The design of concrete elements strength under local compression based on the variational method in the plasticity theory

Volodymyr Pohribnyi^{1*}, *Oksana Dovzhenko*¹, *Leonid Karabash*¹, and *Iryna Usenko*¹

¹Poltava National Technical Yuri Kondratyuk University, Building Department, Pershotravneviy sq., 24, Poltava, 36011, Ukraine

Abstract. The two-dimensional concrete elements under the action of one-sided central crushing often occurs. For them, the strength problems are solved on the basis of the concrete ideal plasticity theory variational method with application of discontinuous velocities functions. The method takes into account the influence on concrete elements strength the compressive strength and tensile strength of concrete. It also takes the ratio of a sample height to the crushing platform length and friction coefficient between a surface of element and a load punch if it's necessary. The results of experimental researches were confirmed the assumed in the theoretical solution the kinematical schemes of elements failure. Higher convergence of theoretical strength with tested is received. The calculation sequence of the concrete element under one-sided central crushing is described. The joint calculation of all factors that determine the strength opens the possibility of more accurate assessment and improvement on its basis of structural solutions of concrete elements under the local compressive load.

1 Introduction

The concrete and reinforced concrete constructions and their elements under the action of a local compression are widely used (for example the joints of columns, the end faces of prestressed structures, the horizontal joints of panels of outside walls and floors, walls and supports of bridges in places of beams bearing, girders, lintels).

According to Eurocode 2, the crushing elements design is made by Baushinger's empirical relationship in which the ratio of the so-called "rated" area A_{cl} to the crushing area A_{c0} is assumed as the determining factor. However the value A_{cl} is conditional as there is no precise definition to find it.

The two-dimensional elements failure character under one-sided central crushing [1] confirms the influence on their strength of concrete axial tensile strength f_{ct} and ratio h/l_{loc} (h – a sample height, l_{loc} – the crushing platform length), not taken into account in Eurocode 2.

* Corresponding author: V.V.Pohribnyi@gmail.com

The reference strength design is not based on consideration of elements limiting stress-strain state (SSS) which is the cause of the above lacks.

Significant experimental material about concrete elements strength at local compression and various factors influence for strength now is saved, the numerous empirical dependences for determination of ultimate load (Table 1) are offered [2].

Table 1. The design dependence for determination of elements strength under one-sided central crushing.

№	The formula for determination φ_c	Authors
1	$\sqrt[3]{A_{cl}/A_{c0}}$	J. Baushinger (1876)
2	$4 - 3\sqrt{A_{cl}/A_{c0}}$	V. Chervonobaba (1959)
3	$1.2(2.3 + 5l_{loc}/l) / (\chi l_{loc}/h)$,	B. Gladyshev (1968)
4	$(2.75 + 6l_{loc}/l) / (\chi l_{loc}/h)$	B. Gladyshev, M. Dyuzhenko (1970)
5	$2.7\sqrt{A_{cl}/A_{c0}} / \sqrt{\chi}$	G. Tsyskrel (1970)
6	$(A_{cl}/A_{c0})^{-0.3424}$	I. Rohlin (1972)
7	$0.42\left(\frac{1}{l_{loc}/l} + 2\right) - 0.29\sqrt{\left(\frac{1}{l_{loc}/l - 1}\right)^2} + 5.06$	S. Niyogi (1973)
8	$0.2 + 0.8\sqrt{A_{cl}/A_{c0}}$	B. Jensen (1977)
9	$6.92(A_{cl}/A_{c0})^{-0.47} / \chi$	A. Williams (1979)
10	$\varphi_c = 12.5\sqrt{40/f_{c,prizm}}, \quad \text{if } A_{cl}/A_{c0} \leq 320$ $\varphi_c = 0.7\sqrt{A_{cl}/A_{c0}}, \quad \text{if } A_{cl}/A_{c0} > 320$	K. Lieberum, H Reinhardt (1989)
11	$1/[1 - 0.55(1 - A_{cl}/A_{c0})] \leq 1.7$	A. Griezic (1998)
12	$1 + 0.064(\sqrt{A_{cl}/A_{c0}} - 1)$	N. Rak (2002)

It is difficult to choice one method of all for practical use, not only due to different quantitative results of calculation (Fig. 1) and difference of the basic preconditions.

In our opinion it is necessary to prefer a design method, which would be based on a general basis. A variational method of the concrete ideal plasticity theory is developed in Poltava National Technical Yuriy Kondratyuk University [3] may be such basis.

2 Design assumptions and design sequence

For development of a method the following preconditions were used:

- for concrete in a destruction stage the assumption about ideal plasticity is applied. Determining for applicability of the specified precondition is the condition of simultaneous existence of a limiting condition in more fragile (tensed) and more plastic (compressed) of concrete destruction zones;

- G. Geniyev's concrete strength condition [4] is assumed and considered as a plastic potential;

- relationship of deformation velocities to stresses is found from the associated low of plastic yielding;

– the model of a rigid-plastic body and problems solution in discontinuous velocities functions are used.

Fig. 1. The value of coefficient φ_c in accordance with formulas of Table 1 (for the ratio $h/l_{loc} = 4$ and concrete C20/25).

The calculation is carried out in the following sequence:

– for the being considered case of element crushing the kinematical possible element destruction scheme is assumed, that is the outline of the destruction surface (gaps of velocities) dividing before save element into being considered absolutely rigid parts, making mutual movement with some velocities in a destruction stage, is set. Thus geometrical parameters g_i determining a destruction surface and movement velocities of an element parts V_j are introduced; assumed cinematically possible scheme of the element destruction reflects the specificity of SSS in a destruction stage of the being considered element or construction;

– on a destruction surface S_i there are gaps of a normal ΔV_n and tangent ΔV_t to S_i velocity components expressed by parameters g_i, V_j ;

– enters the functional of a principle of possible velocities speeds and stress [5], which on the valid SSS reaches to a minimum;

– limit load F_u expression is found through the parameters g_i and velocities ratio $k_j = V_j / V_F$, where V_F – velocity of a load application point;

– unknown parameters g_i and k_j are determined out of the condition of a limit load minimum. Then F_u is calculated.

Let's consider the calculation of the concrete element under one-sided central crushing (Fig. 2).

Cinematically possible destruction scheme includes disks 1 and disk 2, which move with velocities accordingly V_1 and $-V_1, V_2$. Disks are divided between themselves by the destruction surface (or surface of velocities gaps), which consists of parts A-B and B-C. On parts A-B the limit normal σ_u and tangent τ_u stresses act, which are determined according to the G. Geniyev's concrete strength condition. The area B-C is principal area with tensile stress $\sigma_u = f_{ct}$.

The being considered problem unknown quantities are the limit load F_u or resistance $f_{c,loc} = F_u / A_{c0}$, the angle γ (inclination angle of the destruction area A-B to a vertical) and velocities ratio $k = V_1 / V_2$.

Gaps of velocities normal and tangent components on the part A-B

$$\Delta V_n = V_1 \cos \gamma - V_2 \sin \gamma, \quad \Delta V_t = V_1 \sin \gamma + V_2 \cos \gamma. \quad (1)$$

Fig. 2. Cinematically possible destruction scheme of element loaded with strip bilateral central load.

The part's area A-B length is

$$S = l_{loc} / 2 \sin \gamma. \quad (2)$$

Accordingly on part B-C:

$$\Delta V_n = 2V_1, \quad \Delta V_t = 0. \quad (3)$$

Equal to zero the functional for the two-dimensional stressed state

$$I = m \left[\int_{S_f} 2B \left(1 + 0.25 \left(\frac{(V_1 \sin \gamma + V_2 \cos \gamma)}{(V_1 \cos \gamma - V_2 \sin \gamma)} \right)^2 \right)^{0.5} - 1 \right] \times \\ \times (V_1 \cos \gamma - V_2 \sin \gamma) l_{loc} / \sin \gamma + f_{ct} V_1 (h - l_{loc} / 2 \tan \gamma) - f_{c,loc} b l_{loc} V_2 = 0. \quad (4)$$

The formula for determining the resistance of local compression was obtained after simple mathematical transformations

$$f_{c,loc} / m = \left\{ \left[2B \left((k - \tan \gamma)^2 + 0.25 (k \tan \gamma + 1)^2 \right)^{0.5} - k + \tan \gamma \right] / \tan \gamma + \right. \\ \left. + f_{ct} k (2 \alpha \tan \gamma - 1) / \tan \gamma m \right\} / (1 - k f_{fric}), \quad (5)$$

where $B^2 = (1 + \chi / (1 - \chi)^2) / 3$, $\chi = f_{ct} / f_{c,prizm}$, $m = f_{c,prizm} - f_{ct}$, $\alpha = l_{loc} / h$, f_{fric} – the friction coefficient between a surface of element and a load surface.

To simplify the calculation, the charts are offered. They make it possible to determine ultimate load for definite α and χ (Fig. 3).

Fig. 3. The charts for determination of ultimate load for definite α and χ .

3 Experimental researches and convergence of theoretical strength with experimental

In a fig. 4 the comparison of experimental and theoretical strength is given at absence of friction forces between a surface of a sample and basic plates press [6-9].

Fig. 4. Comparison of theoretical and experimental strength at one-sided central crushing of a concrete plate: $\square$ – B. Gladyshev; $\circ$ – TsNIISK (Russia); $\blacklozenge$ – G. Shapiro; $\diamond$ – S. Krylov, L. Zaytsev, I. Ulbieva; $\blacktriangle$ – A. Piradov.

The experimental researches includes tests 44 samples - cubes with the size of an edge 150 and 200 mm and samples of prisms 200x200x800 mm from heavy concrete. The tests

were made on press PG-125. The load from a plate press was transferred through metal stamp.

The samples failure character, observable in experiments, has confirmed the kinematical possible failure schemes assumed in the theoretical solutions. At test of samples for one-sided crushing-splitting at stamps a shear cracks allocated wedges of condensation, on which ends a vertical tension crack is formed later.

The convergence analysis of theoretical strength F^{calc} with experimental F^{test} on size of the relation F^{calc} / F^{test} was carried out on the basis of data processing 36 samples, for which $h / l_{loc} = 1.5 - 6.7$, $f_{c,prism} = 1 - 30.0$ MPa and $f_{ct} = 0.4 - 2.39$ MPa. It has shown $\bar{x} = F^{calc} / F^{test} = 0.985$ with factor of a variation $\nu = 11.9\%$, that testifies to sufficient affinity of theoretical strength to experimental and serves a substantiation of applicability of a variational method of the ideal plasticity theory to the decision of the considered types of tasks.

4 Conclusion

The two-dimensional concrete elements strength problems under the action of crushing are solved on the basis of a variational method of the concrete theory ideal plasticity with application of discontinuous velocities functions. The received relationship is exacter in comparison with normative at the expense of the account to the stress-strain station in the case of one-sided central crushing. This account is carried out by means of kinematical scheme of failure displaying specificity of concrete task and introduction in account of a number of the factors: ratio h / l_{loc} , friction forces between the surface of a sample and weight surface etc.

The results of experimental researches confirmed the kinematical possible failure schemes assumed in the theoretical solution, and also influence of the factors, determining strength.

The theoretical strength of concrete elements at crushing, found on an offered method, well enough converges with tested.

References

1. D.H. Hladyshev, Resursoekonomni materialy, konstruktsiyi, budivli ta sporudy, Rovno, **29**, 94-101, (2014).
2. V.V. Tur, N.A. Rak, *Prochnost i deformatsii betona v raschetah konstruktsiy*, Brest, (2003).
3. V.P. Mitrofanov, Stroitel'naya mehanika i raschet sooruzheniy, Moskov, **6**, 23-28, (1990).
4. G.A. Geniev, V.N. Kissyuk, G.A. Tyupin, *Teoriya plastichnosti betona i zhelezobetona*, Moscow, (1974).
5. V.L. Kolmogorov, *Mehanika obrabotki metallov davleniem*, Moscow, (1986).
6. B.M. Gladyshev, *Mehanicheskoe vzaimodeystvie elementov struktury i prochnost betonov*, Kharkov, (1987).
7. S.M. Kryilov, L.N. Zaytsev, I.S. Ulbieva, *Novyye konstruktsii selskohozyaystvennykh zdaniy industrialnogo izgotovleniya i metody ih raschyota*, Moscow, **3**, 66-75, (1984).
8. B.P. Filipov, O.P. Vinokurov, *Beton i zhelezobeton*, Moscow, **1**, 42-43, (1983).
9. A.B. Piradov, *Konstruktivnyie svoystva legkogo betona i zhelezobetona*, Moscow, (1973).