present a synthesis theorem on the LMI-based solution of the continuous-time H_∞ model matching problem with integral control by two degrees of freedom static output feedback:

Theorem 3.1 A 2 DOF static output feedback plus integral controller $[L \ \ M] \in R^{mx2m}$ exists for the continuous-time H_∞ MMP if and only if there is a matrix X>0 such that,

$$\begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix}^T \left\{ \begin{bmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{bmatrix}^T X + X\begin{bmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} C^TC & 0 & -C^TH \\ 0 & 0 & 0 \\ -H^TC & 0 & H^TH \end{bmatrix} \right\} \cdot \begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & \begin{matrix} 0 & 0 \end{matrix} \\ 0 & \begin{matrix} I_m & 0 \\ 0 & I_m \end{matrix} \end{bmatrix}^T \begin{bmatrix} \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix} X^{-1} + X^{-1}\begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix}^T & X^{-1}\begin{pmatrix} -C^T \\ 0_m \\ H^T \end{pmatrix} & \begin{pmatrix} 0_{n_s xm} \\ I_m \\ G \end{pmatrix} \\ \begin{pmatrix} -C & 0_m & H \end{pmatrix} X^{-1} & -\gamma I_m & J \\ \begin{pmatrix} 0_{mxn_s} & I_m & G^T \end{pmatrix} & J^T & -\gamma I_m \end{bmatrix} \cdot \begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & \begin{matrix} 0 & 0 \end{matrix} \\ 0 & \begin{matrix} I_m & 0 \\ 0 & I_m \end{matrix} \end{bmatrix} < 0 \tag{26}$$

where N_c and N_o are full rank matrices with

$$ImN_c = KerB^T \tag{27}$$

$$ImN_o = KerC. \tag{28}$$

Proof: From the Bounded Real Lemma, $K = [L \ \ M] \in R^{mx2m}$ is the two degrees of freedom static output feedback controller in Figure 2 if and only if the LMI

$$\begin{bmatrix} A_{cl}^T X + XA_{cl} & XB_{cl} & C_{cl}^T \\ B_{cl}^T X & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{29}$$

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holds for some X>0 in $R^{(ns+nm+m)x(ns+nm+m)}$. Using the expressions A_{cl}, B_{cl}, C_{cl} and D_{cl} in (21), (22), (23) and (24), this LMI can also be written as,

$$H_X+Q^TK^TP_X+P_X^TKQ<0 \tag{30}$$

where

$$P_X=\begin{bmatrix}B_2^TX & 0 & 0\end{bmatrix} \tag{31}$$

$$Q=\begin{bmatrix}C_2 & D_2 & 0\end{bmatrix} \tag{32}$$

$$H_X=\begin{bmatrix}\underline{A}^TX+X\underline{A} & XB_1 & C_1^T\\ B_1^TX & -\gamma I & D_1^T\\ C_1 & D_1 & -\gamma I\end{bmatrix}. \tag{33}$$

Lemma 1.2 eliminates the matrix K in (30). Hence, the linear matrix inequality (30) holds for some K if and only if

$$N_{Px}^TH_XN_{Px}<0 \quad \text{and} \quad N_Q^TH_XN_Q<0 \tag{34}$$

where

$$ImN_{Px}=KerP_X \tag{35}$$

$$ImN_Q=KerQ \tag{36}$$

$$X>0. \tag{37}$$

Since

$$P_X=\begin{bmatrix}B_2^TX & 0 & 0\end{bmatrix}=\begin{bmatrix}(B & 0 & 0)X & 0 & 0\end{bmatrix} \tag{38}$$

we can write

$$P_X=P\begin{bmatrix}X & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix} \tag{39}$$

and

$$P=\begin{bmatrix}B^T & 0 & 0\end{bmatrix}. \tag{40}$$

We can take

$$N_{P_X}=\begin{bmatrix}X^{-1} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}N_P. \tag{41}$$

Consequently,

$$N_{Px}^TH_XN_{Px}<0 \tag{42}$$

is equivalent to

$$N_P^T\left\{\begin{bmatrix}X^{-1} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}\begin{bmatrix}\underline{A}^TX+X\underline{A} & XB_1 & C_1^T\\ B_1^TX & -\gamma I & D_1^T\\ C_1 & D_1 & -\gamma I\end{bmatrix}\begin{bmatrix}X^{-1} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}\right\}N_P<0 \tag{43}$$

or

$$N_P^TT_XN_P<0 \tag{44}$$

where

$$T_X=\begin{bmatrix}\underline{A}^TX^{-1}+X^{-1}\underline{A} & B_1 & X^{-1}C_1^T\\ B_1^T & -\gamma I & D_1^T\\ C_1X^{-1} & D_1 & -\gamma I\end{bmatrix}. \tag{45}$$

Meanwhile, from (41) the bases of KerP are

$$N_P=\begin{bmatrix}N_c & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix} \tag{46}$$

where $ImN_c=KerB^T$. Therefore the condition (43) can be reduced to the inequality (26).

On the other hand, let us obtain the first inequality (25): From $ImN_Q=KerQ$ we can write,

$$Q=\begin{bmatrix}C_2 & D_2 & 0\end{bmatrix}=\begin{bmatrix}C & 0_m & 0_{mxn_m} & 0_m & 0_m\\ 0_{mxn_s} & 0_m & 0_{mxn_m} & I_m & 0_m\end{bmatrix} \tag{47}$$

and so

$$N_Q=\begin{bmatrix}\begin{pmatrix}N_o & 0 & 0\\ 0 & I_m & 0\\ 0 & 0 & I_{n_m}\end{pmatrix} & 0\\ 0_{mx(m+n_m+r)} & 0\\ 0 & I_m\end{bmatrix} \tag{48}$$

where the matrix N_o is defined by,

$$ImN_o=KerC \tag{49}$$

and

$$r=dim(KerC). \tag{50}$$

When the state space equations (17), (18), (19) and the matrix N_o are used in $N_Q^TH_XN_Q<0$, the following LMI is obtained:

$$\begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 \\ 0 & I_m \end{bmatrix}^T \cdot \begin{bmatrix} \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix}^T X + X \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix} & X \begin{pmatrix} 0_{n_s xm} \\ I_m \\ G \end{pmatrix} & \begin{pmatrix} -C^T \\ 0_m \\ H^T \end{pmatrix} \\ \begin{pmatrix} 0_{mxn_s} & I_m & G^T \end{pmatrix} X & -\gamma I_m & J^T \\ \begin{pmatrix} -C & 0_m & H \end{pmatrix} & J & -\gamma I_m \end{bmatrix}$$

$$\cdot \begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 \\ 0 & I_m \end{bmatrix} < 0. \tag{51}$$

Therefore we have,

$$\begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & I_m \end{bmatrix}^T \cdot \begin{bmatrix} \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix}^T X + X \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix} & \begin{pmatrix} -C^T \\ 0_m \\ H^T \end{pmatrix} \\ \begin{pmatrix} -C & 0_m & H \end{pmatrix} & -\gamma I_m \end{bmatrix}$$

$$\cdot \begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & I_m \end{bmatrix} < 0 \tag{52}$$

and if we use Schur complement argument in Lemma 1.3, the linear matrix inequality (52) can be converted to form of as follows:

$$\begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix}^T \left\{ \begin{bmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{bmatrix}^T X + X \begin{bmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{bmatrix} + \frac{1}{\gamma} \begin{bmatrix} C^T C & 0 & -C^T H \\ 0 & 0 & 0 \\ -H^T C & 0 & H^T H \end{bmatrix} \right\}$$

$$\cdot \begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix} < 0. \tag{53}$$

The proof is completed. ■

4 Controller construction

Although Theorem 3.1 is about the solvability conditions of the continuous-time H_∞ MMP by the 2 DOF static output feedback with integral control, it also provides a controller construction procedure. And The MATLAB LMI Control Toolbox [11] can be also used to solve LMIs. The controller construction procedure can be summarized as follows:

Step 1: Find a solution X>0 to the LMIs (25) and (26) for γ_{opt} which is the minimal of γ.

Step 2: Solve a 2 DOF static output feedback control law $K=[L \quad M] \in R^{mx2m}$ from LMI

$$H_X + Q^T K^T P_X + P_X^T K Q < 0 \tag{54}$$

where

$$P_X = \begin{bmatrix} (B^T & 0_m & 0_{mxn_m})X & 0_m & 0_m \end{bmatrix} \tag{55}$$

$$Q = \begin{bmatrix} C & 0_m & 0_{mxn_m} & 0_m & 0_m \\ 0_{mxn_s} & 0_m & 0_{mxn_m} & I_m & 0_m \end{bmatrix} \tag{56}$$

$$H_X = \begin{bmatrix} \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix}^T X + X \begin{pmatrix} A & B & 0 \\ -C & 0 & 0 \\ 0 & 0 & F \end{pmatrix} & X \begin{pmatrix} 0_{n_s xm} \\ I_m \\ G \end{pmatrix} & \begin{pmatrix} -C^T \\ 0_m \\ H^T \end{pmatrix} \\ \begin{pmatrix} 0_{mxn_s} & I_m & G^T \end{pmatrix} X & -\gamma I_m & J^T \\ \begin{pmatrix} -C & 0_m & H \end{pmatrix} & J & -\gamma I_m \end{bmatrix}. \tag{57}$$

5 Numerical example

Theorem 3.1 and the controller construction algorithm will be used to design a controller to achieve model matching for the given system. Consider the second-order unstable system

$$G(s) = \frac{s+0.6}{(s-0.7)(s+0.5)}. \tag{58}$$

The model system is taken as

$$G_m(s) = \frac{1}{s+1}. \tag{59}$$

The state-space equations of G(s) are expressed in the minimum phase as

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.35 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) \tag{60}$$

$$y_S(t) = \begin{bmatrix} 0.6 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \tag{61}$$

The state-space equations of $G_m(s)$ are again expressed as

$$\frac{dq(t)}{dt} = -q(t) + w(t) \tag{62}$$

$$y_m(t) = q(t). \tag{63}$$

The state-space equations of P(s) in Figure 1 can be written as

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.35 & 0.2 & 1 & 0 \\ -0.6 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t) \tag{64}$$

$$z(t) = \begin{bmatrix} -0.6 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} \tag{65}$$

$$y(t) = \begin{bmatrix} 0.6 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \hat{x}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t). \tag{66}$$

When we search for γ_{opt}, the matrix X>0 and the 2 DOF static output feedback controller, the results are obtained as follows:

$\gamma_{opt}=0.775$, *L= -5.9578 and M=2.6904.*

$$X = \begin{bmatrix} 0.7179 & 0.3004 & -0.1531 & -0.2494 \\ 0.3004 & 0.6626 & -0.1936 & -0.2317 \\ -0.1531 & -0.1936 & 0.6540 & -0.1464 \\ -0.2494 & -0.2317 & -0.1464 & 1.0785 \end{bmatrix} > 0$$

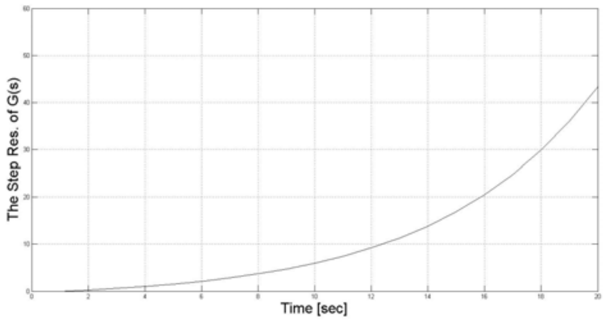

Fig. 3. The step response of the open loop system G(s).

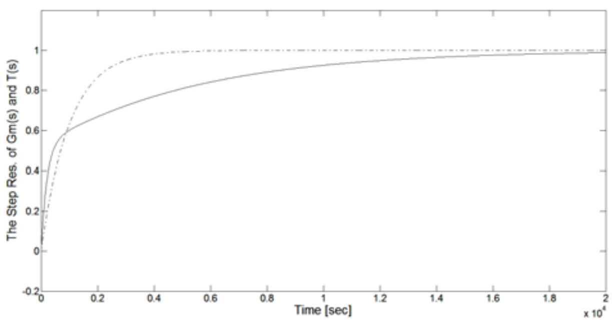

Fig. 4. The step responses of the model system $G_m(s)$:-.- and the closed system T(s):---

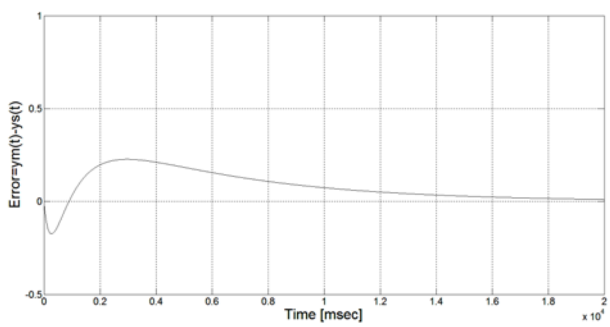

Fig. 5. The error function between the model and closed loop system outputs.

6 Conclusions

In this paper, we have studied the continuous-time H_∞ model matching problem with two degrees of freedom static output feedback. We have induced integral controller to this classical problem. The introduction of integral type of controller to this configuration naturally forces the steady-output error to zero. Moreover, the nearly proposed block diagram reduces the problem to an H_∞ optimal control problem and a theorem is proposed which provides a procedure to design the controller. Numerical simulation results have shown the expected results; that is; the steady-state error has approached to zero. However, we suppose that the two LMI conditions provided in the Theorem 3.1 can be simplified in future works.

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References

1. M. Akın, A. Bir, Turkish J. of Electrical Eng. and Computer Sciences, **11**, (1), 35-44, (2003)
2. M. Akın, A. Bir, *Proceedings on the 1st International Conference on Mathematics and Informatics for Industry*, Thessaloniki, Greece, 151-161, (2003)
3. M. Akın, A. Bir, *Proceedings on the 3rd IFAC Workshop DECOM-TT 2003, Automatic Systems for Building the Infrastructure in Developing Countries*, Istanbul, Turkey, 323-328, (2003)
4. M. Akın, T. Acarman, Int. J. of Control Systems and Robotics, **1**, 113-122, (2016)
5. J. C. Doyle, B. A. Francis, A. R. Tannenbaum, *Feedback Control Theory*, Macmillan Publishing Company, (1992)
6. G. R. Duan, H. H. Yu, *LMIs in Control Systems*, CRC Press, (2013)
7. G. E. Dullerud, F. Paganini, *A Course in Robust Control Theory, A Convex Approach*, Springer-Verlag, (2000)
8. B. A. Francis, *A Course in H_∞ Control Theory, No.88, Lecture Notes in Control and Information Sciences*, Springer-Verlag, (1987)
9. B. A. Francis, J. C. Doyle, SIAM J. on Control and Optimization, **4**, 815-844, (1987)
10. P. Gahinet, P. Apkarian, Int. J. of Robust and Nonlinear Control, **4**, 421-448, (1994)
11. P. Gahinet, A. Nemirovski, A. J. Laub, M. Chilali, *The LMI Control Toolbox For Use with MATLAB*, The MathWorks Inc., (1995)
12. M. J. Grimble, *Robust Industrial Control, Optimal Design Approach for Polynomial Systems*, Prentice-Hall International, (1994)
13. V. Kucera, *Analysis and Design of Discrete Linear Control Systems*, Prentice-Hall International, (1991)
14. W. A. Wolovich, *Linear Multivariable Systems*, Springer-Verlag, (1974)