The estimation of transition curves geometry in railway engineering from measured data

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Abstract. The position of the track is defined as a course of the track in its ground plan. [1] In the simplest way the course of track can be defined as an axis, which is constructed not only from straight parts, but from the arcs and transit curves as well. In consideration of the normative changes it is necessary to change the point of view on measuring of track geometry parameters and hand in hand to adjust the way of processing and the results interpretation.

1 The current status of the geodesy in the railway engineering

Through track geometry measurements, which are defining the course of the track, some normative changes enter into force. From 1st July 2015 there is in force norm STN 73 6360-1: *Railway applications. Track. Part 1: Geometrical position arrangement of 1 435mm gauge railways* and norm 73 6360-2 *Railway applications. Track. Part 2: Acceptance of construction works, maintenance works and assessment of service condition track gauges 1 435mm*, which replaced norm STN 73 6360. These norms are interested in the requirements necessary for the designing, the construction, the reconstruction and the modernization of the normal track gauge with speed to 300km/h, than the technical parameters of the structural and the geometrical setting of the track, the rail branches and their spatial position, the taking over of the construction and the maintenance works. Through the track condition assessing is needed to check the geometrical position of track by the geodetic instrument with the continual recording. There are measured the position coordinates of track axis and the elevation of the non-overridden tracks roof. In special, reasonable occasions it is possible to control the position by conventional geodetic method. In other reasonable occasions it is possible to check track gauge and overriding by manual measuring method without continual recording. [2] The usage of these suppositions needs the technology of measurement and calculations to be adjusted depending on the design, the distance and the location of the object. [3] The possibility of the movements' estimation is the analysis of measurement as the parametrical defined geometrical parts. These can be after the measurement of displacement and deformation stages compared between stages. The biggest benefit of this method is closely total elimination of errors depending from centration and signalisation of points. In railway engineering, there are not expected discreet deformations, on the other hand deformations of whole parts are.

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2 The characteristic of measured area

Hand in hand with the trans-European corridors modernisation, Railways of Slovak republic realized the reconstruction of track Nové Mesto nad Váhom- Púchov, where in one part- Turecký vrch, first time ever in Slovakia was designed ballastless track system RHEDA 2000®. [3] The whole length of Turecký vrch tunnel in the axis is 1775 m, the length of tunnels tube is 1738.5 m plus 25m continuing part from south side and 10m continuing part from the North. The double rail track in the tunnel is designed for speed 200km/h with opposite arcs with radius 2000 m. [4] The measurement was done not only in the part of ballastless track system but on the north side of the tunnel as well. In the transition part was first time used new type of construction in total length of 20 m that is using standard components of the rail superstructure without its´ stabilisation. [5]

3 The estimation of the track positioning from the measured data

The easiest is to estimate the parameters of the straight part. For the most precise interpretation of rails´ course is sufficient to fold the regression line through measured data (it is better to estimate the rail axis but it is possible to estimate one of the track as well). This regression line can be described algebraically by linear equation [6]

$$y = ax + b. \tag{1}$$

Through the estimation of arc parameters are estimated coordinates of the arc centre and its´ radius. In Cartesian coordinate system with the arc centre S [Y_0, X_0] and radius R is the set of points defined by linear equation

$$(Y - Y_0)^2 + (X - X_0)^2 = R^2. \tag{2}$$

3.1 The comparison of measured data with the different types of transition curves

The troubles through estimation rise because of its parametrical definition. The basic functions of transition curves are:

- a smooth transition between the straight section and the circular arc, the curvature gradually changes from zero to the final value;
- provide space for smooth change when creating or changing an override;
- ensure a smooth curvature change in complex arcs or between opposite arcs.

The transition curve can be only the curves that meet these conditions:

- one end of the curve has a cross-section with a downstream straight line;
- the other end of the curve has a common cross-section with a downstream arc;
- the curvature of the transition point at the point following the circular arc have to be the same as the curvature of the circular arc;
- the curvature of the transition point at the point following the straight section have to be equal to zero;
- the course of the change of the curvature over the entire length of the transition period corresponds to the course of the override change.

3.1.1 Parabola

A parabola is a two-dimensional mirror-symmetrical curve. The parabola is described as a lotus of points that are equidistant from both a directrix (line) and a focus (point). The focus does not lie on the directrix.

The parabola can be algebraically described by equation

$$y = ax^2 + bx + c\,. \tag{4}$$

The comparing calculation was done by the least squares method it is necessary to create design matrix, which contains from partial derivatives of the *a, b, c* parameters with total length n x r (n- number of measured points, r- number of unknown parameters), the weight matrix could be neglected, every point was measured with same precision. From the results of the calculation, parameter a=0.000508 b= -0.866551and c= 0.102152. The mean error m_0 calculated from equation

$$m_0 = \sqrt{\frac{v^T v}{n-k}}\,, \tag{5}$$

where k means the number of needed measures (3) and v means the vector of residuals. The value of m_0 is the best comparative criterion. The size of parabolas´ mean error is 0,147m.

3.1.1 Klothoide

Whereas a Klothoide is forbidden for Railway Engineering in Slovakia, it is used in every other country in European Union but it is often used worldwide. The klothoide is also known as the curve with linear increase of curvature. From the Figure 1 it is possible to see that the growing in axis x and y are marked by dx and dy:

$$dx = dl.\cos\tau\,, \tag{6}$$

$$dy = dl.\sin\tau\,. \tag{7}$$

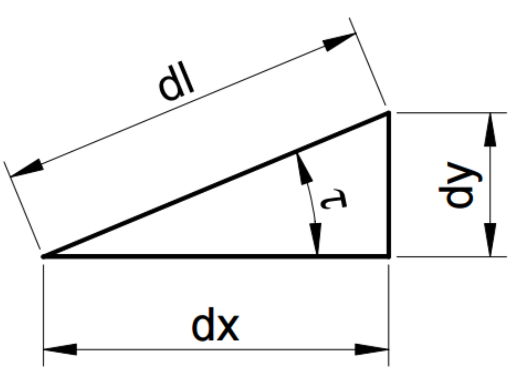

Fig. 1. The growing of x and y coordinates in klothoide

$$\tau = \frac{l^2}{2.L_K R} \tag{8}$$

The l means the distance from the start of transition curve, LK is the distance of whole curve and R is the radius of klothoide curvature with is equal to radius of arc, which transition curve is connected to.

To possible use of the least square method it is necessary to derivate equations (6) and (7) so these functions must be linearized by Taylor series.

$$\begin{aligned} dx &= dl.\cos\tau = dl.(1 - \frac{\tau^2}{2!} + \frac{\tau^4}{4!} - \frac{\tau^6}{6!} + ...) = \\ &= dl.(1 - \frac{l^4}{8.L_K^2 R^2} + \frac{l^8}{384.L_K^4 R^4} - \frac{l^{12}}{46080.L_K^6 R^6} + ...) \end{aligned} \tag{9}$$

$$\begin{aligned} dy &= dl.\sin\tau = dl.(\tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \frac{\tau^7}{7!} + ...) = \\ &= dl.(\frac{l^2}{2.L_K R} - \frac{l^6}{48.L_K^3 R^3} + \frac{l^{10}}{3840.L_K^5 R^5} - \frac{l^{14}}{645120.L_K^7 R^7} + ...) \end{aligned} \tag{10}$$

After the integration by the distance l we become equations:

$$\begin{aligned} dx &= \int (1 - \frac{\tau^2}{2!} + \frac{\tau^4}{4!} - \frac{\tau^6}{6!} + ...)dl = \\ &= (l - \frac{l^5}{8.L_K^2 R^2} + \frac{l^9}{384.L_K^4 R^4} - \frac{l^{13}}{46080.L_K^6 R^6} + ...) \end{aligned} \tag{11}$$

$$\begin{aligned} dy &= \int (\tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \frac{\tau^7}{7!} + ...)dl = \\ &= (\frac{l^3}{2.L_K R} - \frac{l^7}{48.L_K^3 R^3} + \frac{l^{11}}{3840.L_K^5 R^5} - \frac{l^{15}}{645120.L_K^7 R^7} + ...) \end{aligned} \tag{12}$$

After the adjustment by the least square method it is necessary to astimate the values, l_i is distance of every point from the beginning, other valuea ase L_K- the lenght of whole curvature and the radius of curvature R. The number of unknown is equal to number of meausred points n+2. The number of equation that are used in method is 2.n, every coordinate has it own equation.

The shape of desing matrix is

$$A_{2n,n+2} = \begin{vmatrix} \frac{\partial f(x_1)}{\partial R} & \frac{\partial f(x_1)}{\partial L_K} & \frac{\partial f(x_1)}{\partial l_1} & \frac{\partial f(x_1)}{\partial l_2} & \cdots & \frac{\partial f(x_1)}{\partial l_n} \\ \frac{\partial f(y_1)}{\partial R} & \frac{\partial f(y_1)}{\partial L_K} & \frac{\partial f(y_1)}{\partial l_1} & \frac{\partial f(y_1)}{\partial l_2} & \cdots & \frac{\partial f(y_1)}{\partial l_n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial f(x_n)}{\partial R} & \cdots & & & & \frac{\partial f(x_n)}{\partial l_n} \\ \frac{\partial f(y_n)}{\partial R} & \cdots & & & & \frac{\partial f(y_n)}{\partial l_n} \end{vmatrix} =$$

$$= \begin{vmatrix} A_{1,1} & A_{1,2} & A_{1,3} & \cdots & A_{1,n+2} \\ A_{2,1} & A_{2,2} & A_{2,3} & \cdots & A_{2,n+2} \\ \vdots & \vdots & \vdots & & \vdots \\ A_{2n-1,1} & \cdots & & & A_{2n-1,n+2} \\ A_{2n,1} & \cdots & & & A_{2n,n+2} \end{vmatrix}, \qquad (13)$$

To simplify the calculation it was not needed to calculate with more than first three units of Taylor series because of their low impact to calculations presented by thousandths of millimetre. Through calculations were the values of R, L_K and l_i estimated and there was only calculation of the supplements. The values were R_0=1997m, L_{KO}=450m and l_{i0} was calculated from equation $l_{i_0} = \sqrt{x_i^2 + y_i^2}$.

The final values of R=1997.362m and L_K=432.233m. The equations of coordinate calculations look like:

$$y = \frac{l^3}{5{,}179954.10^6} - \frac{l^7}{3{,}088626.10^{19}} + \frac{l^{11}}{2{,}302050.10^{31}}, \qquad (20)$$

$$x = l - \frac{l^5}{2{,}9981326.10^{13}} + \frac{l^9}{1{,}919873.10^{27}}. \qquad (21)$$

The size of klothoides´ mean error is 0,220m.

Conclusion

After the legislative changes in present age, it is necessary to change the access of measurement adjustment. It is needed not only to change the methods of measurement from discrete to continual but to change the method of adjustment as well. One possibility is to divide the track into the parts with same directional guidance (line, transition curve and arc) and calculate their parameters. These can be compared between epochs and show better conclusion without impact of pointing error etc.

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