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Analysis of dynamic coefficients for damage to the middle support of two-span and three-span continuous beams

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Annotation. The paper deals with the operation of continuous two- and three-span beams with damage to the middle support. For a two-span continuous beam, the dynamic coefficients for the action of concentrated forces over the middle support and span of the beam are theoretically substantiated. The results of numerical studies of double-span beams at different loads and time intervals of damage to the middle support are presented. The method of dynamic calculation, which can be used to calculate the survivability of steel structures, has been worked out. A good correspondence of the dynamic coefficients obtained numerically with theoretical values is established. Depending on the order of the formation of the design, the dynamic coefficients vary from 1 to 3. For survivability of the design, the pre-bending of the beams above the middle support is most dangerous. With the use of a well-grounded numerical technique, the dynamic coefficients for a continuous three-beam beam are investigated. For continuous beams, if the middle supports are damaged, it is recommended to use a dynamic coefficient of 2 for the load in the spans adjacent to the damaged support

1 Introduction

In Russia and other countries, a scientific direction is developing, which studies the behavior of damaged structural structures. In the design practice, the analysis of the load-carrying capacity of such structures is called the calculation for progressive destruction or the calculation of survivability. The problem of studying the bearing capacity of damaged structures is very relevant because of the adverse consequences of the destruction of buildings [1, 2]. In Russia, when designing unique buildings and structures, the calculation of load-bearing structures is performed with damage to their individual elements [3-18]. One of the ways to calculate survivability is to apply a modified long-term load with a dynamic coefficient to the changed design. The magnitude of the dynamic coefficient with a quick turn-off of the element can be taken as equal to 2. For spatial systems, local damage, even with an instantaneous failure of the element, does not always lead to dynamic effects corresponding to the dynamic coefficient 2. Determination of the dynamic coefficients for common designs for various damage schemes is of great practical importance .

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2 Theoretical part

For a theoretical justification of the dynamic coefficient, simple beam structures can be used [19, 20]. Consider a beam on two supports with hinged support at the ends and an intermediate support in the middle of the span. In the middle of the span the beam is loaded with concentrated force. Damage to the structure consists in the destruction of the middle support (Fig.1).

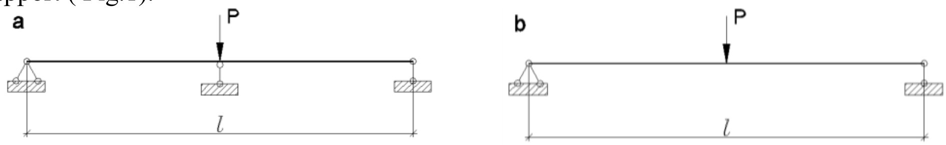

Fig.1. The beam is loaded with a concentrated force in the middle of the span: a - the original structure; B- damaged construction.

Perhaps several options for the interaction of the middle support with a beam:

1 variant. With the static action of the force of the beam the span of the span l curved to the deflection, then the middle support was fed. The reaction of the support is zero.

2 variant. The support is fed at a load equal to the force fraction, at the moment of contact of the beam with the middle support, the deflection of the beam.

3 variant. The beam is supported by an intermediate support before the load is applied, the initial deflection is zero

4 variant. The beam is supported on the intermediate support, while the beam is given the initial bend up equal to the deflection.

To determine the dynamic coefficient for each of the variants, one can use the well-known dependence [21]:

$$k_d = 1 + \sqrt{1 + 2H / w_c}\,, \tag{1}$$

where H - the height from which the load P falls on the beam, w_c - the deflection of the beam by a span l of a hinged-at-end force from the static action.

For the connection between force and deflection, we use the following relationships [21]:

$$P = cw_c \text{ и } P_d = cw_d\,, \tag{2}$$

where c - system stiffness (the same for static and dynamic load action), P_d - dynamic load, w_d - dynamic deflection.

1 variant

In the first version, the beam initially has a deflection equal to the full static while supporting it at the ends and in the middle of the force P. The absence of an intermediate support during its damage does not affect the operation of the beam, $k_d = 1$.

2 variant

With the 2nd variant of the damage, the part of the full load P_o is perceived due to deformations of the beam, the rest of the load is transferred to the middle support. With the initial deflection w_o, the fraction of force perceived by the deformation of the beam is:

$$P_o = P\, w_o / w_c \tag{3}$$

After removing the support on the beam, it will act as an instantly applied, unbalanced fraction of the force, and the total load on the beam will be:

$$P_d = P_o + 2\Delta P = P\left(w_o / w_c + 2 - 2w_o / w_c\right) = P\left(2 - w_o / w_c\right) \tag{4}$$

The dynamic coefficient in this case is:

$$k_d = P_d / P = 2 - w_o / w_c \tag{5}$$

3 variant

With the 3rd variant of the damage, the full load is transferred to the middle support. After the instant removal of this support, the entire load is immediately applied to the beam, and the dynamic coefficient is $k_d = 2$. This result is obtained both from formula (1) for, and from formula (5).

4 variant

With the 4th variant of structural damage, to determine the dynamic coefficient, let us consider the energy of the system deformation U_d, the deformation energy of the pre-curved beam U_o and the work A done by the load P after removal of the support. Energy balance of the system:

$$U_o + A = U_d , \tag{7}$$

where

$$U_o = Pw_c / 2 \tag{8}$$

$$A = P(w_c + w_d) \tag{9}$$

$$U_d = P_d w_d / 2 = cw_d^2 / 2 = Pw_d^2 / (2w_c) \tag{10}$$

w_d - the dynamic deflection of the beam, measured from the rectilinear axis of the beam.

After the simplest transformations, we get the quadratic equation:

$$w_d{}^2 - 2w_c w_d - 3w_c^2 = 0 \tag{11}$$

The solution of the quadratic equation has the form:

$$w_{d_{1,2}} = w_c \pm \sqrt{w_c^2 + 3w_c^2} \tag{12}$$

The roots of the quadratic equation:

$$w_{d_1} = 3w_c , \quad w_{d_2} = -2w_c \tag{13}$$

The second root contradicts the physical meaning of the problem, therefore $k_d = 3$. The same value k_d can also be obtained from expression (5), if we take: $w_o = -w_c$.

Set the initial bend over the middle support $0.5w_c$ And carry out similar transformations. Then:

$$w_d = 2.581w_c , \; k_d = w_d / w_c = 2.581 \tag{14}$$

The dynamic coefficient obtained from expression (5) at: $w_o = -0.5w_c$, is 2.5, which differs from k_d from the expression (14) 3%.

Thus, the coefficient of dynamism depends on how the structure was formed. It is necessary to take into account the presence of gaps between the supports and the structure, as well as the presence of preliminary bends during the construction of supports. More dangerous for the survivability of the design is its preliminary bending.

Consider the construction in the form of a continuous two-span beam, loaded in the middle of each of the spans by concentrated force P (Fig.2).

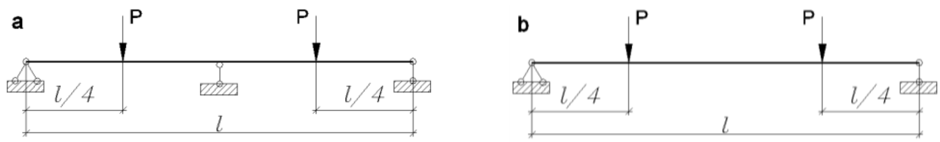

Fig.2. A double-span beam loaded with two concentrated forces in the middle of the span: a - the original structure; b - damaged construction

The reference reactions for the original construction are:

- on extreme supports:

$$F_1 = 5P/16; \tag{15}$$

- on medium support:

$$F_2 = 22P/16. \tag{16}$$

Deflection of the beam under each of the concentrated forces in the original structure:

$$w_o = 0.001139Pl^3/(EI) \tag{17}$$

The deflection of the beam under each of the concentrated forces with their static action in the damaged structure, measured from the rectilinear beam axis, is equal to:

$$w_c = 0.02083Pl^3/(EI) \tag{18}$$

After examining the energy balance of the system (7) and performing transformations similar to those carried out for the 4th variant, we obtain a dynamic deflection of the beam in the places of application of forces, measured from the rectilinear axis of the beam:

$$w_d = 1.972w_c. \tag{19}$$

The dynamic coefficient in this case:

$$k_d = w_d / w_c = 1.972 \tag{20}$$

3 Numerical studies

Numerical studies are performed using finite element models of a damaged design in a static and dynamic setting [22-25]. With dynamic calculations, the damaged element is removed, and the internal forces arising in the removed element are applied to the structure, which decrease to zero during the element's damage time [22-24].

Numerical studies were carried out using the Nastran computer complex. The beam was modeled by the beam elements "beam". Each beam span was broken into 6 elements. To take into account the dynamic mass calculations in places where concentrated forces are applied, an element of the "mass" type is used. Calculations are made for a beam from steel I-beam 20B1.

The study of the dynamic coefficients under the action of concentrated force over the middle support is carried out for several design schemes:

1 scheme: force 48.94 kN is applied over the middle support to the undeformed beam;

2 scheme: the force is applied to the beam by a span of 12 m, after reaching a half of the beam deflection by a span of 12 m from the force of 48.94 kN, the middle support is brought under the beam and the load is brought to 48.94 kN;

3 scheme: the beam is bent by displacing the middle support by an amount equal to half the beam deflection by a span of 12 m from the concentrated force of 48.94 kN, after which a concentrated force of 48.9 kN is applied to the beam to the middle support, the reference reaction is 73.41 kN.

Variants of the investigated structures are presented on Fig.3.

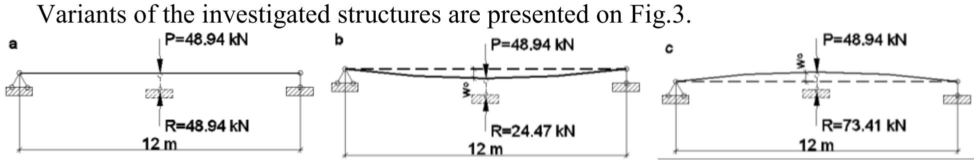

Fig.3. Schemes of the structures under investigation: a-1 scheme; b- 2 scheme; c- 3 scheme

With dynamic calculations, the weight of the beam (21 kg / m) and the concentrated mass of 4894 kg (equivalent to 48.94 kN force) are taken into account as the distributed masses. The frequency and period of the first form of natural oscillations were: 0.712 Hz and 1.404 seconds. The following time intervals for the reduction of the reference reaction are considered, sec .: 0.05, 0.1, 0.14, 0.4, 0.7, 1.0, 1.404, 2.0, 3.0. When calculating the 1 circuit, the external load is constant, and the reference reaction decreases for a specified period of time to zero. In the calculation of schemes 2 and 3, the external load and the reference reaction increase for 20 seconds. From zero to the full value, the external load then does not change, and the reference reaction decreases to zero within a specified time interval.

Table 1 shows the forces in a two-span beam, depending on the time of reduction of the support reaction in the event of damage to the middle support. In addition to the efforts in Table 1, the dynamical coefficients are equal to the ratio of the maximum dynamic forces to the corresponding forces calculated for the static loading of the damaged structure. Static load forces are equal: maximum bending moment *M*=48.94*12/4=146.82 кН·м, Maximum lateral force *Q*=48.94/2=24.47 кН.

Table 1. Efforts and coefficients of dynamism in two span beam

Parameter	Efforts and coefficients of dynamism at the time of reaction decrease								
	0.05	0.10	0.14	0.40	0.70	1.00	1.401	2.00	3.00
1 scheme									
M, kN·m	273.88	273.08	272.05	257.78	228.17	191.77	152.04	171.38	154.26
k_d	1.87	1.86	1.85	1.76	1.55	1.31	1.04	1.17	1.05
Q, kN	45.95	45.82	45.65	43.24	38.23	32.07	25.35	28.62	25.35
k_d	1.88	1.87	1.86	1.76	1.56	1.31	1.03	1.17	1.03
2 scheme									
M, kN·m	210.25	209.95	209.43	202.14	187.49	169.30	149.43	159.10	150.54
k_d	1.43	1.43	1.43	1.38	1.28	1.15	1.02	1.08	1.03
Q, kN	35.22	35.15	35.06	33.86	31.37	28.28	24.92	26.55	25.11
k_d	1.44	1.43	1.43	1.38	1.28	1.15	1.02	1.08	1.02
3 scheme									
M, kN·m	333.79	332.67	331.24	310.63	267.17	213.51	154.95	182.92	157.87
k_d	2.27	2.27	2.26	2.12	1.82	1.45	1.06	1.25	1.08
Q, kN	56.12	55.94	55.70	52.20	44.85	35.76	25.85	30.58	26.34
k_d	2.29	2.28	2.27	2.13	1.83	1.46	1.06	1.25	1.08

It is established, the faster the support reaction decreases, the greater the dynamism coefficient. The maximum design dynamic coefficients differ from the theoretical ones by 4-12% (the theoretical coefficients of dynamism are somewhat larger). For the considered constructive schemes, if the time of reduction of the reference reaction coincides with the period of the first form of natural oscillations k_d is close to unity.

The work of beams with load in the flights of a continuous beam (Fig.4) is numerically investigated: a beam with concentrated forces in the middle of each span and a beam with a uniformly distributed load.

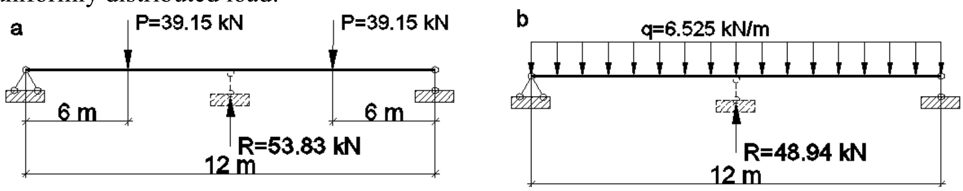

Fig.4. Schemes of the structures under study with a load in the span: a - concentrated forces; b- distributed load

As the dynamic masses, the weight of the beam (21 kg / m) is taken into account, in the places of action of the concentrated forces of mass 3915 kg, for the distributed load the distributed masses 652.5-21 = 631.5 kg / m are taken into account. For circuits with load in the spans, the frequency and period of the first form of natural oscillations were: with a concentrated load of 0.794 Hz and 1.260 seconds, with a distributed load of 0.812 Hz and 1.232 seconds. The following time intervals are considered for which the reference reactions decrease, sec .: 0.05, 0.1, 0.14, 0.4, 0.7, 1.0, 1.260 (concentrated forces), 1.232 (distributed load), 2.0, 3.0. When calculating beams with load in the span, the external load does not change with time, the reference reaction at the removal of the middle support decreases for the above-mentioned time intervals to zero.

Table 2 shows the forces and coefficients of dynamism in a double-span beam loaded in spans, with damage to the middle support. Efforts from the static load with concentrated forces in the span are equal: the maximum bending moment *M*=39.15*3=117.45 kN·m, the maximum lateral force *Q*=39.15 kN. With a uniformly distributed load, static forces: *M*=6.525*12^2/8=117.45 kN·m, *Q*=6.525*12/2=39.15 kN.

Table 2. Efforts and coefficients of dynamism in a double-span beam under the action of load in the span

Parameter	Efforts and coefficients of dynamism at the time of reaction decrease								
	0.05	0.10	0.14	0.40	0.70	1.00	1.260 (1.232)	2.00	3.00
Concentrated forces in flight									
M, kN·m	217.97	217.38	216.07	200.93	170.39	137.05	123.39	132.90	128.57
k_d	1.86	1.85	1.84	1.71	1.45	1.17	1.05	1.13	1.09
Q, kN	72.57	72.30	71.94	66.91	56.84	45.57	41.13	44.28	42.85
k_d	1.85	1.85	1.84	1.71	1.45	1.16	1.05	1.13	1.09
Evenly distributed load									
M, kN·m	221.81	219.99	219.80	204.40	173.69	139.26	122.87	133.86	128.62
k_d	1.89	1.87	1.87	1.74	1.48	1.19	1.05	1.14	1.10
Q, kN	66.14	65.92	65.64	61.58	53.63	44.70	40.42	43.28	41.91
k_d	1.69	1.68	1.68	1.57	1.37	1.14	1.03	1.11	1.07

With a decrease in the time of damage to the middle support, the dynamic coefficient increases. The maximum coefficient of dynamism under the action of concentrated forces in the middle of the spans, obtained numerically, differs from the theoretical one by 6% (theoretical value $k_d = 1.972$ more numerical).

The operation of continuous three-span beams is considered in case of damage to one of the middle supports (Fig.5). The removed support is marked in the figures with a dotted line. The value of the support reaction of the removed support is 1.1*6.525*6=43.07 kN.

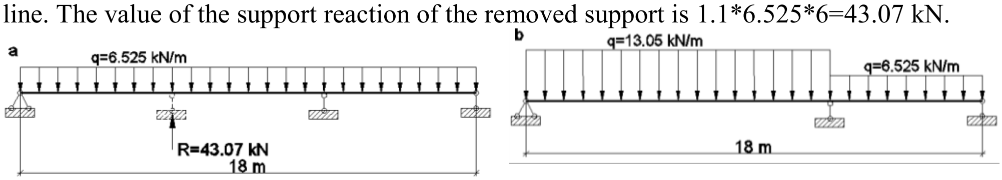

Fig.5. Calculation schemes for the destruction of the internal support of the three-span beam: a- for the calculation in a dynamic setting; b- for calculation in a static setting with k_d=2

In a three-span continuous beam, the maximum bending moment occurs in the region of the remote support. The dynamic moment is 141.92 kN·m, on and 159.05 kN·m According to static calculation with a dynamic coefficient 2. The maximum reference reaction occurs in an undamaged middle support and is 113.5 kN (dynamic calculation) and 139.5 kN (static calculation). Bending moments and support reactions in the three span beam, obtained by dynamic calculation are less, obtained by static calculation for 11-22%.

4 Conclusions

Depending on the initial state of a continuous two-span beam, the theoretical dynamic coefficients vary in the range from 1 to 3. The possibility of numerical calculation of damaged structures in a dynamic setting using the Nastran computer complex has been developed, and a numerical calculation method for the action of concentrated and distributed load has been worked out. The discrepancy between the theoretical coefficients of dynamism and the coefficients determined by the numerical calculation is 4-12%. For the constructions considered, the shorter the time of destruction of the middle support, the greater the coefficient of dynamism. If the average support is damaged during a time equal to the period of the first waveform, there is no significant increase in effort in the structure, while the dynamic coefficients are 1.02 - 1.06.

Taking into account the carried out researches for structures in the form of cut and continuous beams, in the absence of initial gaps or bends, it is recommended to use in the calculations in the static setting the load with the dynamic coefficient equal to 2 in the spans adjacent to the damaged support.

For simple beam structures (in the form of individual beams, girders, consisting of main and secondary beams), in the load capacity reserve it is allowed to perform a static calculation in the design using loads with the corresponding dynamic coefficients.

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