Comparative assessment between GR model and tank model for rainfall-runoff analysis using Kalman filter- application to Algerian basins-.

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Abstract. Modeling the rainfall-runoff relationship with conceptual models has always been a fascinating subject for hydrologists in view of its practical importance and complexity. This study presents a comparative assessment of the performance of two well established rainfall-runoff conceptual models. A first model called: Model 'Genie Rural' (i.e. Agricultural Engineering) and abbreviated GR, developed by Cemagref has been extensively tested in the Mediterranean watersheds and some basins in African countries. When applied to Algerian basins, the different version of the GR model gave satisfactory results, particularly for long time steps (monthly and annual data). In this work, the tank Model by Sugawara using Kalman filter for adaptive calibration is developed and tested for the first time to assess rainfall-runoff in Algerian basins. The results appear to be very prominent and far better than those given by the GR models including daily time steps. Indeed, a comparison between the two models established for daily and monthly data was performed on the three (03) Algerian Basins (i.e. Isser, Zardezas basin and Cheffia). Calibration of the Tank model parameters was performed by Kalman filter. Furthermore, the structure of tank model (i.e: number of tanks, number of outlets in each tank, and their location) was determinated for the studied basins.

1 Introduction

The rainfall-runoff relationship is undeniably useful in the field of hydrology and water projects. The design of structures and dam management can't be achieved without the use of rain and runoff parameters. Conceptual models have taken a large part in modeling this relationship. The "tank model" Sugawara [1,2] is one of these methods. Indeed, the major difficulty in this model is the calibration of parameters in order to achieve a good estimate of the observed data.

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2 Formulation

2.1 Tank model

Since its introduction in early 1950's by Sugawara [3], the tank model has found growing and wide applications in the field of hydrology including flood control and groundwater modeling. Figure 1 introduces the general structure of a four-stage tank model widely applied in catchment characterization. The tank model is a conceptual rainfall-runoff model characterized by several interconnected storage units, called tanks, to account for surface, intermediate, and groundwater runoff components.

The first tank usually consists of two side outlets and infiltration outlet, while the tanks of intermediate and groundwater zones are equipped with one side and one infiltration outlet, respectively. The runoff components from the outlets are usually expressed as a linear function of the storage amount, i.e., water level in a tank. Whenever a linear relation does not generate satisfactory result a simple square relation can also be used (Parsad, 1967; Jermar, 1987 , [4,5]).

Fig. 1- Four-stage tank model.

The discharge, i.e., runoff component, from a side outlet is defined by:

$$Q_{ij}(t) = A_{ij} (H_i(t) - C_{ij})^\gamma U(H_i(t) - C_{ij}) \tag{1}$$

where $\gamma=1$ for linear approximation, Q_{ij} the runoff height component from side-outlet j of tank i (unit of length/time), A_{ij} the runoff coefficient of side-outlet j of tank i (unit of 1/time), and C_{ij} the height of side-outlet j of tank i (unit of length). As for the bottom tank, the value of C_{ij} (i.e., C_{41}) usually equals 0, whereas the side-outlet represents the continuous base flow.

The infiltration to the lower tank, or deep percolation in case of the bottom tank expressed by:

$$I_i = B_i H_i \tag{2}$$

where l_i is the infiltration from tank i to lower tank $i+1$ in height (unit length/time), and B_i the infiltration outlet coefficient.

The total discharge $QT(t)$, affected by a given observation noise $w(t)$ for each time period t , is given by:

$$QT(t) = \sum_{i=1}^N \sum_{j=1}^{J_i} Q_{ij} + w(t) \quad (3)$$

In control theory Eq. (3) is termed the observation equation, where i is the tank index ($i=1, \dots, n=4$), and j the side-outlet index in tank i ($j=1, \dots, J_i$). According to the schematic structure of the four-stage model given in Figure 1, $J_i=2$ if $i=1$, and $J_i=1$ otherwise ($i=2, \dots, 4$).

The system equation of the model is formulated by the equation system representing the changing of the water level in each tank by considering a given system noise $u(t)$:

$$\begin{aligned} \frac{dH_1}{dt} &= \left(\text{Rain}(t) - \text{Evt}(t) - Q_{11}(t) - Q_{12}(t) - I_1(t) \right) + u_1(t) \quad \text{for } i=1; j=1. \\ \frac{dH_i}{dt} &= \left(I_{i-1}(t) - Q_{ij}(t) - I_i(t) + u_i(t) \right) \quad \text{for } i=2,3 \text{ or } 4\text{th tank, } j=1. \end{aligned} \quad (4)$$

where $\text{Rain}(t)$ and $\text{Evt}(t)$ is the rainfall and evapotranspiration in [length/time], respectively. The parameters A_{ij} , B_i , C_{ij} , and water depth H_i are the unknown variables of the tank model to be calibrated. The developed methodology integrates Kalman filtering technique to optimize the model parameters in adaptive mode as discussed later.

2.2 Kalman Filter

Kalman filtering is the most commonly applied method for estimating the state of a linear system with known system dynamics and known statistics distribution of the assumed Gaussian error (i.e., noise), and it was developed by Kalman in early 1960's (Athans et al., 1968; Balakrishnan, 1984) [6,7] .

The system state and observation equations of the Kalman filtering are expressed in discrete form as:

$$\text{System state equation:} \quad X(k+1) = \Phi(k)X(k) + \alpha(k) + u(k) \quad (5)$$

$$\text{Observation equation:} \quad Y(k) = H(k)X(k) + \beta(k) + w(k) \quad (6)$$

Where, at each time step k , $X(k)$ is the system vector to be estimated, $H(k)$ and $\Phi(k)$ the known transition matrices, $\alpha(k)$ the known constant vector, $u(k)$ the white Gaussian system noise vector, $Y(k)$ the observed vector, $\beta(k)$ known constant vector, and $w(k)$ the white Gaussian observation noise vector. Details on the Kalman filtering procedure can be found in, e.g., Kitanadis and Bras (1980) [8], Anderson and Burt (1985) [9], and Bras and Rodriguez-Iturbe (1985) [10] .

2.3 Formulation of tank model by Kalman filtering:

In application to the tank model [11,12], the system state equation and observation equation (Eqs. 5 and 6) are given by Eqs. (3) and (4). The state system vector X for the four-stage tank (**Figure 1**) model is given by:

$$\begin{aligned} X &= [x_1 \ x_2 \ \dots \ x_{17}]^T \\ &= [H_1 \ H_2 \ H_3 \ H_4 \ A_{11} \ A_{12} \ A_{21} \ A_{31} \ A_{41} \ B_1 \ B_2 \ B_3 \ B_4 \ C_{11} \ C_{12} \ C_{31} \ C_{41}]^T \end{aligned} \quad (7)$$

If we define by $F(X)$ a vector function of the state system vector X given by:

$$F = [f_1, f_2, \dots, f_{17}]^T \quad (8)$$

the transition equation $\frac{dX}{dt}$ representing the system dynamics of X may be expressed as:

$$\frac{dX}{dt} = F(X) \quad (9)$$

In the specification of the four-stage tank model system state equation (Eq.4), the discrete form of (Eq. 5) is given by:

$$\begin{aligned} f_1(X) &= Rain(t) - Evt(t) - x_5(x_1 - x_{14})U(x_1 - x_{14}) - x_6(x_1 - x_{15})U(x_1 - x_{15}) - x_{10}x_1 + u_1(t) \\ f_2(X) &= x_{10}x_1 - x_7(x_2 - x_{16})U(x_2 - x_{16}) - x_{11}x_2 + u_2(t) \\ f_3(X) &= x_{11}x_2 - x_8(x_3 - x_{17})U(x_3 - x_{17}) - x_{12}x_3 + u_3(t) \\ f_4(X) &= x_{12}x_3 - x_9x_4 - x_{13}x_4 + u_4(t) \\ f_i(X) &= 0 \quad (i=5, 6, \dots, 17) \end{aligned} \quad (10)$$

2.4 Formulation of GR model:

2.4.1 GR2M model:

The GR2M model (i.e :2-parameters model of Rural Engineering) is a global model Rainfall-runoff with two parameters for monthly scale. Its development was initiated at Cemagref in the late 1980s, with application targets in the field of water resources and low flows periods.

This model has known several versions, successively proposed by Kabouya (1990), Kabouya and Michel (1991), Makhoulf (1994), Makhoulf and Michel (1994), Mouelhi (2003), Mouelhi and al. (2006b), which have gradually improved the performance of the model.

Its structure, although empirical, class it in the group of conceptual tanks models with a procedure for monitoring the moisture state of the basin which appears to be the best for taking into account the previous conditions and to ensure continuous operation of the model. Its structure combines a production tank with a tank of Routing as well as an opening on the outside other than the atmospheric environment. These Three functions simulate the hydrological behavior of the basin [13].

Fig. 2 - Architecture of GR2M model.

The model has two optimizable parameters :

X1 : capacity of tank production (mm)

X2 : exchange underground coefficient (-).

On a large sample of watersheds, the values given in Table 1.

Table 1 - Values of GR2M parameters obtained on a large sample of watersheds.

Parameter	Median	At confidence interval (90%)
X1(mm)	380	140- 2640
X2(-)	0.92	0.21 -- 1.31

2.4.2 GR4J Model:

The GR4J model (i.e : 4 parameters Rural Engineering model for daily scale) is a global model Rainfall-runoff with four parameters. Its development was initiated at Cemagref In the early 1980s, with a view to developing a Robust and reliable rainfall-runoff simulation for applications for Management of water resources and engineering (dimensioning of works, forecasting of floods and low flows, management of tanks, impact detection, etc.).

This model has had several versions, successively proposed by Edijatno and Michel (1989), Edijatno (1991), Nascimento (1995), Edijatno and al. (1999), Perrin (2000), Perrin (2002) and Perrin and al. (2003) which have improved the performance of the model [13].

Fig. 3 - Architecture of GR4J model.

GR4J model has four parameters to be calibrated :

- $X1$: capacity of production tank (mm).
- $X2$: exchange underground coefficient (mm)
- $X3$: daily capacity of routing tank (mm)
- $X4$: base time of hydrograph unit $HU1$ (day)

On a large sample of watersheds, the values given in Table 2, [13] :

Table 2 - Values of GR4J parameters obtained on a large sample of watersheds.

Parameter	Median	At confidence interval (80%)
$X1(mm)$	350	100 -- 1200
$X2(mm)$	0	-5--3
$X3(mm)$	90	20--300
$X4(day)$	1.7	1.1--2.9

3 Studied basins presentation

The input data used are the daily data of rainfall and concomitant hydrometric and evapotranspiration data (ETP). The data of rainfall and hydrometric issued by ANRH (the National Agency of Water Resources) and the ANBT (National Agency of Dams and Transfers).

In the basin of the Cheffia, we will use the data for daily evapotranspiration from the city of Annaba, calculated by the method of Penmann. For Zardezas basin they are calculated by a Colorado tray (coefficient $K = 0.7$) [14]. For Isser catchement we used the formula Oudin (2004), [15].

Table 3. Morphological characteristics of the studied watersheds.

Basin	Cheffia (03-15)	Isser (09-04)	Zardezas (03-09)
Area (km2)	575	1044	345
Coefficient Kc	1.20	1.25	1.26
Average Altitude (m)	420	665	628
Roche slope index	0.11	0.11	0.17

Table 4- hydrometric stations studied.

Stations (code)	Basin	Lambert coordinates	Installation date	Area (km ²)
Cheffia(031501)	Cheffia	X=977.70 -Y=381.32	1978	575 km ²
Lakhdaria (090501)	Isser	X=579.10 - Y=368.75	1954/65	1044 km ²
Zardezas (030902)	Zardezas	X=878.75 - Y=370.67	1989	345 km ²

Table 5 Period of observations used in this study.

Rainfull stations(code)	Hydrometric Stations	Daily observations Periods	Calibration/ validation -Daily	Calibration/ validation -Monthly
Chefia (031501)	Cheffia(031501)	1986-1990	03 years / 2 years	1978-1986/ 1987-1991
Isser(090403)+(090413)+(090419)	Isser (090501)	1992-1994	02 years / 1 year	1980-1992/ 1993-1996
Zardezas (030903)	Zardezas (030902)	1990-1995	04 years / 02 years	1990-1993/ 1994-1996

Fig. 4- Position of Zardezas (code :03 09) and Cheffia (code:03 15) Algerian basins. .

Fig. 5- Position of Isser (code: 09 04) Algerian basin.

4 Results and comments

4.1 Definition of validation criteria

To assess the quality of our results we calculated the following numbers: Nash Sutcliffe (*N.S.E*), R^2 . These parameters are defined in the following: Considering that: *qiobs*: observed flow, *qsim*: simulated flow, *q0obs*: the average of the observed flows , *q0sim* : the average of the simulated or estimated flows [16,17] .

- *N.S.E* : Nash Sutcliffe number:
$$N.S.E = 1 - \left[\frac{\sum_n^1 (qiobs-qsim)^2}{\sum_n^1 (qiobs-q0obs)^2} \right] \tag{11}$$

- Determination coefficient:
$$R^2 = \left[\frac{\sum_n^1 (qsim-q0sim)(qiobs-q0obs)}{\sqrt{\sum_n^1 (qsim-q0sim)^2} \cdot \sqrt{\sum_n^1 (qiobs-q0obs)^2}} \right]^2 \tag{12}$$

4.2 Results and interpretations

4.2.1 Not-updating approach

In this approach the structural parameters of the tank are fixed, only the dynamic parameters change, in our case the water levels in the tanks are (H1, H2, H3 and H4).

4.2.2 Results Summary

Table 6 - Summary results of all models studied.

Basin	Step	GR4J		Tank by Kalman filter	
		Nash	R ²	Nash	R ²
Isser	Calibration	0.72	0.73	0.97	0.97
	Validation	0.43	0.44	0.91	0.91
Cheffia	Calibration	0.71	0.72	0.98	0.98
	Validation	0.78	0.82	0.99	0.99
Zardezas	Calibration	0.81	0.81	0.998	0.998
	Validation	0.74	0.82	0.99	0.99
Basin	Step	GR2M		Tank by Kalman filter	
		Nash	R ²	Nash	R ²
Isser	Calibration	0.28	0.37	0.82	0.83
	Validation	0.11	0.20	0.97	0.97
Cheffia	Calibration	0.75	0.76	0.96	0.96
	Validation	0.74	0.88	0.96	0.96
Zardezas	Calibration	0.83	0.82	0.78	0.80
	Validation	0.58	0.83	0.95	0.95

4.2.3 Daily results

Fig. 6- Runoff with tank model by Kalman filtering Model and GR4J -Calibration - Isser basin.

Fig. 7- Simulated runoff as a function of observed runoff with GR4J model-Calibration of Isser basin.

Fig. 8- Estimated runoff as a function of observed runoff with tank model by Kalman filtering-Calibration of Isser basin.

Fig. 9- Runoff with tank model by Kalman filtering Model and GR4J -Validation- Isser basin.

Fig. 10- Simulated runoff as a function of observed runoff with GR4J model-Validation of Isser basin.

Fig. 11- Estimated runoff as a function of observed runoff with tank model by Kalman filtering- Validation of Isser basin.

4.2.4 Monthly results

Fig. 12- Runoff with tank model by Kalman filtering Model and GR2M -Calibration - Cheffia basin.

Fig. 13- Simulated runoff as a function of observed runoff with GR2M model-Calibration of Cheffia basin.

Fig. 14- Estimated runoff as a function of observed runoff with tank model by Kalman filtering- Calibration of Cheffia basin.

Fig.15- Runoff with tank model by Kalman filtering Model and GR2M-Validation- Cheffia basin.

Fig. 16- Simulated runoff as a function of observed runoff with GR2M model-Validation of Cheffia basin.

Fig. 17- Estimated runoff as a function of observed runoff with tank model by Kalman filtering- Validation of Cheffia basin.

5 Adopted models

In previous results after , the tank model by Kalman filter with the approach "Not updating" seems best for modeling our three catchments. We have thus reduced the number of tank to 2 upper tanks which reduces the number of parameters to 10: (A11, A12, C11, C12, A21, C21, B1, B2, H1, H2).

The first 08 parameters are fixed and present the structural parameters of the Tank, and for a watershed given the variable parameters are only reduced to levels : H1 and H2, this greatly simplifies the problem and is in itself a very useful results.

Fig. 18 - Adopted simplified structures in modeling of the three basins.

a)-Daily model ; b) Monthly model

We found a similarity between the tanks of the GR model and those of the adopted tank model from the present study. This similarity appears in the number of tanks as well as in the meaning of outlet's discharge.

6 Conclusion

Through this study we were able to model three (03) basins by GR and tank model using Kalman filter for two different time scales: daily and monthly. The graphical analysis and that of the validation parameters (Nash and R²) showed that the modeling results by the tank model using Kalman filter are better than those given by the GR models on these Algerian basins.

In view of their simplicity and practicality, especially for manual calibration, GR models have been widely used in North Africa and particularly in Algeria. The excellence of the results given by tank model using Kalman filter leads us to take more interest in these models in this region of the world, especially, for the prediction, simulation, filling and correction of hydrometric series.

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