General aspects regarding the kinematic and dynamic analysis of the fabric take up/ cloth roller

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Abstract. The aim of this paper is to presents different aspects regarding the kinematic and dynamic behaviour of the fabric winding at the weaving loom. In this paper shall be presented the kinematic diagram for operating the cloth roll on the weaving machine, making the study of kinematic and dynamic cloth behaviour, setting up the expression of fabric tension. Finally, where drawn the charts of the variation of the fabric load depending by kinematic and constructive parameters.

1 Introduction

In present days, the request for the high quality garments imposes certain trends in the weaving loom development/construction. The modern weaving loom uses positive take-up mechanisms, acted by individual c.c. motors and clutches, protecting the fabric for over loading.

The paper presents a kinematical and a dynamical analysis of the take-up and cloth roller mechanism. The command schema and the kinematic chain that acting the take up cylinder contain the computer, gears, chain and frictional clutch as is shown in Figure 1 [1-2], where: 1 – loading cell, 2 – computer, 3 – d. c. motor, 4 - zi gears, 5 – cloth take up roller, 6 – chain and chain wheels, 7 – clutch, 8 – cloth roller.

2 Theoretical kinematical and dynamical aspects of the fabric loading

In the fabric take up and rolling process, the fabric element is extracted with a constant velocity. The characteristics of the cloth (from the paper's point of view) are:

- specific weight \( \rho \);
- fabric thickness \( \delta \);
- the length of the cloth roller \( l_0 \);
- the diameter of the fabric roller \( d_0 \);
- the axial moment of inertia \( J_0 \).

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Fig. 1. The command schema of the take up and the fabric rolling mechanism (1- loading cell, 2- computer, 3- d. c. motor, 4- z₁ gears, 5- cloth take up roller, 6- chain and chain wheels, 7- clutch, 8- cloth roller).

In the process of the rolling cloth, its axial momentum of inertia ($J_\Delta$) represents a sum of the both empty fabric roller and fabrics' axial momentum of inertia (1) [3–4].

$$J_\Delta = J_0 + J_{\text{tes}}$$  \hspace{1cm} (1)

The axial moment of inertia of the empty fabric roller ($J_0$) could be calculated using:

$$J_0 = J_{\text{cil}^{\text{plin}}} - J_{\text{cil}^{\text{gol}}} = \frac{m_{\text{cil}^{\text{plin}}}}{2} r_0^2 - \frac{m_{\text{cil}^{\text{gol}}}}{2} r_{\text{int}}^2$$  \hspace{1cm} (2)

Where: $m_{\text{cil}^{\text{plin}}}$, $m_{\text{cil}^{\text{gol}}}$ – the mass of the filled/empty fabric roller, [kg], $r_0$, $r_{\text{int}}$ – external/internal radius of the fabric roller, [m].

Using the constructive particularities of the beam and the density of its material (aluminium), the equation (2) became:

$$J_{\text{cil}} = \frac{\pi \cdot \rho \cdot h}{2} (r_0^4 - r_{\text{int}}^4)$$  \hspace{1cm} (3)

Where: $\rho$ – Aluminium density [2700 kg/m³], $h$ – Length of the fabric beam [m], $r_0$, $r_{\text{int}}$ – external/internal radius of the fabric beam, [m].

The fabrics' axial momentum of inertia, $J_{\text{tes}}^{\text{tes}}$ depends by time (indirectly) and could be calculate using the relation:

$$J_{\text{tes}}^{\text{tes}} = \int r^2 dm$$  \hspace{1cm} (4)

Where: $r_0$ – the radius of the empty fabric beam, [m], $R$ – maximal radius of the fabric roller, [m], $r$ – the radius of the fabric roller at a specific time, [m], $dm$ – elementary mass, [kg].
And

\[ dm = \rho \cdot dV \]  
\[ dV \] – elementary volume, Figure 2.

It results:

\[ dm = \rho \cdot dV = \rho \cdot dl \cdot dA = \rho \cdot dl \cdot dl_1 \cdot dr = \rho \cdot dl \cdot r \cdot d\theta \cdot dr \]  
\[ (6) \]

for \( dl \in [0, l_0]; d\theta \in [0, 2\pi]; dr \in [r_0, r] \)

Where \( dl \) – elementary length of cloth roller.

Fig. 2. The elementary volume calculus, \( dV \).

dA – surface element;

d\( \theta \) – the elementary angle;

dl_1 = r \cdot d\theta

dl_1 – elementary length of the circles' arch.

Replacing (6) in (4), it results:

\[ J^{les}_\Delta = \rho \cdot \int_{r_0}^{r} r^3 dr \cdot \int_{0}^{l} dl \cdot \int_{0}^{2\pi} d\theta \]  
\[ (7) \]

Where

\[ r = r_0 + n \cdot \delta \]
\[ n = \frac{\theta}{2\pi} \]  
\[ (8) \]

where

\[ n \] – the number of cloth layers at a specific moment.

Finally, will obtain:

\[ J^{les}_\Delta = \frac{\pi \mu A}{2} \left[ \left( r_0 + \delta \cdot \frac{\theta}{2\pi} \right)^4 - r_0^4 \right] \]  
\[ (8) \]

\( \theta \) – the warping angle, [rad];
\( \delta \) – fabrics' thickness, [m].

The elements of the fabric winding are show in the figure 3.

The theorem of the kinetic moment variation, calculated in relation with the rotation axis of the fabric beam, is given by:
\[ \dot{K}_\Delta = M_\Delta \] (9)

Where

\[ K_\Delta = J_\Delta \cdot \omega \] (10)
\[ \dot{K}_\Delta = \dot{J}_\Delta \cdot \omega + J_\Delta \cdot \dot{\omega} \] (11)
\[ M_\Delta = M_a - T \cdot r \] (12)

It results:

\[ \dot{J}_\Delta \cdot \omega + J_\Delta \cdot \varepsilon = -T \cdot r + M_a \] (13)

Where \( M_a \) – drive momentum, [N·m],

\[ M_a = 9.55 \cdot 10^3 \frac{P}{n_s} \] (14)

\( P \) – the power of the driving motor [W];
\( n_s \) – speed of the fabric roller [rpm].

Fig. 3. Elements of the fabric winding [5].

From the equation (13), we can determine the fabric tension:

\[ T = \frac{1}{r} \left( M_a - \dot{J}_\Delta \cdot \omega - J_\Delta \cdot \varepsilon \right) \] (15)

Where: \( \varepsilon \) – angular acceleration, [rad/s²].

Deriving the equation (1), we get:

\[ \dot{J}_\Delta = \dot{J}_0 + \dot{J}_{\Delta es} \] (16)
And

\[ J_0 = 0 \]  

(17)

Deriving and successive processing the equation (8), we get:

\[ J_{\Delta}^{\text{tes}} = \frac{4\pi\rho l}{2} \left( r_0 + \frac{\delta \cdot \Theta}{2\pi} \right)^3 \cdot \frac{\delta \cdot \vartheta}{2\pi} = \rho \cdot l \cdot r^2 \cdot v_T \]  

(18)

From figure 3 results the angular acceleration deriving the (9) in relation to time:

\[ \varepsilon = \dot{\omega} = \frac{d\omega}{dt} \]  

(19)

Realizing a change of variable in (19), we'll obtain:

\[ \varepsilon = \frac{d\omega}{dr} \cdot \frac{dr}{dt} \]  

(20)

Where:

\[ \frac{d\omega}{dr} = \frac{d}{dr} \left( \frac{v_i}{r} \right) = -\frac{v_i}{r^2} \]  

(21)

\[ \frac{dr}{dt} = 0 + \delta \cdot \dot{n} = \delta \left( \frac{\dot{\vartheta}}{2\pi} \right) = \frac{\delta}{2\pi} \cdot \dot{r} = \frac{\delta}{2\pi} \cdot (\dot{\omega} \cdot r) = \frac{\delta \cdot v_i}{2\pi \cdot r} \]  

(22)

Replacing in (20), it became:

\[ \varepsilon = -\frac{v_i}{r^2} \cdot \frac{\delta \cdot v_i}{2\pi \cdot r} = -\frac{\delta \cdot v_i^2}{2\pi \cdot r^3} \]  

(23)

Replacing the calculated sized in relation (15), the final relation will be:

\[ T = \frac{1}{r} \left[ M_a - \rho \cdot l \cdot r^2 \cdot \frac{v_i^2}{r} + \frac{\delta \cdot v_i^2}{2\pi \cdot r^3} \cdot \frac{\pi \cdot \rho \cdot l}{2} \left( r^4 - r_{\text{int}}^4 \right) \right] \]  

(24)

3 Experimental aspects to calculate fabric loading

The theoretical values were applied to the Somet Thema weaving loom, with the main parameters shown in table 1.

The working parameters of the loom being optimized by the loom's computer, the transmission of the movement to the fabric roller presents different technological capabilities:
- \( z_1=48/24/17 \), \( z_2=80/96/103 \), \( z_3=16 \), \( z_4=48 \), \( z_5=16 \), \( z_6=48 \), \( z_7=2 \), \( z_8=90 \), \( z_9=45 \), \( z_{10}=31 \);
- main motor speed, max. 2350 rpm.;
- take up roller speed, 100 – 1400 rpm;
- pick density, 5 - 150 picks/cm.
### Table 1. Combination of gears to calculate maximal fabric density.

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<tr>
<th>Type</th>
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<th>yarns/cm</th>
<th>yarns/inch</th>
<th>Comment</th>
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<td>5 – 40</td>
<td>12,7 – 101,6</td>
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<td>(z_2 = 80)</td>
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<tr>
<td>C</td>
<td>(z_1 = 24)</td>
<td>10 – 80</td>
<td>25,4 – 203,2</td>
<td>Standard</td>
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<td></td>
<td>(z_2 = 96)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>D</td>
<td>(z_1 = 17)</td>
<td>15 - 150</td>
<td>38,1 - 381</td>
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<td></td>
<td>(z_2 = 103)</td>
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### Table 2. Fabric loading.

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<tr>
<th>(n_e) [rpm]</th>
<th>(M_a) [Nm]</th>
<th>(\delta) [m]</th>
<th>(J_0) [kg·m²]</th>
<th>(p_s) [kg/m²]</th>
<th>(l_0) [m]</th>
<th>(r_0) [m]</th>
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Fig. 4. The variation of the fabric loading depending by winding radius.

### 4 The Clutch's influence

As is seen in the weaving process, the fabric load is falling down according with the rising of the radius, as shown in table 4 and figure 4.
On the other hand, using the gear ratio between take up roller and the cloth beam, the peripheral speed of the fabric roller is rising, overloading the fabric (the gear ratio is greater than 1). This overload of fabric could be lowered using a frictional clutch. Because the same element of fabric extracted by the take up roller must be wrapped on the cloth roller, and keeping in mind that the both peripheral speeds of the rollers are different, we could calculate the clutch's slippage.

Starting from the equation of the rollers' peripheral velocity depending by its speed, we obtained:

\[ l_e = l_s \]  \hspace{1cm} (25)

Where:
\( l_{s,b} \) – the length of a fabric element, [m];

On the other hand, the expression of the cloth roller's speed, based on the take up roller's speed becomes:

\[ n_x = n_t \cdot \frac{z_9}{z_{10}} \cdot \frac{100-a}{100} \]  \hspace{1cm} (28)

Where:
\( z_9,z_{10} \) – sprockets' size;
\( a \) – clutch slippage, [%].

\[ n_x = \frac{v_t}{2 \cdot \pi \cdot r_x} = n_t \cdot \frac{z_9}{z_{10}} \cdot \frac{100-a}{100} \]  \hspace{1cm} (29)

Where:
\( v_t \) – peripheral speed of the take up roller, [m/s];
\( r_x \) – the radius of the cloth roller at a specific time, [m].

According with (29), we can calculate the radius of cloth roller:

\[ r_x = \frac{100 \cdot v_t}{2 \cdot \pi \cdot (100-a) \cdot z_9} \]  \hspace{1cm} (30)

The slippage of the clutch will result according with (30):

\[ a = 100 - \frac{100 \cdot v_t}{2 \cdot \pi \cdot n_t \cdot r_x \cdot z_9} \]  \hspace{1cm} (31)

Considering that \( a=0\% \) (the working moment of clutch), the cloth roller's radius is:

\[ r_x = \frac{v_t}{2 \cdot \pi \cdot n_t \cdot z_9} \]  \hspace{1cm} (32)

At limit, when \( r_x \to \infty \),

\[ a = 100 \]  \hspace{1cm} (33)
Table 3. Clutch's slippage.

<table>
<thead>
<tr>
<th>( n_{\text{et}} ) [rpm]</th>
<th>( n_t ) [rpm]</th>
<th>( v_t ) [m/s]</th>
<th>( r ) [m]</th>
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Based on the Table 3, the graphical variation of the slippage is presented in figure 5.

![Graph of Clutch's Slippage](image)

**Fig. 5.** The clutch's slippage.

### 5 Conclusions

Starting from kinematical and dynamical analysis, it results a calculus formula of the fabric loading in the wrapping process, depending by the geometrical dimensions of the cloth roller.

Also, we obtained the mathematical equation and diagrams of speed variation and clutch's slip, depending by the radius of the cloth roller, too.

As is seen in the weaving process, the fabric load is falling down according with the rising of the radius, as shown in table 2 and figure 4.
On the other hand, using the gear ratio between take up roller and the cloth beam, the peripheral speed of the fabric roller is rising, overloading the fabric. This overload of fabric could be eliminated by the worker by using a clutch (raising the slippage of the clutch's discs).

The worker's action in the fabric rolling is more often with the radius raising and it decrease the number of weaving loom surveyed by a worker.

The state-of-the-art weaving looms use an electronic controlled take up and individual motors [6-7].

References

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