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Vibration isolation systems, considered as systems with single degree of freedom

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Abstract: The research considers and analyzes vibration isolation systems, whose design schemes are single degree of freedom systems, including nonlinear elements - displacement limiter and viscous damper. Presented are calculation formulas in closed form for linear systems in operational modes (for harmonic and impulse loads), algorithms and examples of calculation of linear and nonlinear systems in operational and transient modes. The calculation method and the above dependences are written using the transfer (TF) and impulse response functions (IRF) of linear dynamical systems and dependencies that determine the relationship between these functions. The effectiveness of 2 options of vibration isolation systems in transient modes is analyzed. There is significant reduction of load from the equipment to the supporting structures in the starting-stopping modes by the use of displacement limiter.
Keywords: vibration isolation, displacement limiter, viscous damper, nonlinear system, Duhamel integral.

1. Introduction

Vibration-active equipment stationed close to buildings or installed on structures cause such structures to undergo oscillation, which often go above the limits set in building standards and design manuals [1]. High displacements can also damage springs and flexible elements (for example, pipelines) which are connected to such equipment. Ways of reducing such dynamic loads interest a lot of scientists and engineers when putting up structures with vibration-active equipment. Assessment of vibration isolation system using auxiliary mass damper to reduce structural vibration was investigated in [2].

There has been growing interest in variable stiffness isolation systems[3-9]. These systems work by changing the structural stiffness thereby altering the natural period of the structure and thus escaping resonance. The changing of the structural stiffness is done by switching devices which are controlled by control laws [10-12]. Some of these devices incorporate dampers such that there is either stiffness or damping added to the structure, depending on the feedback response[11,13]. In [5] smart rubber material with capability of changing its stiffness was studied to be used in variable stiffness base isolation system.
In order to reduce vibrations in transient modes in many fields of technology, including the operation of vibration isolated equipment, systems with non-linear elements - displacement limiters and viscous dampers are widely used. In a significant part of the design process, dampers are constantly associated with an oscillating body.

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Such dampers, including a stator and a rotor, are tuned to the frequency of oscillations of the object and are effective in damping vibrations in transient modes. The disadvantage of such dampers is that their natural frequencies depend on the viscosity of the liquid, which varies with the ambient temperature. Such dampers, including a stator and a rotor, are tuned to the frequency of oscillations of the object and are effective in damping vibrations in transient modes. The disadvantage of such dampers is that their natural frequencies depend on the viscosity of the liquid, which varies with the ambient temperature. More stable in the operation of the damper are the options in which the damper is activated in operation directly in areas close to resonance, in particular, during start-up and shutdown modes. Such a scheme is shown in Fig. 1 [14].
This paper outlines the algorithm and calculation method to be used in analyzing vibration isolation system with nonlinear elements for a single degree of freedom system. Using the relationship between transfer function and impulse response function, the method could be used for 2 or more degrees of freedom systems [15,16] .
Two kinds of nonlinear elements are studied - nonlinear stiffness element (displacement limiters), and non-linear viscous damper.

In this study the control information (feedback structural information) is the reference structural displacement (y0). The structure and the standby stiffness engage together as soon as the displacement peaks to the reference point (y0). It maintains in this state until the displacement goes below the reference point. Below the reference point, the standby stiffness and the structure disengage separately.

These types of vibration isolation could be employed in reducing the levels of oscillation of equipment with rotating parts (Fans, compressors) and on screen machines in transition modes - starting and stopping modes[16].

2. Algorithm of calculation

The calculation schemes of single degree of system with displacement limiter and viscous damper are shown in figure 1. The algorithms of solutions of such systems in operational and transient modes are considered.

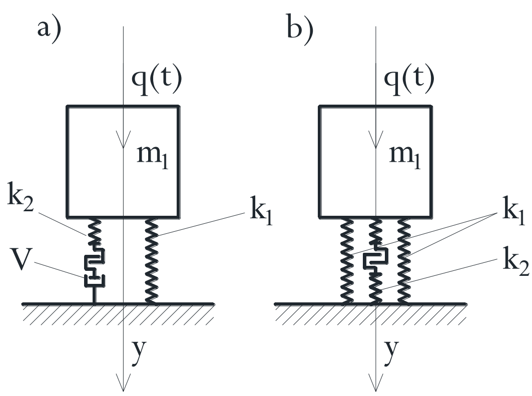

Fig. 1: a) system with displacement limiter ;b) system with viscous damper

2.1Algorithm of solving vibration isolation system with displacement limiter (fig. 1a):

The equation of motion of vibration isolation in fig. 1a is of the form:

$$m\ddot{y}+\left(1+2v\frac{d}{dt}\right)c(y)y=q(t) \tag{1}$$

The characteristics of the non-linearity considered in this systems is:

$$c(y)y = k_1 y \text{ при } y \le y_0;$$
$$c(y)y = k_1 y_0 + (k_1 + k_2)(y - y_0) \text{ при } y > y_0 \quad (2)$$

Taking the nonlinear components of equation (1) to the right side, we have:

$$\ddot{y} + \left(1 + 2v\frac{d}{dt}\right)p_1{}^2 y = \frac{q(t)}{m} + \left(1 + 2v\frac{d}{dt}\right)p_1{}^2 y - \left(1 + 2v\frac{d}{dt}\right)\frac{c(y)y}{m}; \quad (3)$$

After steps of simplification of equation (3), we have:

$$\ddot{y} + \left(1 + 2v\frac{d}{dt}\right)p_1{}^2 y = \frac{q(t)}{m} - \left(1 + 2v\frac{d}{dt}\right)\frac{k_2(y - y_0)}{m}; \quad (4)$$

The solution of equation (4) is of two parts; from the linear aspect from the applied dynamic load (y_{lin}) and from fictitious load which depends on the type of nonlinearity (y_{nonlin}).

$$y = y_{lin} - y_{nonlin} \quad (5)$$

The solution from the applied dynamic load is expressed in terms of Duhamel integral [16,17]:

$$y_{lin} = \frac{1}{p_1^* m}\int_0^t q(\tau)e^{-n_1(t-\tau)}\sin p_1^*(t-\tau)d\tau;$$

$$= \frac{1}{p_1^* m}\int_0^t q(\tau)e^{-n_1(t-\tau)}(\sin p_1^* t\cos p_1^*\tau - \cos p_1^* t\sin p_1^*\tau)d\tau$$

$$= \frac{1}{mp_1}\left[d_1(t)F_2(t) - d_2(t)F_1(t)\right]; \quad (6)$$

$$\text{где } 2n_1 = 2vp_1^2,\ p_1^* = \sqrt{p_1^2 - n_1^2},$$

$$d_1 = e^{-n_1 t}\sin p_1 t;\ d_2 = e^{-n_1 t}\cos p_1 t \quad (7)$$

$$F_1 = \int_0^t q(\tau)\cdot e^{n_1\tau}\sin p_1^*\tau d\tau;\ F_2 = \int_0^t q(\tau)\cdot e^{n_1\tau}\cos p_1^*\tau d\tau; \quad (8)$$

The solution from the fictitious load is also expressed in terms of Duhamel integral. For the reason that the integrand contains the expression of displacement (y), the second-order integral is nonlinear.

$$y_{nonlin} = \frac{1}{mp_1^*}\int_{t_0}^t \left(1 + 2v\frac{d}{dt}\right)k_2(y - y_0)e^{-n_1(t-\tau)}\sin p_1^*(t-\tau)d\tau \quad (9)$$

where t_0 – time of switching on the additional stiffness

Following equations (6) – (8), we have:

$$y_{nonlin} = \frac{k_2}{mp_1^*}\int_{t_0}^t (y - y_0)e^{-n_1(t-\tau)}(\sin p_1^* t\cos p_1\tau - \cos p_1^* t\sin p_1^*\tau)d\tau$$

$$=\frac{k_2}{mp_1^*}\left[d_1(t)F_2(t_0,t)-d_2(t)F_1(t_0,t)\right]. \tag{10}$$

where $F_2(t_0,t)=\int_{t_0}^{t}(y-y_0)\cdot e^{n_1\tau}\cos p_1^*\tau d\tau$;

$$F_1(t_0,t)=\int_{t_0}^{t}(y-y_0)\cdot e^{n_1\tau}\sin p_1^*\tau d\tau. \tag{11}$$

The total displacement is found from equation (5), which is solved for each time step by iterations at each step [18,19].

The force of reaction to the supporting structure is calculated using:

$$Q_{\sup}=Ak_i \tag{12}$$

where A - amplitude of displacement, k_i - system stiffness

2.2 Algorithm of calculation of system with viscous damper (fig. 1b):

The dissipative force arising when the piston moves in a working medium with a velocity in the vertical direction is determined by the formula:
$P_{gk}=h_k\upsilon_k=\mu l_p\Psi$, where h_k- the coefficient of resistance, in which $\mu(Pa.s)-$ the dynamic viscosity of the medium; l_p- Working height of a layer of viscous liquid; $\Psi(t)-$ Coefficient determined from Fig. 35 [14].

In the presence of a viscous damper according to the scheme in Fig. 1b, the equation of motion of a system with one degree of freedom takes the form (in particular, during start-up and in operating mode)

$$m\ddot{y}+\left(1+2\mu\frac{d}{dt}\right)\kappa y+h_k\left|\frac{dy}{dt}\right|=q(t). \tag{13}$$

At $y_0<y<y_1$, where $y_0(t_0)$ and $y_1(t_1)-$ the boundary of the zones of inclusion in the work of damping and shutdown. In the rest of the zone h_k equals to zero.

Taking into account the above conditions, using the Duhamel integral, the solution of equation (13) can also be represented as the sum of two solutions: a linear system for the applied load $q(t)$ and a fictitious load, which takes into account the nonlinear dependence of the dissipative forces. The solution of the linear systems for the load $q(t)$ is given above and can be rewritten as follows:

$$y_{lin}=\frac{1}{p_1^*m}\int_0^t q(t)V_1(p_1^*,t-\tau)d\tau; \tag{14}$$

where $V_1(p_1^*,t)=e^{-n_1t}\sin p_1t-$ Impulse response function.

$$y_{nonlin}=\frac{1}{p_1^*m}\int_{t_0}^{t}h_k\frac{dy}{d\tau}V_1(p_1^*,t-\tau)d\tau-\text{when } t\le t_1 \tag{15}$$

Integrating (15) by parts, we write (in the interval $t_0\le t\le t_1$)

$$y_{nonlin}=\frac{1}{p_1^*m}\left\{h_kyV_1(p_1^*,t)\Big|_{t_0}^{t}-\int_{t_0}^{t}h_kyV_2(p_1^*,t-\tau)d\tau\right\}; \tag{16}$$

$V_2=\frac{d}{d\tau}V_1$, after similar simplification (see expressions 6-11) takes the form:

$$y_{nonlin}(t)=\frac{h_k}{p_1^*m}\left\{\left[n_1d_1(t)-p_1d_2(t)\right]F_2(\tau)+\left[n_1d_1(t)-p_1d_2(t)\right]F_1(\tau)\right\}. \tag{17}$$

The nonintegral terms are 0 for $\tau=t$; and

$$y_{non\,\mathrm{int}} = \frac{h_k}{mp_1^*} y(t)V_1(p_1^*, t-t_0) \text{ при for } \tau = t_0 . \quad (18)$$

$d_i, F_i;\ i = 1,2$ see formula (8), (9)

Without giving numerical solutions, we note that this algorithm corresponds to the start solution, and the numerical algorithm is similar to the algorithm used in the first problem. In the operating mode, the upper limit of the integral is to be put $t = t_1$. The displacements of the systems in the stop mode are determined by the algorithm given above in the interval $t_3 - t$, where t_3 – the time for switching on the damper.

3. Numerical example

A single degree of freedom of mass 10 ton with displacement limiter is investigated, amplitude of excitation force (Q) –350kN, frequency of excitation force (ω) – 78rad/s, damping coefficient - 0.1, reference structural displacement (y0) = 0.015m.

For the linear displacement, four values of stiffness k1 were used as input to get the displacement for the start-up and shut-down modes. Additional stiffness (k2) with values 500, 1000 and 1500 kN/m were used for each value of k1 to get the nonlinear displacement. Results are shown in table 1.

The support reactions were computed and are shown in table 2.

The effect of values of the reference structural displacement(y0) was investigated and results are in table 3.

The input dynamic load is taken from [2]. The time taken is 12seconds for start-up, 60 seconds for operational and 45 seconds for shut-down mode.

3.1 Results and discussion

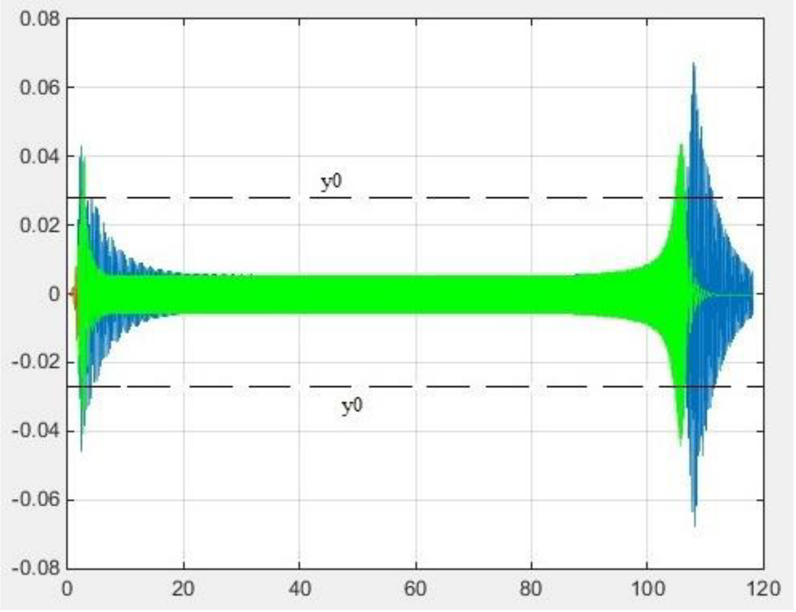

Fig. 2: displacement of system with traditional vibration isolation (black) and with displacement limiter (grey).

Table 1: displacement of system with traditional vibration isolation and with displacement limiter

	displacement of system with traditional vibration isolation (m)			displacement of system with displacement limiter (m)	
k1 (kN/m)	in start-up	in shut-down	k2 (kN/m)	in start-up	in shut-down
2500	0.0497	0.0772	500	0.0535	0.0700
			1000	0.0530	0.0537
			1500	0.0717	0.0328
3000	0.054	0.0818	500	0.0566	0.0760
			1000	0.0569	0.0636
			1500	0.0565	0.0458
3500	0.0561	0.0853	500	0.0596	0.0804
			1000	0.0604	0.0704
			1500	0.0548	0.0545
4000	0.0592	0.0881	500	0.0624	0.0841
			1000	0.0629	0.0771
			1500	0.0575	0.0652
4500	0.0608	0.0905	500	0.0642	0.0872
			1000	0.0654	0.0817
			1500	0.0602	0.0739

For the k1 values of 2500kN/m and 3000kN/m there a decrease in displacement in the start- of up to 57% and 44% in the shut down mode. There is however an increase in displacement in the startup mode. Therefore the optimum parameters of k1 and k2 are 3500 kN/m and 1500kN/m, where the reduction in displacement is 36% in the shut-down mode and marginal decrease in start-up mode (table 1).

There is a 32% reduction in the support reaction in the shut-down mode where k1 and k2 are 2500kN/m and 1500kN/m. In the start-up mode there is increase in the support reaction for all values of k1 and k2. (table 2).

The optimum value of y0 is 0.025m for k2 value of 1500kN/m which results in the most reduction of shut-down support reaction (table 3).

Table 2: support reaction of system with traditional vibration isolation and with displacement limiter

	support reaction of system with traditional vibration isolation(kN)			support reaction of system with displacement limiter (kN)	
k1 (kN/m)	in start-up	in shut-down	k2 (kN/m)	in start-up	in shut-down
2500	124.25	193.00	500	160.50	210.00
			1000	185.50	187.95
			1500	286.80	131.20
3000	162.00	245.40	500	198.10	266.00
			1000	227.60	254.40
			1500	254.25	206.10
3500	196.35	298.55	500	238.40	321.60
			1000	271.80	316.80
			1500	274.00	272.50
4000	236.80	352.40	500	280.80	378.45
			1000	314.50	385.50
			1500	316.25	358.60
4500	273.60	407.25	500	321.00	436.00
			1000	359.70	449.35
			1500	361.20	443.40

Table 3: Variation of values of reference structural displacement(y0) on displacement

k1(kN)	k2 (kN)	**y0**(m)	displacement(m)		support reaction (kN)	
			in start-up	in shut-down	in start-up	in start-up
3500	500	0.015	0.0596	0.0804	238.4	321.6
		0.025	0.0579	0.0745	231.6	298
		0.035	0.0563	0.0723	225.2	289.2
	1000	0.015	0.0604	0.0704	271.8	316.8
		0.025	0.0583	0.0609	262.35	274.05
		0.035	0.0561	0.0676	252.45	304.2
	1500	0.015	0.0548	0.0545	274	272.5
		0.025	0.0573	0.0512	286.5	256
		0.035	0.0558	0.093	279	465

4. Conclusion

The paper looked at variants of nonlinear systems of vibration isolation (with displacement limiter and viscous damper) and algorithms for their calculation as systems with single degree of freedom under the action of harmonic load (in operating mode) and in transient modes. From the numerical example it was observed that the system with the displacement limiter could reduce the maximum values of displacements in transient modes by 30-35%. The algorithm for calculating the system with a viscous friction dampener is illustrated by the example of calculating the system in the starting mode.

References

1. TSNIISK im. V. A. Kucherenko Nits construction, (2015).
1. M.V Osipov, Seismic construction. Seismic resistant construction. Safety of buildings, **1**, 31 - 34 (2014)
2. Ya. M. Eisenberg, Stroiizdat, (1976)
3. S.A. Alotaibi, M.A. Al-Ajmi, International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering, **8, 3,** 565-567 (2014).
4. Y. Li, J. Li, Australasian Conference on the Mechanics of Structures and Materials (ACMSM23). Byron Bay, Australia. **II,** 913-918 (2014).
5. Y. Xinghua, Model and analysis of variable stiffness semi-active control system. *12WCEE*. (2000)
6. D.Y. Kim, J. Joung, Experimental and Numerical Studies of a Newly Developed Semi-Active Outrigger Damper System. *CTBUH 2011 World Conference*. Seoul. Korea. (2011)
7. S. Nagarajaiah, S. Ertan, Journal of Structural Engineering, **33, 1**, 67-77 (2007).
8. S. Nagarajaiah, S. Sahasrabudhe, Earthquake Engineering and Structural Dynamics, **35, 2**, 177-197 (2006)
9. S. Kamagata, T. Kobori, Proceedings of the 1st World Conference on Structural Control. Los Angeles, U.S.A. (1994).
10. J.N. Yang, J.H. Kim, A.K. Agrawal, Journal of Structural Engineering, 1427-1433 (2000)
11. J.W. Joung, A.W. Smyth, L. Chung, Smart Materials and Structures, **19** (2010)
12. K. Kawashima, Proc.of International Workshop on Structural Control. (1993).
13. TSNIISK im. V. A. Kucherenko. Rekomendatsii po vibrozashchite nesushchikh konstruktsiy proizvodstvennykh zdaniy. [Recommendations on vibration isolation of load-bearing structures of industrial buildings]. (1986).
14. V.V. Solodovnikov, Staticheskaya dinamika lineynykh sistem avtomaticheskogo upravleniya [Static dynamics of linear automatic control systems]. Moscow: Fizmatgiz, (1960).
15. J.T. Chernov **2**nd ed. Moscow: Publisher ASV, 384 (2011).
16. A.K. Chopra, Pearson Education Asia Ltd and Tsinghua University Press, 846 (2004)
17. I.A. Petrov, M.V. Osipova, Internet-Bulletin VolgGASU. Ser.: Polit., 3 (23), (2012).
18. A.S. Gorodetsky, I.D. Evzerov, Kompyuternyye modeli konstruktsiy [Computer models of constructions] Moscow: Publishing Association of Construction Universities, 360 (2009).