The influence of self-stress on the behavior of tensegrity-like real structure

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Abstract. Full-scale tensegrity skeleton of White Rhino is considered in this paper. The influence of self-stress state on structure behaviour is studied. Few models that consist of 15, 16, 17 or 18 elements are analysed. For the models self-stress states and infinitesimal mechanisms are considered. The impact of the level of self-stress state on displacement is investigated. Moreover a failure of White Rhino is considered. The influence of a damage of one, two or three cables on displacements is examined. Analyses are performed using the second order theory in the *Mathematica* environment and the third order theory using the *Sofistic* program.

1 Introduction

The term tensegrity concerns complex systems which consist of elements which are only in compression (struts) or tension (cables) [1]. Cables are foldable, light and strong. Thanks to these properties tensegrity structures gain the potential to be light but as well as deployable. There are many benefits of tensegrities for example tension stabilization, efficient structures development, deployability and easily tunable properties [2]. That are the main reasons of wide applications of tensegrity structures is appropriate in various areas of civil engineering [3]. Due to construction tensegrities can be clasiffied as linear systems (1D) like towers, sufrace systems (2D) like plates or shalls and single modules (3D). The example of the last one is building which is covered with membrane roofs supported by two tensegrity skeletons called White Rhino [4, 5].

White Rhino has been constructed at Chiba in Japan in June, 2001. The name of a structure, comes from the exterior appearance of the roofs. The membrane roof is white colour and it is supported by two tensegrity trusses varying in height which looks like two "horns" (Fig. 1a). The height of the bigger tensegrity structure is about ten meters, the smaller is seven meters in high. Both skeletons are modified simplex trusses (Fig. 1b). The simplex is three dimensional truss consists of the three struts and nine cables. This truss is the simplest example of tensegrity. It has one infinitesimal mechanism which is stabilized by self-stress state. The constructors of White Rhino added seven additional members – six

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cables and one strut. Three additional strings there are between unconnected six points of Simplex. These members do not change the appearance of the tensegrity but they improve much the rigidity of the structure. An isolated strut which is connected with truss by three another cables are a kind of support for the roof. These elements absorb large deformation of membrane roof and transmits the force from membrane roof to the tensegrity skeleton. Moreover White Rhino's general shape differs from Simplex model. It is rather trapezoidal than prismatic. The upper triangle is smaller than the bottom triangle.

a)

b)

Fig. 1. White Rhino: a) exterior appearance, b) view inside [4].

Extended Simplex tensegrity skeleton of White Rhino is considered in this paper. According to the paper [4] real geometry and material characteristics are taking into account. The influence of damage of some cables on behaviour of the truss is discussed. Additionally, the impact of the level of self-stress on displacements in the structure are analysed.

Analyses were performed using the second order theory and the third order theory. In the first case calculations were made in the *Mathematica* environment – the computational program based on the finite element method was written. In the second case calculations were made using the program *Sofistik* that served to accomplish fully geometrical non-linear analysis.

2 Formulation of the problem

The aim of this paper is study behaviour of structure stabilized by cables tension. To do this infinitesimal mechanisms and self-stress states should be determined and next the influence of the level of self-stress on displacements should be analysed. Identification infinitesimal mechanisms and a self-stresses in structures is possible by using the second order theory. In order to take account the effect of additional compression the third order theory should be used.

2.1 Mathematical formulation

The equilibrium equation for geometrically nonlinear model (third order theory) is presented in the form:

$$\left(\mathbf{K}_{L} + \mathbf{K}_{NL}^{I} + \mathbf{K}_{NL}^{II}\right)\mathbf{q} = \mathbf{P} \tag{1}$$

where **q** is displacement vector and **P** is the load vector, $\mathbf{K}_L$ is the linear stiffness matrix and $\mathbf{K}^I_{NL}$ is the pre-stress stiffness matrix called the geometric stiffness matrix and $\mathbf{K}^{II}_{NL}$ is the initial strain matrix. For trusses the linear stiffness matrix can be write as:

$$\mathbf{K}_L = \mathbf{B}^T\mathbf{E}\mathbf{B} \tag{2}$$

where **B** is the compatibility matrix and **E** is the elasticity matrix.

Based on singular value decomposition of compatibility matrix **B** infinitesimal mechanisms and self-stress states are identified [6, 7]. Next the pre-stress stiffness matrix depended on the self-stress state is calculated and spectral analysis taking into account this matrix is used. Geometric non-linear analysis, including the initial strain matrix, is made using the program *Sofistic*.

2.2 Model of the White Rhino

A modified model of the full-scale extended Simplex tensegrity skeleton of the White Rhino is considered. This model consists of 18 elements. An isolated strut which transmits the force from membrane roof to the tensegrity skeleton is omitted. This rod is modelled as a load. Several models of the structure are analysed.

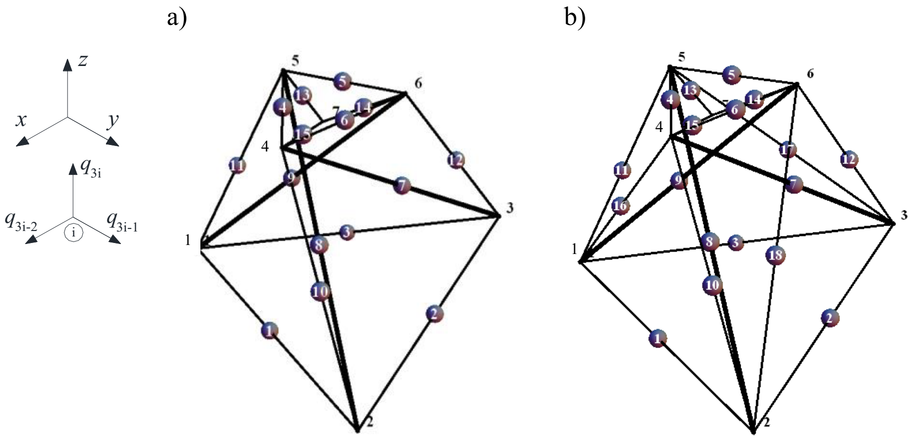

Fig. 2. Scheme of the White Rhino: a) model I, b) model IV.

Table 1. Node coordinate of the White Rhino model.

No. of node	*x* [m]	*y* [m]	*z* [m]
1	-15.283	0.000	3.000
2	-8.868	-3.748	4.000
3	-8.868	3.748	4.000
4	-11.805	- 1.709	8.796
5	-13.267	0.854	8.568
6	-10.343	0.854	9.020
7	-11.805	0.000	8.300

The first examined model (model I) consists of the 15 elements (Fig. 2a). It is a modified Simplex truss by added three cables (13, 14 and 15), which carry load from membrane roof. The next models are built by adding additional cables. Model II consists of the 16 elements (16th cable was added). Model III consists of the 17 elements (16th and 17th cables were added) and last – model IV consists of the 18 elements (16th, 17th and 18th cables were added) (Fig. 2b). The structure is supported in nodes 1, 2, 3 (six degrees of freedom are locked, in the direction of displacements: q_1, q_2, q_4, q_6, q_8, q_9). For all models self-stress states and infinitesimal mechanisms are defined. For the model I and the model IV the impact of the level of self-stress state on displacements is investigated.

Table 2. Member properties of steal (Young's modulus E = 210GPa).

No. of elements	Kind of elements	Diameter [m]	Sectional area [m^2]	Length [m]	Yielding stress [MPa]	Load capacity [kN]
1, 2, 3	cables	0.032	$8.04 \cdot 10^{-4}$	7.497	460	369.8
4, 5, 6	cables	0.028	$6.16 \cdot 10^{-4}$	2.959	460	283.4
7, 8, 9	struts	0.216	$5.36 \cdot 10^{-3}$	7.836	320	-938.0
10, 11, 12	cables	0.032	$8.04 \cdot 10^{-4}$	5.982	460	369.8
13, 14, 15	cables	0.025	$4.91 \cdot 10^{-4}$	1.840	460	225.9
16, 17, 18	cables	0.025	$4.91 \cdot 10^{-4}$	6.972	460	225.9

In the next step of the analysis the failure of the White Rhino is considered. The impact of a damage of one, two or three cables of (1, 2, 3, 4, 5, 6, 10, 11, 12, 16, 17, 18) on displacement q_{10} is examined (Table 4,5).

Real geometry (Table 1) and material characteristics (Table 2) are taking into account according to the paper [1]. Concentrated force P_{21} at the node 7, which is a simulation of the load from the roof, is considered. Since the model is nonlinear three values of this force are taking into account: P_{21}=-20 kN, P_{21}=-60 kN and P_{21}=-100 kN.

3 Results of the analysis

3.1 Identification of self-stress states and infinitesimal mechanisms

For all models self-stress states (Tab. 3) and infinitesimal mechanisms are defined. For the model I and the model IV the impact of the level of self-stress state on displacements for different values of external load is presented in Fig. 3-5.

Analysing the first model one infinitesimal mechanism which is stabilised by one identified self-stress state (SS) is determined. One additional cable (model II) causes that the structure loses some of features which are typical for tensegrities. There is no mechanism identified but there is still one self-stresses state (SS) the same like in the model I. In the case of the model III no mechanisms are identified but there are two self-stress states (1th SS, 2th SS – in the Tab. 3). These states need to be considered together. Superposition and normalisation of self-stress states are performed and the self-stress state (SS – in the Tab. 3) is obtained which is equal to self-stresses identified in previous models. The analysis of the model IV shows that there are no mechanisms, like previously, but there are three self-stress states (1th SS, 2th SS, 3th SS). In this case a superposition and a normalisation of three states allow to gain the one self-stress state (SS). This self-stress state is the same like in previous models. Further analyses are performed for this self-stress state.

Dependence of self-stress factor *S* on displacement q_{10} for the model I and model IV is shown in Fig. 3, 4 and 5. Three values of the external force are analysed: P_{21}=-20 kN (Fig. 3) P_{21}=-60 kN (Fig. 4) and P_{21}=-100 kN (Fig. 5). Values of the self-stress factor *S* are limited by load capacity of struts.

Table 3. Self-stress states [*S*].

No. of element	Model I	Model II	Model III			Model IV			
	SS	**SS**	1 th SS	2 th SS	**SS**	1 th SS	2 th SS	3 th SS	**SS**
1	0.22	0.22	-0,09	0,08	0.22	-0,17	0,03	0,08	0.22
2	0.22	0.22	-0,09	0,08	0.22	0,10	-0,02	0,16	0.22
3	0.22	0.22	-0,09	-0,17	0.22	-0,02	-0,18	-0,05	0.22
4	0.55	0.55	-0,23	0,42	0.55	-0,10	0,16	0,44	0.55
5	0.55	0.55	-0,23	-0,21	0.55	-0,41	-0,24	-0,09	0.55
6	0.55	0.55	-0,23	-0,21	0.55	0,28	-0,37	0,12	0.55
7	-1.00	-1.00	0,42	0,01	-1.00	-0,27	0,35	-0,41	-1.00
8	-1.00	-1.00	0,42	-0,38	-1.00	0,37	-0,06	-0,48	-1.00
9	-1.00	-1.00	0,41	0,39	-1.00	0,32	0,51	0,04	-1.00
10	0.76	0.76	-0,32	0,29	0.76	0,03	-0,02	0,46	0.76
11	0.76	0.76	-0,32	0,00	0.76	-0,42	-0,14	0,12	0.76
12	0.76	0.76	-0,31	-0,30	0.76	0,07	-0,45	0,07	0.76
13	0.00	0.00	0.152	0.152	0.00	0.00	0.00	0.00	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	-	0.00	0,00	-0,34	0.00	0,20	-0,29	-0,17	0.00
17	-	-	0,00	0,34	0.00	0,16	0,22	0,28	0.00
18	-	-	-	-	-	-0,37	0,07	-0,11	0.00

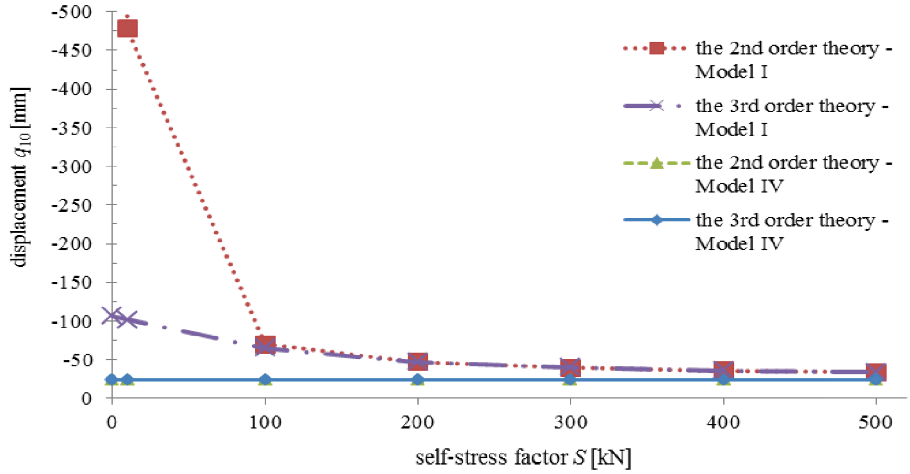

Fig. 3. Values of displacement q_{10} for a nodal load P_{21}=-20kN.

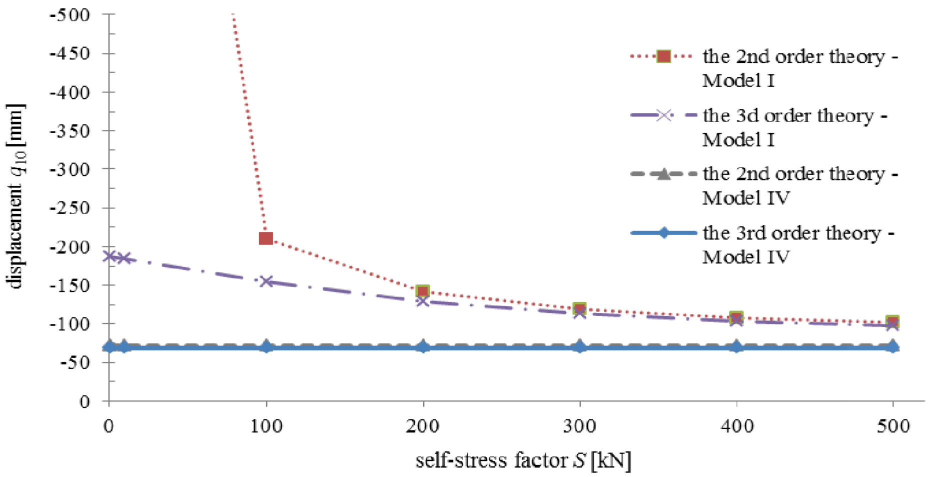

Fig. 4. Values of displacement q_{10} for a nodal load P_{21}=-60kN.

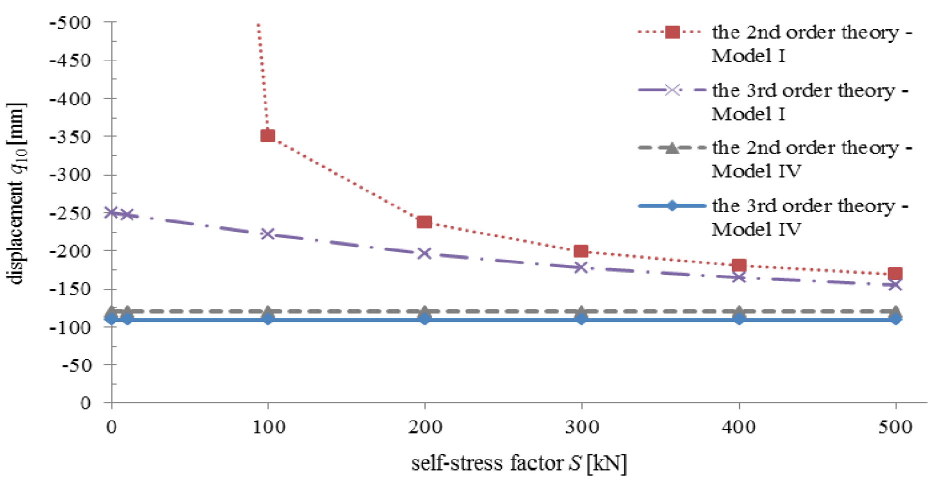

Fig. 5. Values of displacement q_{10} for a nodal load P_{21}=-100kN.

The analysis of selected displacement in case of damage of some elements are presented in the Tab. 4 for the structure supported with six d.o.f. locked on the support. The displacement is relatively large, then the analysis with nine d.o.f. locked is performed and presented in the Tab. 5. The displacement is significantly smaller in the second model of damage.

Table 4. Values of displacement q_{10} [mm] in dependence on the self-stress factor S for a nodal load P_{21}=-60kN (six degrees of freedom are locked).

The number of damaged elements	**No. of damaged elements**	**The self-stress factor *S***	
		S=0 kN	S=500 kN
1	1 or 2 or 3	-586.273	-835.759
	4 or 5 or 6	-72.494	-61.959
	10 or 11 or 12	-69.263	-83.482
	16 or 17 or 18	-68.996	-68.933
2	1,2 or 2,3 or 1,3	-665.817	-762.567
	4,5 or 5,6 or 4,6	-76.850	-42.715
	10,11 or 11,12 or 10,12	-73.081	-52.496
	16,17 or 17,18 or 16,18	-68.946	-68.897
3	1, 2, 3	-663.711	-741.911
	4, 5, 6	-75.974	-66.571
	10, 11, 12	-75.580	-71.866
	16, 17, 18	-187.673	-97.552

Table 5. Values of displacement q_{10} [mm] in dependence on the self-stress factor S for a nodal load P_{21}=-60kN (nine degrees of freedom are locked).

The number of damaged elements	**No. of damaged elements**	**The factor of self-stress *S***	
		S=0 kN	S=500 kN
3	1, 2, 3	-0.772	-0.768
	4, 5, 6	-1.400	4.057
	10, 11, 12	-1.151	-8.731
	16, 17, 18	-109.210	-26.341

4 Conclusions

The influence of self-stress on the behaviour of the tensegrity-like structure is studied in this paper. The full-scale tensegrity skeleton of White Rhino is considered. Four models which consist of 15, 16, 17 or 18 elements are analysed.

For the model I, the influence of the self-stress factor on nodal displacements is very noticeable. Nodal displacements decrease significantly with increase of internal forces of self-stress as well as of the influence of geometrical nonlinearity is clearly seen. If the value of self-stress equals zero, displacements tend to the infinity. An adjusting of prestressing forces allows to control displacements of nodes.

Additionally the analysis shows that the effect of nonlinearity is the most significant at low values of self-stress forces. With increasing of self-stress the differences in the values of displacements calculated according to the second and third order theory are getting smaller.

In the case of the model IV three extra cables cause that the structure lost some of features which are typical for tensegrities. There are no mechanisms identified but there are three self-stresses states which is just one both in the case of the original Simplex and in the model I. Elimination of the mechanism makes the skeleton very rigid. The model IV is much less sensitive for changes of the self-stress factor.

Moreover, comparing obtained graphs (Fig. 3-5) it can be noticed that nodal displacements increase nonlinearly with the increase of external load.

In the analysis of damage of White Rhino, the failure of one, two and three cables is considered. Lower cables (1,2,3), upper cables (4,5,6), diagonal cables (10,11,12) and additional cables (16,17,18) are analysed (Tab. 4). The analyses performed reveal that the destruction of one, two or even three cables from (4,5,6,10,11,12,16,17,18) does not damage the whole structure. In such cases, the higher factor of self-stress improves a reliability slightly.

However, the failure of one of the lower cables (1,2,3) would be equivalent to the damage of the truss. In case of such the damage, the pre-stress of the elements additionally increases nodal displacements. The values of displacements are relatively large in case of the failure lower tendons what was the reason that the model supported by nine ties was analysed (Tab. 5). Safety of the structure increases significantly in this case. That is why it is a recommended way to support the structure.

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