Numerical study of the applicability of the Saint-Venant principle

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Abstract. The limits of the applicability of the Saint-Venant principle are investigated while using the LIRA-CAD app. A large number of bars loaded at the end with different loads are considered. A graph of the distribution of the zones of influence of the Saint-Venant principle is plotted, depending on the size of the bars. It can be concluded that the stress-strain state near the perturbation zones can be represented by three components, two of which are linear, and the third is self-balanced in force and moment.

1 Introduction

One formulation of the Saint-Venant principle allows us to assume that the stress-strain state of the bar depends on the mode of load applications only near the load placement location [1]. In the resistance of materials, it is usually believed that the "perturbations" of the stress-strain state decay approximately at a distance of one transverse dimension of the bar. Calculations were carried out with various grids on the LIRA-CAD software complex.

2 Research

In calculation No.1 (fig. 1, a), a rigidly clamped bar with dimensions of 800x100x10 cm, loaded with a central tensile force $F = 100$ kN, was considered. In calculation No. 2 (fig.1, b) two forces $F/2 = 50$ kN were applied along the edges of the free end of the bar.

In the calculation of No. 3, a central compressive force $F = 1000$ kN (fig.3) was applied to the end of the bar.

The criterion for determining the zone where the results of the numerical calculation coincide with the results obtained from the material resistance formulas was the area where the discrepancy reached no more than 5%.

Fig. 3 demonstrates the diagrams of normal stresses over the five sections indicated in fig. 2. Irregular distribution of stresses decays with distance from the place where the load is applied (from section 1 to section 4 in 20 cm spacing). An error of 5% with the results obtained from the material resistance formula $\sigma = F/A$ is indicated for the section 5 at a distance of 92 cm from the application point. In this case, cross sectional dimension was 100 cm.

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Fig. 1. Results of calculations No.1 (a) and No.2. (b). Distribution of normal stresses.

Fig. 2. To calculation No.3.

In the calculation No. 4 (Fig. 4), the same bar was tested for bending to determine the zone on which the formula $\sigma = (M/J)y$ is observed. The bending moment was formed by a pair of forces $F = 1000$ kN of the opposite direction.

From fig. 5 it follows that the form of the normal stress diagram tends to a linear function when the load is displaced from the place of application of the load at a distance of half the cross-section of the beam (section 4).

About a hundred numerical experiments on these loads were held, and the ratio of the transverse dimension of the beam to its length and the dimensions of the grid changed. The criterion for the matching of the results was the 5% discrepancy deviation barrier. For such types of loading, the perturbation of the stressed state did cease at a distance not exceeding one transverse dimension.

Fig. 3. Diagrams of normal stresses (to calculation No. 3)

Fig. 4. To calculation No. 4

Fig. 5. Diagrams of normal stresses along sections 1 - 4 to calculation No. 4.

- Additionally, the bars were calculated for the following loads:
- a). Two opposing horizontal forces applied at a distance 20 cm at the free end (fig. 6, a).
 - b). One vertical force at the free end (fig. 6, b).
 - c). Two vertical forces at the free end (fig. 6, c).
 - d). Eccentric force at the free end (fig. 6, d).

Fig 6. Calculation schemes and the results of experiments a, b, c, d.

Fig. 7 shows normal-stress diagrams in five sections, obtained from the calculation for eccentric tension (fig. 6, d).

Fig. 7. Diagrams of normal stresses in five sections for calculation of eccentric tension

3 Conclusions

Based on the results of the research, the following schedule was constructed.

Fig. 8. Dependency graph of the implementation of the principle of Saint-Venant with a 5% discrepancy to the ratio of linear dimensions.

The graph (fig. 8) indicates: a – length of the disturbance zone, h – transverse dimension, l – length of the bar.

Loading:

- 1 - Central extension by force F (fig.1, a).
- 2 - Tension by two forces $F/2$ (fig. 1, b).
- 3 - Bending by a pair of forces F of the opposite direction (fig. 4).
- 4 - Bending by two forces $F/2$ (fig.6, c).

In all numerical experiments, the problem of studying the nature of the change in the stress-strain diagram along different sections near the end of the bar was also posed. The stress state near the places of application of loads is usually ignored in engineering calculations. However, it is possible to refine the state of stress. Near the place where the load is applied, the diagram of normal stresses can be represented as a sum of three components:

1. $\sigma = \text{const}$, the area of which is equal to the normal force:

$$N = \int_A \sigma dA;$$

2. $\sigma = ay$ – linear function corresponding to the condition:

$$M = \int_A \sigma y dA;$$

3. A self-balanced function for which the following conditions are valid:

$$N = \int_A \sigma dA = 0;$$

$$M = \int_A \sigma y dA = 0.$$

Studies in this direction can be continued to create a library of self-balanced functions corresponding to different loads, which will allow them to be superimposed on a linear diagram of normal stresses in order to clarify the stress state in places close to the points of application of loads.

References

1. V. Andreev, A. Paushkin, A. Leontiev, *Technical mechanics*, 18 (2013)