

Limitation of Stresses in Concrete According to Eurocode 2

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Abstract. When stresses in concrete in reinforced concrete beams with the reinforcement ratio being high due to the ultimate limit state are calculated – according to the linear elasticity theory – from strains of a characteristic combination or even a quasi-permanent one, in some cases compressive stresses in extreme concrete fibres can be even higher than the characteristic compressive strength of concrete. In the following report, it is postulated to calculate these stresses with the help of the non-linear stress-strain relationship for concrete. It has also been demonstrated that the requirements specified in points 7.2(1)P and 7.2(2) of the standard *PN-EN 1992-1-1:2008/NA:2010 Eurocode 2:Design of concrete structures. Part 1-1: General rules and rules for buildings*, are useless and do not suit the limit state analysis, on which Eurocodes are based.

1 Introduction

Eurocode 2 [1] comprises regulations – not completely unambiguous ones – regarding stresses in concrete. Point 7.2(1)P of the norm [1] states ‘the compressive stress in the concrete shall be limited in order to avoid longitudinal cracks, micro-cracks or high levels of creep, where they could result in unacceptable effects on the function of the structure’. Point 7.2(2) further specifies that ‘longitudinal cracks can appear if the stress level under a characteristic combination of loads exceeds a critical value’. Such cracks can deteriorate the durability of a structure. If no other measures are taken, e.g. an increase in the cover of the reinforcement in the compressive zone or confinement of the concrete by transverse reinforcement, then it may be appropriate to limit compressive stress to a value $k_1 f_{ck}$ in areas exposed to the influence of environments of exposure classes XD, XF and XS. The recommended value is $k_1 = 0.6$, but the norm [1] allows the specification of another value of this factor in a National Annex. The Polish National Annex NA.5 assumes that $k_1 = 1.0$.

As noted by M. Knauff in [3], the above regulation is made ambiguous by the phrase: ‘it may be appropriate (does it mean mandatory?) to limit the compressive stresses to a value $k_1 f_{ck}$ ’. It is also uncertain whether an increase in the thickness of the cover up to the one required because of more unfavourable environments, such as XD, XF and XS, can be equated with the increase which abolishes the obligation to monitor compressive stresses in concrete.

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Before Eurocode 2 [1] was adopted in Poland, when designing reinforced concrete structures according to the Polish standard PN-B-03264:1984 [2], it was necessary to check the ultimate limit state (ULT) as well as bendings and widths of cracks caused by quasi-permanent loadings. Meanwhile, compressive stresses in concrete caused by a characteristic combination of loads were not monitored. This approach did not lead to any damage to a designed structure.

2 Calculation of Compressive Stresses in Compressed Concrete

As in [4], stresses in concrete for non-linear structural analysis were calculated as recommended by Eurocode 2 [1]. By employing the stress-strain relation in concrete (3a – 3c), given in point 3.1.5 of Eurocode 2 and shown in fig. 1, as well as the values illustrated in fig.2, two equilibrium equations in a cross-section being in the serviceability state can be written, i.e. the sum of projection forces onto the axis of a beam (1) and the sum of bending moments of the cross-section relative to the resultant force of the concrete's compression zone (2).

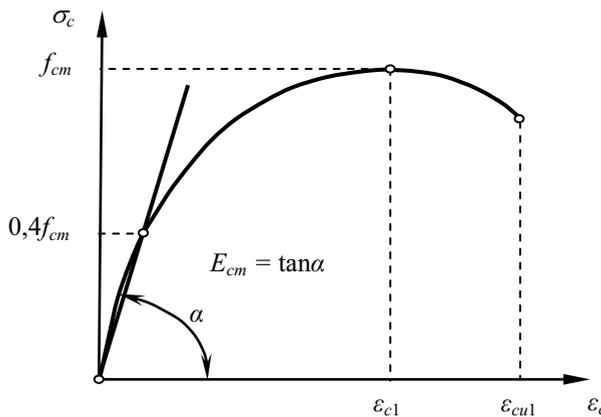


Fig. 1. Schematic representation of the stress-strain relation in concrete for structural analysis

$$A_{s1} \sigma_{s1} = P_c = \int_0^x \sigma_c(\varepsilon_c) b dr \quad (1)$$

$$A_{s1} \sigma_{s1} (d - x + r_0) = M, \quad (2)$$

where:

$$\sigma_c(\varepsilon_c) = f_{cm} \left[\frac{k\eta - \eta^2}{1 + (k-2)\eta} \right] \quad (3a)$$

$$k = 1,05 E_{cm} \frac{\varepsilon_{c1}}{f_{cm}} \quad (3b)$$

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}}, \tag{3c}$$

$$r_0 = \frac{\int_0^x \sigma_c(\varepsilon_c) b r dr}{\int_0^x \sigma_c(\varepsilon_c) b dr}, \tag{4}$$

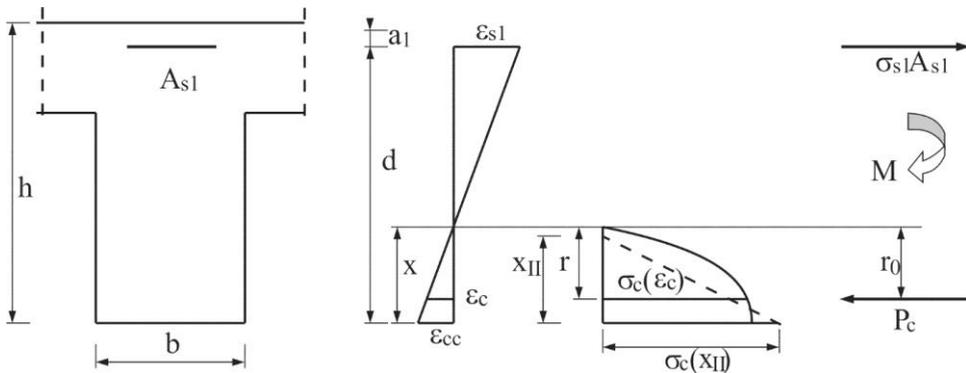


Fig. 2. Stress-strain diagram in a bending cross-section in the serviceability state. For comparison, the picture shows the location of the cross-section centre of gravity in phase II - x_{II} and the diagram of strains in concrete determined according to the linear theory - $\sigma_c(x_{II})$.

In line with the Bernoulli's hypothesis regarding flat cross-sections, the following are calculated from the relationships between strains:

$$\varepsilon_c = \varepsilon_{cc} \frac{r}{x}, \quad \varepsilon_{s1} = \varepsilon_{cc} \frac{d-x}{x}, \tag{5}$$

And afterwards, stresses σ_{s1} in reinforcement A_{s1} are derived:

$$\sigma_{s1} = E_s \varepsilon_{s1}. \tag{6}$$

By substituting expressions (3) ÷ (6) to (1) and (2), we obtain a set of two equations with two unknowns x and ε_{cc} . The problem can be solved numerically (integration leads to the formulation of implicit equations). In a programme written in the Fortran language, for a given $\varepsilon_{cc} \in (0, \varepsilon_{c1})$ that changes by $0.001\varepsilon_{c1}$ at a step, the subsequent values $x \in (0, 0.5d)$ changing by $0.5d/1000$ at a step are assumed. Then, it is checked whether the left side of equation (1) is equal to the right side, and when this agreement is sufficient, the appropriateness of the achieved values x and ε_{cc} is verified with equation (2). If the verification fails, the programme resumes the search (at the values of ε_{cc} and x at which it stopped) for a new pair of x and ε_{cc} , again solving equation (1), then verifying whether a new pair of values satisfies equation (2). Integers present in expression (4) are calculated by the programme with the Simpson method. Having derived the value of ε_{cc} from equations (1) and (2), equations (3a – 3c) are used to determine the stress in the extreme compressive fibre of the cross-section $\sigma_c(\varepsilon_c = \varepsilon_{cc})$.

3 Examples

The solid reinforced concrete flooring shown in fig. 3, of the thickness 0.2 m and the span 5.1 m, rests on the main T cross-section beam, which is a two-span continuous beam of the span length equal 8.1 m. The cross-section of the main beam: height $h=0.70$ m, width of the web of the T beam $b=0.35$ m, distance from the reinforcement centre of gravity to the tensile edge $a_1=0.05$ m; material: concrete C16/20, the secant modulus of elasticity of concrete $E_{cm} = 29$ GPa, design yield strength of reinforcement $f_{yd} = 350$ MPa, the modulus of elasticity of reinforcing steel $E_s = 200$ GPa. The live action of the warehouse equals 4.0 kN/m², the value of the quasi-permanent coefficient of variable action is $\psi_2 = 0.8$. The following were determined in the main beam above the support: design value of the applied bending moment $M_{Ed} = 591.6$ kNm and the cross-sectional area of reinforcement $A_{s1} = 34.00$ cm². The planned reinforcement consisted of $7\phi 25$ with $A_{s1} = 34.96$ cm², which yields the reinforcement ratio $\rho_L = A_{s1}/(bd) = 0.0154$.

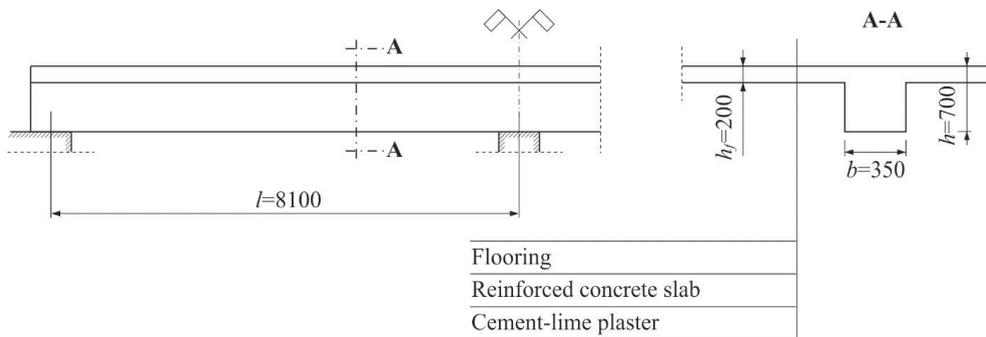


Fig. 3. Cross-sections of the floor

The moment from quasi-permanent actions is $M = 430.65$ kNm. Having solved a set of equations (1) and (2), the following were determined: $x = 0.2535$ m and $\varepsilon_{cc} = 0.0007011$, and when ε_{cc} was substituted to equations (3a – 3c) the value $\sigma_c = 15.70$ MPa was obtained.

It is worth underlining that calculations of widths of cracks of strains in compressed concrete and tensile steel at A_{s1} – in the cracked cross-section (being in phase II) - are traditionally made having assumed the linear elasticity theory (e.g. [5]). However, in cross-sections exposed to very heavy actions, stresses in concrete are impossible to be calculated based on the assumption of their linear distribution, as this would generate absurd results, i.e. not only higher than the ones derived from the set of equations (1) and (2), but also, and not infrequently, even exceeding the characteristic compressive strength of concrete, as it is illustrated underneath – treating actions as short-term ones (immediately after the application of the moment $M = 430.65$ kNm), for which $\alpha_e = E_s/E_{c,eff} = 200/29 = 6.90$.

$$x_{II} = \frac{\alpha_e}{b} [-A_{s1} + \sqrt{(A_{s1})^2 + \frac{2b}{\alpha_e} A_{s1} d}] = 0.238m \quad (7)$$

$$I_{II} = \frac{bx_{II}^3}{3} + \alpha_e A_{s1} (d - x_{II})^2 = 0.00567m^4 \quad (8)$$

$$\sigma_c = \frac{M}{I_{II}} x_{II} = 18.10MPa > f_{ck} = 16MPa \quad (9)$$

An example of the difference in the distribution of stresses in the compression zone of concrete σ_c calculated from the set of equations (1) and (2) and in agreement with the linear theory is shown in figure 2.

While proceeding with further calculations, it was assumed that RH = 50% and $h_0 = 233$ mm from [3] (as it is simpler and more accurate than according to [1]), after which the creep coefficient was calculated: $\varphi(\infty, 28) = 3.04$, and next, the following were calculated at $\sigma_c = 15.70$ MPa: $\varphi_{nl}(\infty, 28) = 3.04 \cdot 2.218 = 6.74$ (from equation (3.7) from [1]), $E_{c,eff} = 3.74$ GPa (equation (7.20) from [1]), $\alpha_{e,t} = E_s/E_{c,eff} = 200/3.74 = 53.41$, as well as:

$$x_{II} = \frac{\alpha_{e,t}}{b} [-A_{s1} + \sqrt{(A_{s1})^2 + \frac{2b}{\alpha_{e,t}} A_{s1} d}] = 0.4556m \quad (10)$$

$$\sigma_{s1} = \frac{M}{(d - \frac{x_{II}}{3}) A_{s1}} = 247.3MPa \quad (11)$$

The above calculations – from equations (10) and (11) – of stresses in tensile reinforcement σ_{s1} in the elasticity range are sufficiently accurate because – having accounted for the creep - the depth of the compression zone x_{II} increased significantly, i.e. from 0.238 m to 0.4556 m.

Next, according to [1] in fig. 7.1 and from equations (7.10), (7.9), (7.11) and (7.8), the following were calculated: $h_{c,ef} = 0.0814$ m, $\rho_{p,eff} = 0.1226$, $(\varepsilon_{sm} - \varepsilon_{cm}) = 0.00118$, $s_{r,max} = 162$ mm and $w_k = 0.191$ mm.

In the analysed cross-section, the moment of the characteristic combination was $M = 464.11$ kNm. The stress calculated in line with the procedure described in point 2 was $\sigma_c = 16.58$ MPa, which is higher than $f_{ck} = 16.0$ MPa! Should we calculate these stresses in the elasticity range, as soon as the actions are applied ($\alpha_e = 6.90$), an even higher excess of the characteristic compressive strength of concrete would be achieved, which is shown below ($x_{II} = 0.238$ m, $I_{II} = 0.00567$ m⁴):

$$\sigma_c = \frac{M}{I_{II}} x_{II} = 19.48MPa > f_{ck} = 16MPa \quad (12)$$

If the concrete in the above example is C30/37 concrete, for which $E_{cm} = 32$ GPa, then the required reinforcement with the same steel will be $A_{s1} = 29.39$ cm². Assuming that it is $6\phi 25$ with $A_{s1} = 29.45$ cm², the lower reinforcement ratio is achieved: $\rho_L = 0.0129$.

For the moment from quasi-permanent actions equal $M = 430.65$ kNm, the following were determined from equations (1) and (2) as well as (3): $x = 0.21905$ m, $\varepsilon_{cc} = 0.000649$ and $\sigma_c = 18.80$ MPa.

When calculating the stresses from the above moment that appear immediately after its application (and considering them to be short-lived ones, for which $\alpha_e = 200/32 = 6.25$), the following values were obtained from equations (7), (8) and (9): $x_{II} = 0.2141$ m, $I_{II} = 0.00464$ m⁴ and $\sigma_c = 19.86$ MPa, which is higher than $\sigma_c = 18.80$ MPa.

In the subsequent calculations, it was assumed as before that RH = 50% and at $h_0 = 233$ mm, hence the creep coefficient was calculated from [3] as equal $\varphi(\infty, 28) = 2.33$, and after that the following were calculated for $\sigma_c = 18.80$ MPa: $\varphi_{nl}(\infty, 28) = 2.33 \cdot 1.304 = 3.04$, $E_{c,eff} = 7.92$ GPa and $\alpha_{e,t} = E_s/E_{c,eff} = 25.25$. Equations (10) and (11) yielded: $x_{II} = 0.3544$ m and $\sigma_{s1} = 274.9$ MPa.

Next, according to [1] in fig. 7.1 and from equations (7.10), (7.9), (7.11) and (7.8), the following were determined: $h_{c,ef} = 0.1152$ m, $\rho_{p,eff} = 0.073$, $(\varepsilon_{sm} - \varepsilon_{cm}) = 0.00126$, $s_{r,max} = 186$ mm and $w_k = 0.234$ mm.

For the moment from the characteristic combination $M = 464.11$ kNm, using the procedure described in point 2, the calculated stress was $\sigma_c = 19,91$ MPa, which in this case does not exceed $f_{ck} = 32.0$ MPa. If these stresses are calculated in the elasticity range, for $\alpha_e = 6.25$, $x_{II} = 0.2141$ m, $I_{II} = 0.00464$ m⁴, then, naturally, higher values will be obtained from equation (12), i.e. $\sigma_c = 21.42$ MPa.

4 Conclusions

The above examples suggest that sometimes in a cross-section that fulfils the ultimate limit state and has cracks not exceeding the limit value compressive stresses in concrete caused by a characteristic combination exceed the characteristic compressive strength of concrete f_{ck} . The ambiguously worded requirement in point 7.2(2) [1] – ‘it may be appropriate to limit the compressive stresses to a value $k_1 f_{ck}$ ’ – does not resolve the dilemma whether a given cross-section satisfies the requirements set in Eurocode 2 [1]. The above statement seems to be confounding and it can be claimed to be useless. Limiting stresses down to a level below the limit stresses would be more suitable for the method of designing allowable stresses rather than for the ultimate limit analysis, on which Eurocodes are based.

The excess stresses demonstrated in the discussed example arise solely from certain imperfections of the calculation models, which we have at our disposal. Limiting these stresses to the value in agreement with the original version of Eurocode 2 would be even more absurd, because than the limit stress in the first example would equal merely:

$$\sigma_c = k_1 f_{ck} = 0.6 \cdot 16.0 = 9.6 \text{ MPa},$$

and the width of cracks – in line with Eurocode 2 – is calculated for stresses even equal to f_{ck} (in our case $f_{ck} = 16$ MPa), and these are stresses triggered by a combination from quasi-permanent loads, which causes smaller generalised internal strengths and stresses in a cross-section than a characteristic combination. Also, in the second case – if the original version of Eurocode 2 was binding in Poland – stresses in concrete $\sigma_c = 19.91$ MPa would be above the recommended value:

$$\sigma_c = k_1 f_{ck} = 0.6 \cdot 30.0 = 18.0 \text{ MPa}.$$

References

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