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Estimating reliability of degraded system based on the probability density evolution with multi-parameter

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Abstract. System degradation was usually caused by multiple-parameter degradation. The assessment result of system reliability by universal generating function was low accurate when compared with the Monte Carlo simulation. And the probability density function of the system output performance cannot be got. So the reliability assessment method based on the probability density evolution with multi-parameter was presented for complexly degraded system. Firstly, the system output function was founded according to the transitive relation between component parameters and the system output performance. Then, the probability density evolution equation based on the probability conservation principle and the system output function was established. Furthermore, probability distribution characteristics of the system output performance was obtained by solving differential equation. Finally, the reliability of the degraded system was estimated. This method did not need to discrete the performance parameters and can establish continuous probability density function of the system output performance with high calculation efficiency and low cost. Numerical example shows that this method is applicable to evaluate the reliability of multi-parameter degraded system.

1 Introduction

For high reliability and long life products, it is difficult to obtain failure life time through life test and accelerated life test. The performance degradation data can provide important information for reliability assessment. Therefore the system reliability assessment based on degradation data becomes an important direction in the field of reliability research[1-2]. The performance of the system depends on the performance parameters of its components. According to the degradation law of the main parts and the function relationship between the performance of system and the parameters of various components, Monte Carlo simulation method can provide the degradation law of the performance of system and evaluate system reliability[3]. But this method need the simulation calculation on each discrete time point and large amount of calculation. So the literatures[4-8] put forward the universal generating function method. The method discreted the input parameter and combined the discrete state of the input parameters. Finally, the universal generating function algorithm deduced the reliability that meet the requirements of system output performance. But the precision of the system reliability evaluated by the universal generating function was related with the discrete degree of parameter. And

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with the increasing of the number of discrete state, the precision significantly improves. Because the discrete state combinations increase like geometric model with the increase of the number of discrete state, this method makes the calculation very complex[9] and can not get the probability density function of the system output performance. This paper proposes a system reliability assessment method based on multiple-parameter density evolution. This method can deduce the continuous probability density function of the system output performance parameters and calculate the system reliability. Compared with the Monte Carlo method, this method needs a small amount of calculation, but it provides higher precision than the universal generating function method.

2 Universal generating function method

The universal generating function method defines the contact between the possible value of the random parameters and the corresponding probability. It relates the state of the component performance parameter with the state probability of the system performance parameter. According to the system universal generating function computed by universal generating function algorithms, system reliability is obtained.

System degradation belongs to multiple random degradation process. The distribution characteristics of the degraded component performance parameters change with time. Degraded performance parameters obey normal distribution $X \sim N\left(\mu,\sigma^2\right)$ at the given moment. For discrete random variable, Universal generating function discretes the distribution for J states $\left\{x_1, x_2, ..., x_j\right\}$, The corresponding probability of each state is

$$q_j = \Pr\left\{X = x_j\right\} \text{ and } \sum_{j=1}^{J} q_j = 1 \tag{1}$$

At the time t, universal generating function of the component performance parameters is expressed as

$$U(\mathrm{z}) = \sum_{j=1}^{J} q_j \bullet z^{x_j} \tag{2}$$

When there are n performance parameters degenerating, the parameter vectors are $X = [X_1, X_2, ..., X_n]$ and system output function is $G\left(X,t\right)$。At the time t，the universal formula of the system universal generating function is

$$U_{G(X,t)}\left(z\right) = \Omega\left(U_1(\mathrm{z}), U_2(\mathrm{z}), ..., U_n(\mathrm{z})\right) = \sum_{i_1=1}^{I_1} \cdots \sum_{i_n=1}^{I_n} q_{1i_1} \cdots q_{ni_n} \bullet z^{\mathrm{g}(x_{1i_1}, x_{2i_2}, ... x_{ni_n})} \tag{3}$$

among them， $g\left(\bullet\right)$ is the system output performance value, $I_1, I_2, ..., I_n$ are different discrete states of n system performance degraded parameters at the time t.
Particularly when n=2, the system universal generating function is

$$U_{G(X,t)}\left(z\right) = \Omega\left(U_1(\mathrm{z}), U_2(\mathrm{z})\right) = \sum_{i=1}^{I} \sum_{j=1}^{J} q_{1i} \bullet q_{2j} \bullet z^{\mathrm{g}(x_{1i}, x_{2j})} \tag{4}$$

When the degradation threshold of system output performance parameters is D, system reliability is

$$R(t) = \sum_{g(x_{1i},x_{2j}) \geq D} q_{1i} \bullet q_{2j} \tag{5}$$

The literature[7] defined two special operators for the system output function: π operator and σ operator.The output functions of the system with parallel structure is

$$g(a,b) = \pi(\mathrm{a,b}) = \mathrm{a+b} \tag{6}$$

The output functions of the system with tandem structure is

$$\mathrm{g(a,b)} = \sigma_2(\mathrm{a,b}) = \frac{a \bullet b}{a+b} \tag{7}$$

According to the literature[9], the more the discrete states of component performance parameters, the higher the accuracy of the assessment. But the increase in discrete states will lead the number of the performance parameters state combinations increase like the geometric growth. When the system degraded performance parameters are more and the parameters are not the same type of distribution, it will bring huge computational burden.

3 Probability density evolution method

3.1 Establishing evolution equations

Probability density evolution method evolved from static and dynamic stochastic systems. This method based on systems physics equations researched the law of probability evolution from system multivariate stochastic parameters to the system output response and its theory is based on probability conservation principle. The existing random factors don't disappear and new random factor isn't added in conservative random system. The evolution of the physical state of the random system leads to the migration of probability and causes the evolution of the probability density. Establish the probability density evolution equation as follows [10-12], assuming that the system output function is

$$Z(t) = G(X,t) \tag{8}$$

$Z(t)$ is the system output function and random variables of system are $X=[X_1,X_2,...,X_n]$ .Augmented system $(Z(t),X)$ constitutes a conservative stochastic system. Joint probability density function of $(Z(t),X)$ is $p_{ZX}(z,x,t)$.

According to probability conservation principle, probability density evolution equation can be established [13, 14]:

$$\frac{\partial p_{ZX}(z,x,t)}{\partial t} + \sum_{i=1}^{m} \dot{Z}_i(x,t) \frac{\partial p_{ZX}(z,x,t)}{\partial z_i} = 0 \tag{9}$$

Formula (9) is general probability density evolution equations. When m=1, the general probability density evolution equations are

$$\frac{\partial p_{ZX}(z,x,t)}{\partial t} + \dot{Z}(x,t) \frac{\partial p_{ZX}(z,x,t)}{\partial z} = 0 \tag{10}$$

Typically, the boundary conditions of equation (9) is

$$p_{ZX}\left(z,x,t\right)\Big|_{z_i\to\pm\infty}=0,\ i=1,2,\cdots,m \tag{11}$$

The initial condition is

$$p_{ZX}\left(z,x,t\right)\Big|_{t=t_0}=\delta(z-z_0)p_X(x) \tag{12}$$

For partial differential equation (10), when $\dot{z}(x,t)$ is known, according to the initial and boundary conditions the analytical solution is

$$p_{ZX}\left(z,x,t\right)=\delta[z-z(x,t)]p_X\left(x\right) \tag{13}$$

$p_X\left(x\right)$ is the joint probability density of random parameters. $\delta(\bullet)$ is Dirac function. When using one-dimensional normal distribution density function δ sequence, Dirac function can be approximated by a normal distribution.

$$\delta(z-z(x))=\frac{1}{\sqrt{2\pi}\lambda}\exp[-\frac{1}{2\lambda^2}(z-z(x))^2] \tag{14}$$

λ above is a custom amount in Dirac function. When λ is close to zero, the above equation is theoretically established. Therefore, the following equation can be used to solve the probability density function of system output.

$$p_Z(z,t)=\frac{1}{\sqrt{2\pi}\lambda}\int_{\Omega_x}\exp[-\frac{1}{2\lambda^2}(\frac{z-z(x,t)}{z_{\max}})^2]\bullet\frac{1}{z_{\max}}\bullet p_X(x)dx \tag{15}$$

It is complex to directly solve equation (15). According to the method of multi-dimensional space selected point [15], the approximate calculation formula is:

$$p_Z(z,t)=\frac{1}{N_{sel}}\sum_{i=1}^{N_{sel}}\frac{1}{\sqrt{2\pi}\lambda}\exp[-\frac{1}{2\lambda^2}(z-G(x_i,t))^2]\bullet V_{x_i}\bullet\prod_{j=1}^{n}p_j(x_i) \tag{16}$$

N_{sel} is the total number of selected points.
x_i is the coordinates of the selected point.
$p_j\left(x_i\right)$ is the probability density of performance parameters X_j at the selected point.
V_{x_i} is the volume of the region the point represents.

3.2 System reliability assessment

According to the described method above, the probability density function of system output performance can be established at any time t. Assume that the system performance levels required for normal work is D, the system reliability at the time t is

$$R(t)=\Pr\left\{G(t)\ge D\right\}=\int_D^{+\infty}p_Z(z,t)dg \tag{17}$$

4 Comparison with Monte Carlo simulation

Taking an example of the function of system output performance given in the literature [9], $G(X,t)$ is the parameters of output performance.

$$G(X,t)=\frac{1}{4}\left(\left(\sin\left(X_1(t)-3\right)\right)\bullet\left(X_2(t)-1\right)+\left(X_3(t)-1\right)^2\right)-1 \tag{18}$$

Suppose at time of $t=t_0$, $X_1(t_0)$ and $X_3(t_0)$ obey uniform distribution $U[0,10]$, $X_2(t_0)$ obeys uniform distribution $U[6,16]$.The level of performance required by the system is 20. Using Monte Carlo simulation and the method proposed in this paper estimate the system reliability. And the compared result is shown in Table 1.

Table 1. Compared with Monte Carlo.

$G(X,t_0)$	20	The number of calculations
Monte Carlo	0.9837	100000
This method	0.9865	135
The error	0.28%	--

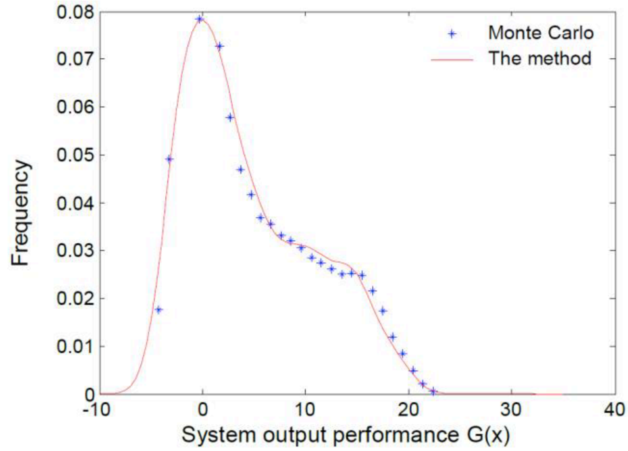

Figure 1. The probability density curve.

Figure 1 is the probability density curve of system output. As is shown in Figure 1, the calculation results based on the multi-parameter probability density evolution and Monte Carlo simulation are very close. Compared with Monte Carlo simulation results, when obtaining the same precision of system output performance parameters probability density function and system reliability, the computational efficiency is nearly 750 times of Monte Carlo simulation. Obviously, the system reliability assessment based on multi-parameter probability density evolution can not only ensure the accuracy of the assessment but also reduce the amount of calculation.

5 Example of system reliability assessment

If the system performance output function is

$$G(X,t)=\frac{X_1(t)\bullet X_2(t)}{X_1(t)+X_2(t)} \tag{19}$$

$X_1(t)$ is performance parameter of the component 1 and its deterioration law is in line with the Wiener process. Its characteristic parameters are

$$\mu_1: \quad 6.0-2\times10^{-5}\bullet t$$

$$\sigma_1: \quad 0.2+3.0\times10^{-6}\bullet t$$

$X_2(t)$ is performance parameter of the component 2 and its deterioration law is in line with the Wiener process. Its characteristic parameters are

$$\mu_2: \quad 5.0-1\times10^{-5}\bullet t$$

$$\sigma_2: \quad 0.3+1.5.0\times10^{-6}\bullet t$$

Table 2. Contrast of results among the three method.

Time/h	Monte Carlo Simulation	Probability Density evolution	Universal Generating Function（3 discrete states）	Universal Generating Function（7 discrete states）
0	0.99921	0.999293	1	0.999585
5000	0.997045	0.99663	1	0.99533
10000	0.991045	0.989008	0.974829	0.990157
15000	0.975943	0.971918	0.974829	0.973631
20000	0.94636	0.939747	0.866516	0.93574
25000	0.895974	0.887415	0.866516	0.86028
30000	0.821404	0.812742	0.758204	0.835575
35000	0.724797	0.718108	0.733032	0.739338
40000	0.612443	0.610291	0.733032	0.572939
45000	0.495858	0.498538	0.733032	0.572901
50000	0.384628	0.391953	0.241796	0.349516
55000	0.287996	0.297411	0.241796	0.349139
60000	0.208795	0.218657	0.241796	0.147861
65000	0.146107	0.156497	0.241796	0.147861
70000	0.099876	0.109638	0.133484	0.123155
75000	0.066361	0.075663	0.025171	0.041518
80000	0.04402	0.051825	0.025171	0.041518
85000	0.028085	0.035552	0.025171	0.03914
90000	0.018184	0.024694	0.025171	0.036762
95000	0.011826	0.017586	0.025171	0.007465
100000	0.007314	0.01301	0.025171	0.007465

The time-varying probability density curve of the system output performance parameters based on multi-parameter probability density evolution method is shown in Figure 2. Table 2 lists the calculated results based on different methods and the calculation accuracy comparison is shown in Figure 3. The result of the system reliability assessment method based on multi-parameter probability density evolution is closest to the result of Monte Carlo simulation through comparison. Therefore, the method presented in this paper is suitable for assessing the reliability of multi-parameter degraded system.

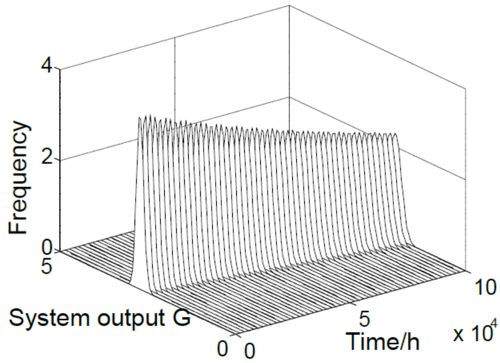

Figure 2. Time-varying probability density curve of the system output.

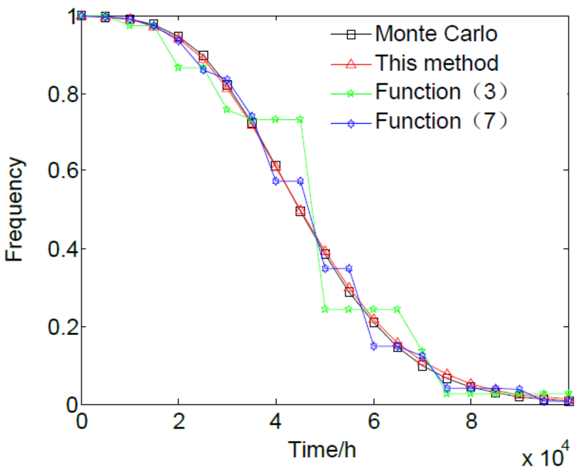

Figure 3. Contrast of reliability curve.

6 Conclusion

This paper presents a system reliability assessment method based on multi-parameter probability density evolution for complex parameter degraded system. This method can deduce the continuous probability density function of the system output performance and calculate the system reliability. This method is more accuracy than universal generating function method. Compared with Monte Carlo simulation, the assessment method proposed in this paper reduces the computational cost in the case to achieve the same accuracy and it provides an efficient way for system reliability assessment.

References

1. J. Tang, T.S. Su, Naval Research Logistics, **55** (3), 265-276 (2008)
2. K.A. Doksum and A. Hoyland, Technometrics, **34** (1), 74-82 (1992)
3. S.K. Zeng, T.D. Zhao, and J.G. Zhang. Tutorial of system reliability design and analysis, Beijing (2001)

4. G. Levitin, A. Lisnianski, H. Ben-Haim, et al., IEEE Transactions on Reliability, **47** (2), 165-172 (1998)
5. A. Azadeh, B. Maleki Shoja, and S. Ghanei, Reliab. Eng. Syst. Safety, **136**, 62-74 (2015)
6. M. Nourelfath, E. Chatelet, N. Nahas, Reliab. Eng. Syst. Safety, **103**, 51-60 (2012）
7. G. Levitin, IEEE Trans. Reliab., **50** (4), 380-388 (2001)
8. V. Ebrahimipour, M. Sheikhalishahi, and B. Maleki Shoja, IEEE, 255-239 (2010)
9. C. Li, Research on reliability analysis and optimization based on the multi-state system theory[D], Changsha(2010)
10. I. Elishakoff, Y.J. Ren, and M. Shinozuka, J. Eng. Mech., **122** (6), 559-565 (1996)
11. R.N. Mantegna, J. Stat. Phys., **70** (3/4), 721-736 (1993)
12. J.B. Chen and J. Li, Probabilist. Eng. Mech., **20** (1), 33-44 (2005)
13. J. Li and J. Chen, Acta Mechanica Sinica, **35** (6), 716-722 (2003) (in Chinese)
14. J. Li and J.B. Chen, Adv Mech, **40** (2), 170-188 (2010) (in Chinese)
15. K.T. Fang and Y. Wang, *Number Theoretic Methods in Statistics* (London,1994)
16. J. Deng, Int J Solids Struct, **43**, 3255-3291 (2006)