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Layered structures mechanical properties assessment by dynamic tests

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Abstract. An effective method of the layered structures mechanical properties determination is presented. The method is based on the non-stationary analysis of the structure response that is acted by impact load on the surface of multi-layered structure. The assessment of the design parameters using a genetic algorithm is performed. During the solving procedure there are several methods for breeding and selection processes are used, that demonstrated different results. The most appropriate parameters for described algorithms are selected and analysed.

1 Introduction

Determination of the mechanical properties of multi-layered structures for various purposes is one of the most important tasks at any stage of their design and operation. In the construction field there are many examples of such structures include coated materials, sandwich panels, ground base construction of roads and others. Depending on the object the methods to determine the structures properties are different. These are indentation methods that used for non-destructive tests metal structures [1,2], determination of the rheological parameters for the creep models [3,4,5], identification of the structures properties [6]. There are many engineering applications in road construction field; methods of analysis of the dispersion properties of surface waves (MASW, SASW), used for soil and foundations.

A semi-analytical approach to the solution of the axisymmetric indentation problem for a multi-layered elastic half-space is presented in [7]. The solution is based on the Hankel transform and numerical computations of the solution are implemented for multi-layered composites.

Assessment of material characteristics using spherical indentation test is described in [8]. Three parameters of the LUDWIG's equation using the spherical indentation test and Neural Networks is determined. A Neural Networks is trained following the spherical indentation test using two parameters that are obtained from the *P-h* curve.

The indentation of a layer by a rigid circular cylinder is considered in [9]. The Green's functions for the indentation of elastic layer lying on or bonded to a rigid base by a line load are found by analytical solution. The peculiarities of this solution under various boundary conditions are considered.

A steady-state dynamic analysis of a response multi-layered transversely isotropic (TI)

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half-space generated by a point load moving along a horizontal straight line with constant speed is investigated in [10]. The solution is obtained using the double inverse Fourier transform with respect to the frequency and one horizontal wavenumber.

There are several ways to determine the mechanical characteristics of layered materials by pressing indenters of various forms. The choice of the shape of the indenter in this case is an important task [11,12]. Most common in modern practice are indenters in the form of a sphere [13,14], cone [15,16], pyramid, but they have one significant drawback – the complexity of the registration of the beginning of plastic flow

In all cases, the problem of the physical parameters determination of the structure uses the solution of the problem of dynamic impact on the area layer type or multi-layered half-space.

2 Problem statement

We consider the action of the impact load that specified subjected in the limited time interval on the surface of a layered viscoelastic half-space. The properties of the medium will determine a set of parameters: - elastic modulus E_i , - Poisson ratio ν_i , - density ρ_i , $tg\alpha_i$, $tg\beta_i$ - the tangents of the angles of the losses for longitudinal and transverse waves.

Because of a sufficiently fast decay time signal due to the spatial scattering of the energy and physical dissipation we use the method of discrete harmonic analysis to find the displacement field of the structure points in the form of a finite set of solutions to the problems of stationary oscillations

$$\mathbf{u}(\mathbf{r},t)=\sum_{k=1}^{N} p_k\left[-\cos\eta_k \operatorname{Im}(\mathbf{U}(\mathbf{r},\omega_k)\exp(-i\omega_k t))+\sin\eta_k \operatorname{Re}(\mathbf{U}(\mathbf{r},\omega_k)\exp(-i\omega_k t))\right], \tag{1}$$

where T- limit time observation, t^* - the decay time of the signal (it is possible to take $t^* = T$);

p_k - the expansion coefficients of the impulse impact in a Fourier series on an interval $\lfloor -T/2, T/2+t^* \rfloor$ with a specified degree of accuracy ε

$$p^{(N)}(t)=\sum_{k=1}^{N} p_k\chi_k(t), \quad \left\| p(t)-p^{(N)}(t) \right\|_{L_2} <\varepsilon$$

$$\chi_k(t) = \sin(\omega_k t + \eta_k);$$

$$\omega_k = k\pi / (T + t^*);$$

$$\eta_k = 0.5 \cdot k\pi \cdot T / (T + t^*).$$

In accordance with this, the displacement field in a layered viscoelastic medium $\mathbf{U}(\mathbf{r},\omega)$ is sought separately for each component of the layered structure under stationary steady state with the frequency of the oscillations ω. In particular for a half-space using the integral transform of the Hankel function $\mathbf{U}(\mathbf{r},\omega)$ can be represented in the form

$$\mathbf{U}(\mathbf{r},\omega) = \int_0^\infty \mathbf{P}(u,z)\cdot \bar{\mathbf{X}}(u) J_k(uR)u du\,, \tag{2}$$

where in cylindrical coordinates $\mathbf{r} = (R,z), \quad R \ge 0, \quad z \le 0$, $\mathbf{X}(R)$ - load distribution on the boundary of half-space and the upper layers,

$$\bar{\mathbf{X}}(u) = \int_0^{+\infty} \mathbf{X}(R)\, J_k(uR) R\, dR\,, \qquad \mathbf{P}(u,z) = \mathbf{U}(u,z) \cdot \mathbf{B}^{-1}(u,0)$$

$$\mathbf{U} = \begin{pmatrix} -u\exp(\sigma_{11}z) & -\sigma_{12}\exp(\sigma_{12}z) \\ \sigma_{11}\exp(\sigma_{11}z) & u\exp(\sigma_{12}z) \end{pmatrix};$$

$$\mathbf{B} = \begin{pmatrix} \varsigma_2^2\exp(\sigma_{11}z) & 2u\sigma_{12}\exp(\sigma_{12}z) \\ -2u\sigma_{11}\exp(\sigma_{11}z) & -\varsigma_2^2\exp(\sigma_{12}z) \end{pmatrix},\ \varsigma_2^2 = u^2 + \sigma_{12}^2,\ \sigma_{1k} = \sqrt{u^2 - \theta_{1k}^2},\ k = 1,2\,; \tag{3}$$

$$\mathbf{B}^{-1}(u,0) = \begin{pmatrix} \zeta_2^2 & 2u\sigma_{21} \\ -2u\sigma_{11} & -\zeta_2^2 \end{pmatrix} \Delta_R^{-1} \equiv \mathbf{B}(u,0)\,\Delta_R^{-1}\,,\ \Delta_R = \zeta_2^4 - 4u^2\sigma_{11}\sigma_{12}\,;$$

$$\theta_{j1}^2 = \omega^2 a^2 / V_{Pj}^2, \qquad \theta_{j2}^2 = \omega^2 a^2 / V_{Sj}^2,$$

- the relative frequency of oscillations j-th component of the medium from the velocities of propagation of longitudinal and transverse waves V_{Pj}, V_{Sj} (for half-space $j = 1$);
a - the radius of the area distribution of surface impact efforts.

It's important that the expression (3) have only exponentially decreasing components (if $z < 0$).

Considering further the j-th layer , assume that on its sides is set to the vectors of stress

$$\mathbf{t}^{(j)}(R,0) = \mathbf{Y}^{(j,1)}(R), \quad \mathbf{t}^{(j)}(R,h_j) = \mathbf{Y}^{(j,2)}(R), \quad R \in (0,+\infty)) \tag{4}$$

The function $\mathbf{U}(\mathbf{r},\omega)$ that satisfies the equations of motion, according to the proposed method, we will seek as

$$\mathbf{U}^{(j,1)}(\mathbf{r},\omega) + \mathbf{U}^{(j,2)}(\mathbf{r},\omega)$$

Here are the components $\mathbf{U}^{(j,n)}(\mathbf{r},\omega),\ \ n = 1,2$ of this representation are solutions of the Lame equations for a homogeneous half-space with satisfaction of the boundary conditions in stresses

$$\mathbf{t}^{(j,1)}(R,0) = \mu_j\,\mathbf{X}^{(j,1)}(R) \text{ for the first half-space } z > 0,$$

$$\mathbf{t}^{(j,2)}(R,h_j) = \mu_j\,\mathbf{X}^{(j,2)}(R) \text{ for second } z < h_j\ .$$

The vectors of displacements $\mathbf{U}^{(j,n)}(\mathbf{r},\omega),\ \ n = 1,2$ and stresses $\mathbf{t}^{(j,n)}(\mathbf{r})$ can be obtained from their submission in the transform of Fourier-Bessel

$$\begin{aligned} \mathbf{U}^{(j,n)}(\mathbf{r},\omega) &= \int_{\Gamma_+} du u J_k(uR)\ \mathbf{P}^{(j,n)}(u,z) \cdot \bar{\mathbf{X}}^{(j,n)}(u) \\ \mathbf{t}^{(j,n)}(\mathbf{r}) &= \int_{\Gamma_+} du u J_k(uR)\ \mathbf{B}^{(j,n)}(u,z) \cdot \bar{\mathbf{X}}^{(j,n)}(u) \end{aligned} \tag{5}$$

For matrix $\mathbf{P},\mathbf{Q}$ the relations are

$$\mathbf{P}^{(j,2)}(u,z) = \mathbf{P}(u,z-h_j)\,,\ \ \mathbf{B}^{(j,2)}(u,z) = \mathbf{B}(u,z-h_j)$$

$$P_{lm}^{(j,1)}(u,z) = P_{lm}^{(1)}(u,-z)(-1)^{\delta_{lm}}\,,\ \ Q_{lm}^{(j,1)}(u,z) = Q_{lm}^{(1)}(u,-z)(-1)^{\delta_{lm}+1}$$

$l,m = 1,2$, δ_{lm} -Kronecker symbol, with the replacement of the half-space parameters by the parameters of the corresponding j -th layer.

Functions $\bar{\mathbf{X}}^{(j,n)}(u)$ in representation (5) are unknown. Their definition is carried out from the boundary conditions of joining the layers, as a rule, requiring the continuity of displacement and stress vectors when passing through interfaces, and the conditions on the surface of the structure. As a result, in the space of Hankel transformations we obtain a system of linear algebraic equations. The matrix of the obtained system has a block diagonal form that is well conditioned. Its determinant $\Delta(u)$ for $u \to +\infty$, $\Delta(u) = \det \mathbf{A}(u) \sim C(E_j, \nu_j) = const$, which ensures the numerical stability of the inversion of the system of equations for arbitrary u. The monotonicity of matrix representations $\mathbf{P}$ and $\mathbf{B}$ also makes it possible to effectively apply the proposed algorithm for constructing wave fields using parallel computations.

3 Solving method

As an example, the implementation of the model we consider the problem of determining the elasticity moduli of the layers of the road structure, consisting of a layer of asphalt, layers of foundation and soil subgrade. The remaining design parameters are assumed to be specified and correspond to the design values.

To assess the moduli of elasticity of the structural elements of the pavement required the comparison of characteristics in time or frequency region obtained from the model and similar experimental characteristics. This task refers to the number of inverse coefficient problems. The realization of the numerical solution of the direct problem in the form (1)-(2) significantly restricts the possibility of using basic methods of solving inverse problems based on analytical approaches.

The result of calculating frequency characteristics of the surface of the multilayer structure (with a short punch small binding energy) shows that the change in the variation of the elastic moduli of the system under study in their real change is relatively small for structures with decreasing elastic modulus with depth and significantly only in the presence of propagating surface waves. In our case, the most sensitive characteristic is the change of the maximum amplitude of vertical displacement at removal of surface design from the point of impact – "the Cup of the maximum dynamic deflections", obtained numerically. This feature can also be obtained on the basis of field measurements, the installation on the surface of the structure group of sensors at different distances from the point of impact.

In the general case the solution of the resulting inverse problem is reduced to a functional equation in the implicit form to determine the elastic moduli of the pavement layers and subgrade

$$W_z\left(R, E_j\right) = \varphi\left(R\right), \tag{6}$$

where $W_z\left(R, E_j\right) = \max_t u_z\left((R,z),t\right)_{z=H}$ - the estimated bowl of the maximum dynamic deflections (obtained numerically for given values of the parameters) on the surface $z = H$; $\varphi\left(R\right)$ - given the shape of the bowl (in the General case is obtained by processing experimental data).

It should be noted that to solve the system of equations (6) at fixed observation points $R = R_k$ the different methods were used: methods of the coordinate and the gradient descent method of conjugate gradients. However, in many cases due to the monotonicity of the influence of parameters investigated E_j on the value of the maximum dynamic deflection (any increase E_j leads to a drop deflection) the solution of system (6) is unstable. As a result, along with the possibility of regularization of the solution of the system in this study used a classical genetic algorithm.

The sequence of actions for solving the inverse problem of identifying properties of pavement structures is the following.

1. Filling of chromosomes the initial population with random values which correspond to values of moduli of elasticity for each layer

$$\mathbf{E}^p = \left\{E_1^p, ..., E_n^p\right\} \quad , \quad E_j^p \in \mathbf{X}, \tag{7}$$

where p is the number of population, n is the population size, $\mathbf{X}$ is physically valid set of design parameters.

2. Calculation of fitness of a population

The adjustment of the population is determined by analysing the values obtained with the help of the fitness function (fitness). The function of fitness you need to pass the values of maximum vertical deflections at the points of observation that chromosome is determined by sequential calculation of amplitude-frequency characteristics amplitude-time characteristics, the magnitude of the dynamic deflection bowls. After these steps will the calculated values of displacements of layers of pavement structures that are passed as arguments to the function adaptation based on the method of least squares

$$S = \sum_{k=1}^{K}\left(W(R_k, E_j^p) - \varphi(R_k)\right)^2, \tag{8}$$

where K – the number of control sensors;

$\varphi(R_k)$ – maximum time of observation of the vertical deflection of the surface at the k point obtained in the course of the experiment;

$W(R_k, E_j^p)$ - values of deflections of the surface of the structure at the k point obtained in the course of the genetic algorithm.

b. If the adjustment of one chromosome satisfies the accuracy of the solution, the genetic algorithm ends and output are the values of the elastic moduli of each layer of pavement structures.

If the fitness of the population does not satisfy the given accuracy of the solution, in this case, you should move on to creating a new generation.

b. The choice of the parents for the next generation.

Selection of parents is based on the tournament selection method or roulette method. The selected parents are then crossed to produce a new generation of chromosomes.

5. Crossing parents

Crossing parents is one of the methods of recombination.

b) Discrete recombination

During the discrete recombination is the exchange of genes (modules of elasticity) between the chromosomes of the parents.

b) Intermediate recombination

During the interim of recombination is determined by the numeric values of the genes of the descendants, which must contain the gene values of the parents. The descendants are generated by the following rule

$$E_1^2 = E_1^1 + \alpha\left(E_2^1 - E_1^1\right), \tag{9}$$

where multiplier α is a random number in the interval $[-d, 1+d]$, $d > 0$,the number d is chosen for each task separately, but it was found experimentally that the optimum value of $d = 0{,}25$.

c) Linear recombination

In the linear recombination, the descendants are created by the same rule used in the intermediate recombination, but the difference is that the multiplier α is chosen at random, and are pre-defined and fixed during the course of the algorithm

6. In the process of life of the population can occur in mutation a random gene (modulus of elasticity) in the chromosome. The mutation probability is specified as a parameter of genetic algorithm. The formula by which the mutation of the chromosome

$$E_j^p = E_j^p \pm \Delta, \tag{10}$$

where the amount of change of a gene chromosome , $\Delta \in R$.

After that, there is the repetition of the algorithm starting from step 2.

On the basis of the presented algorithm was implemented to select optimal parameters of the genetic algorithm.

During the field experiment used 5 sensors – accelerometers at the maximum distance from the point of application of the blow, equal to 2,1 m.

During a series of experimental calculations the comparison was subject to two selection methods - tournament method and the roulette method. The minimum error value in the method of roulette is made up of 0.03. The method of tournament selection showed stable, adequate result. Achieved the desired accuracy of 0,00005.

4 Results and discussion

During a series of experimental calculations when selecting method parameters crossing comparison was subject to three methods – discrete, intermediate and line recombination. In the method of crossing the "intermediate recombination" is used elitist selection, which is that in the new generation of selected chromosome with the best fitness from the previous generation. Further, this chromosome can participate in the crossing and to give their descendants. In the other two methods of crossing an elite selection led to inadequate results of the calculation, therefore, was not used in the calculations. To select the most appropriate method there were 30 test calculations by 10 calculations in each method. As a method of selection the "method of tournament selection" was used.

Best results were recorded for the method of intermediate recombination as accuracy and execution time of calculations. Discrete recombination was better than the linear recombination time calculations.

On the basis of experimental calculations, the following optimal parameters of the genetic algorithm:

- probability of mutation = 0.02,
- standard deviation of the normal distribution - 10,
- number of chromosomes - 30,
- the order of error of 0.0001,
- step enter random chromosome – 3

As an example of the application of the genetic algorithm for calculating the dynamic moduli of the highway layers elasticity, the results of experimental determination using the FWD dynamic deflection unit in the Rostov-on-Don-Taganrog section are considered. The measurements were carried out sequentially at 10 points of dynamic action. As a model, a three-layered half-space model with the specified design properties of the layers was selected (Table 1).

Table 1 Design value of the modulus of elasticity

Layer Number	Layer Material	Design value of the modulus of elasticity, MPa
1	Light loam	80
2	Crushed gravel	120
3	Asphalt-concrete porous coarse-grained	11030

Fig. 1-2 shows the results of restoration of the modulus of elasticity of the asphalt-concrete layer and the ground foundation using the software "RoSy DESIGN NG" and based on the genetic algorithm (GA). In both cases, fairly close results were obtained with the exception of point 3, which is special for both methods and removed from the figures. It can be seen that the genetic algorithm is more stable when restoring the elastic moduli of both the surface layer and the substrate. In this case, the calculation results using the software "RoSy DESIGN NG" for the ground are uniformly overestimated, which is explained by the difference in the mathematical models used in the calculations.

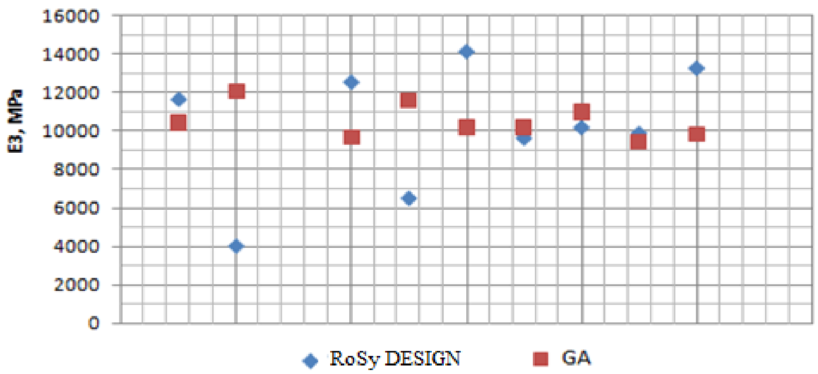

Fig. 1. Modulus E3 recovering

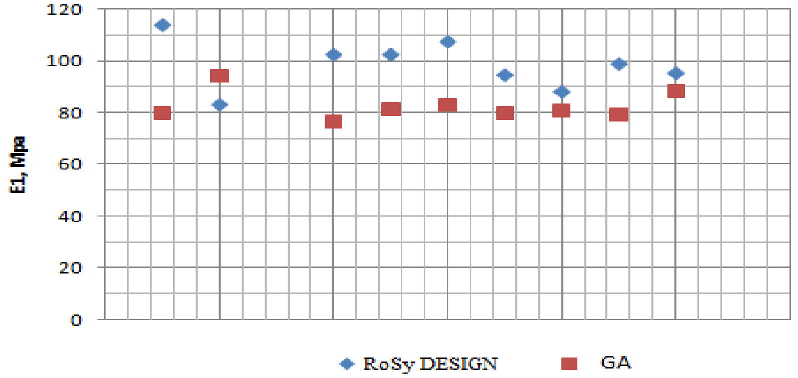

Fig. 2. Modulus E1 recovering

5 Conclusions

The developed approach with the use of the proposed mathematical model of the analysis of the dynamic behaviour of layered structures and the genetic algorithm for determining the properties of the medium on the basis of experimental data has shown good stability of restoring the design parameters.

Genetic algorithms can be used as refining (correcting) methods for solving inverse problems due to significant time costs for the calculation. We should note that the recovery time of the elasticity modules of a three-layer structure at a single measurement point using a genetic algorithm on a dual-core processor with a frequency of 2 GHz is from 2 to 5 minutes, which does not allow it to be used when operating FWD installations in real time.

A significant advantage of these algorithms is the increase in the number of parameters restored from the experiment, including not only the moduli of elasticity of the medium, but also the thickness of the layers.

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