Calculation methodology of reinforced concrete elements based on calculated resistance of reinforced concrete

Dmitriy Kochkarev [1*], and *Tatyana* Galinska[2]

[1]National University of Water and Environmental Engineering, Department Urban building and development, 33028 Rivne, Soborna St.,11, Ukraine
[2]Poltava National Technical Yuri Kondratyuk University, Department Architecture and urban Development, 36011 Poltava, Pershotravnevyi avenue, 24, Ukraine

Abstract. Calculation methodology of reinforced concrete elements based on the calculated resistance of reinforced concrete is presented. The basic dependence which allows setting the strength of bending sections and non-central compressed elements is obtained. The proposed method for calculating reinforced concrete elements is based on the use of nonlinear diagrams of material deformation, the hypothesis of flat sections and deformation criteria for the destruction of materials. The basic equations of strength are reduced to dimensionless quantities and are tabulated. When compiling the tables, the formula proposed in Eurocode 2 was adopted as the diagram of concrete deformation, and for the reinforcement two linear Prandtl diagram was used. The calculated formulas of the proposed method fully correspond to the formulas of the classical resistance of materials, and make it possible to solve the most frequently encountered problems in the practice of modern construction. The reliability of the dependencies is experimentally confirmed. There are calculation examples of bending and non-central compressed elements by the developed methodology.

1 Review of recent studies and publications

Design of reinforced concrete elements based on nonlinear dependencies are listed in the standards of many countries [1, 2, 3], has set a question about their practical calculation for most of the engineers. This is because the usage of the formulas of these rules without a computer [1, 2, 3] is practically impossible. It is generally accepted that any calculation by using computers should be checked or evaluated through classical and practical techniques. Creating such a technique would allow making an assessment, verification and analysis conducted computer calculations. Simplified and approximate methods [4, 5, 6, 7] not correspond to exact solution that is obtained by nonlinear dependencies in most of cases.

Consider the calculation of resistibility of sections of flexural reinforced concrete elements with a single reinforcement. Carrying force of these elements must be determined by following conditions:

* Corresponding author: dim7@ukr.net

1) achievement of extremum of function of carrying force $dM/d\varepsilon_c = 0$ in limits $\varepsilon_{c1} - \varepsilon_{cu}$ and occurrence of fluidity in the stretched reinforcement;
2) achievement of boundary deformation of concrete compressed brink ε_{cu} and limits of fluidity in the stretched reinforcement in the absence of extremum of function of carrying force;
3) occurrence of extremum of function of carrying force $dM/d\varepsilon_c = 0$ in limits $\varepsilon_{c1} - \varepsilon_{cu}$ without achievement fluidity of reinforcement;
4) achievement of boundary deformation of concrete ε_{cu} without achievement fluidity of reinforcement and in the absence of extremum of function of carrying force in limits $\varepsilon_{c1} - \varepsilon_{cu}$;
5) achievement deformations in the stretched reinforcement with value of ε_{cu}.

2 Basic material and results

Calculation of resistibility performed by using the equation of equilibrium of external forces and internal forces, deformation diagrams of concrete and reinforcement and the function of changes of deformation by height section. As a function of diagrams of concretes deformation should be taken like function of stresses in the concrete which would correspond to the conditions of nonlinear deformation of concrete. The following dependences are proposed for calculations of nonlinear structures in the standards [1]: formula of rules Eurocode 2 (3.4), fifth degree polynomial (3.5), two- and three a linear dependence between the stresses and deformations. The dependence of stress-deformation for reinforcement is taken as two linear Prandtl diagrams. The distribution of deformation by height section at the moment of the destruction taking in a linear dependence like:

$$\varepsilon = \frac{1}{r} x \ or \ x = \frac{\varepsilon}{1/r}, \ or \ \frac{1}{r} = \frac{\varepsilon}{x}, \tag{1}$$

where ε – relative deformation of the material at a distance x from neutral line, $1/r$ – curvature of element in section.

In most cases, diagrams of concrete deformation are function of related strength and deformation characteristics: the calculated resistance of concrete to axial junction f_c, concrete deformation module E_c and limits of concrete deformation ε_{c1}, ε_{cu}. This can be expressed in the following functional dependence

$$\sigma_c = f(f_c, E_c, \varepsilon_{c1}, \varepsilon_{cu}). \tag{2}$$

There are many formulas that associate deformation and strength characteristics of concrete. The following expressions can be written by its summarizing

$$E_c = f(f_c), \ \varepsilon_{c1} = f(f_c), \ \varepsilon_{cu} = f(f_c). \tag{3}$$

With taking into account (3) the final dependence (2) will take the following form

$$\sigma_c = f(f_c). \tag{4}$$

The function laid down in the current norms [1, 2] will be examined for further research. It is also called Eurocode function

$$\sigma_c = \frac{E_c\varepsilon_c - f_{cm}\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2}{1+\left(\frac{E_c\varepsilon_{c1}}{f_{cm}}-2\right)\frac{\varepsilon_c}{\varepsilon_{c1}}} \; or \; \frac{\sigma_c}{f_c} = \frac{k\eta - \eta^2}{1+(k-2)\eta}, \tag{5}$$

where $\eta = \varepsilon_c / \varepsilon_{c1}$, $k = 1.05 E_c \varepsilon_{c1} / f_c$.

Here is the expression (4) for the function (5)

$$\sigma_c = f_c \times f(k,\eta) = f_c \times f(f_c). \tag{6}$$

The bending reinforced concrete elements at a single reinforcement are proposed to consider. After putting equilibrium equation and conducting simple transformation with considering the hypothesis of flat sections we will get:
- for non-overreinforced

$$\left(\frac{\int_0^{\varepsilon_c}\sigma_c\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c\right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c}\right)\rho_f{}^2 f_{yd}^2 + \rho_f f_{yd} = \frac{M_{Ed}}{bd^2}; \tag{7}$$

- for overreinforced

$$\left(\frac{\int_0^{\varepsilon_c}\sigma_c\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c\right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c}\right)\rho_f^2 E_s^2 \varepsilon_c^2 + \rho_f E_s \varepsilon_c = \frac{M_{Ed}}{bd^2}. \tag{8}$$

Both sides of equation (7) i (8) are divided into f_c and taking into account (6), we will get:
- for non-overreinforced

$$\left(\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c\right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c}\right)\frac{\rho_f{}^2 f_{yd}^2}{f_c^2} + \frac{\rho_f f_{yd}}{f_c} = \frac{M_{Ed}}{f_c bd^2}; \tag{9}$$

- for overreinforced

$$\left(\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c\right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c}\right)\frac{\rho_f^2 f_{yd}^2 E_s^2 \varepsilon_c^2}{f_c^2 f_{yd}^2} + \frac{\rho_f f_{yd} E_s \varepsilon_c}{f_{yd} f_c} = \frac{M_{Ed}}{f_c bd^2}. \tag{10}$$

The following notations are introduced

$$\omega = \frac{\rho_f f_{yd}}{f_{cd}},\ \eta_s = \frac{E_s \varepsilon_c}{f_{yd}}. \tag{11}$$

The name parameter ω is mechanical reinforcement coefficient [4, 5]. The formulas (9) and (10) with introduced notations (11) will take the following form:
- for non-overreinforced

$$\left(\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c \right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c} \right) \omega^2 + \omega = \frac{M_{Ed}}{f_c b d^2}; \tag{12}$$

- for overreinforced

$$\left(\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c \right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c} \right) \omega^2 \eta_s^2 + \omega \eta_s = \frac{M_{Ed}}{f_c b d^2}. \tag{13}$$

The notation is introduced to both equations

$$k_z = f(f_c, \omega) = \frac{6 M_{Ed}}{f_c b d^2} = \frac{M_{Ed}}{f_c W_c}. \tag{14}$$

Thereby
- for non-overreinforced

$$k_z = \frac{1}{6} \left(\left(\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c \right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c} \right) \omega^2 + \omega \right); \tag{15}$$

- for overreinforced

$$k_z = \frac{1}{6} \left(\left(\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c \right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c} \right) \omega^2 \eta^2 + \omega \eta_s \right). \tag{16}$$

Connection between the calculated resistance of reinforced concrete and characteristics that derived above is following

$$f_{zM} = k_z f_c. \quad (17)$$

Introduced parameter k_z generally depends on the mechanical reinforcement coefficient ϖ and deformation characteristics of concrete. There are more details about the impact of deformation characteristics of concrete on bearing capacity of bending elements. Here is a submission of parameter dependence schedule k_z depending on the mechanical reinforcement coefficient ϖ for different kinds of concrete (Fig.1). The impact of deformation characteristics of concrete (parameters $\eta = \varepsilon_c / \varepsilon_{c1}$, $k = 1.05 E_c \varepsilon_{c1} / f_c$ on bearing capacity of bending reinforced concrete elements for considered kinds of concrete varies with differences within 10% (Fig. 2). This error is completely allowed by normative coefficients of variation of strength for such elements. This makes possible to offer slightly simplified force model of reinforced concrete elements calculation. The impact of deformation characteristics of strength sections is ignored and common functional dependence is taken $k_z = f(\varpi)$ for all kinds of concrete. This dependence is true not only for different kinds of concrete and reinforcement and even for different duration of the load. Formulas for non-central compression can be derived by similar arguments. Dependence of parameter k_z from the mechanical reinforcement coefficient ϖ is presented in tables.

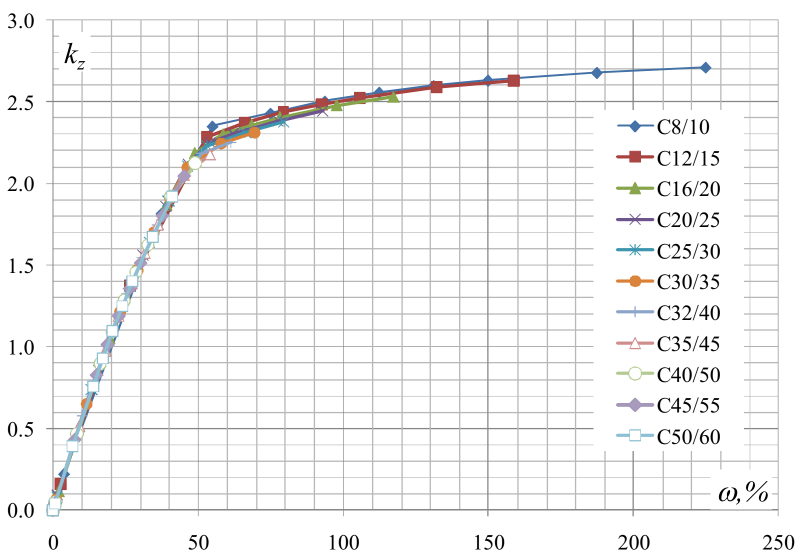

Fig.1. Parameter k_z of reinforced concrete depending on the mechanical reinforcement coefficient at single reinforcement $f_{yd} = 365$ MPa.

The strength conditions by a modified method of calculation resistances are formulated:
- bending elements

$$M_{Ed} = W_c f_{zM}. \quad (18)$$

where f_{zM} – estimated resistance of reinforced concrete on bend, which depends on kinds of concrete and reinforcement, section shapes and percent of reinforcement section that defined by the expression (17). Parameter k_z in expression (7) is calculated by table (Table 1, 2, 3) depending on the mechanical reinforcement coefficient ϖ.
- non-central compressed

$$N_{Ed} = A_c f_{zN}. \tag{19}$$

where f_{zN} – estimated resistance of reinforced concrete on bend, which depends on kinds of concrete and reinforcement, section shapes and percent of reinforcement section, relative initial eccentricity of the force application e_0/d, that defined by the expression $f_{zN} = k_z f_c$, Parameter k_z is also calculated by table (Table 4) depending on the mechanical reinforcement coefficient ϖ; A_c – working sectional area of concrete: for rectangular elements $A_c = bd$; for round ones $A_c = \pi d^2/4$.

The conditions (18, 19) satisfy the conditions of bending element's strength of material's classic resistance and allow solving a number of problems, including:
verification of element's strength; determination of bearing capacity of reinforced concrete elements according defined reinforcement; selection of a reinforcement section of the reinforced concrete element; setting the required sizes of cross-section according defined reinforcement; variant design of bending reinforced concrete elements.

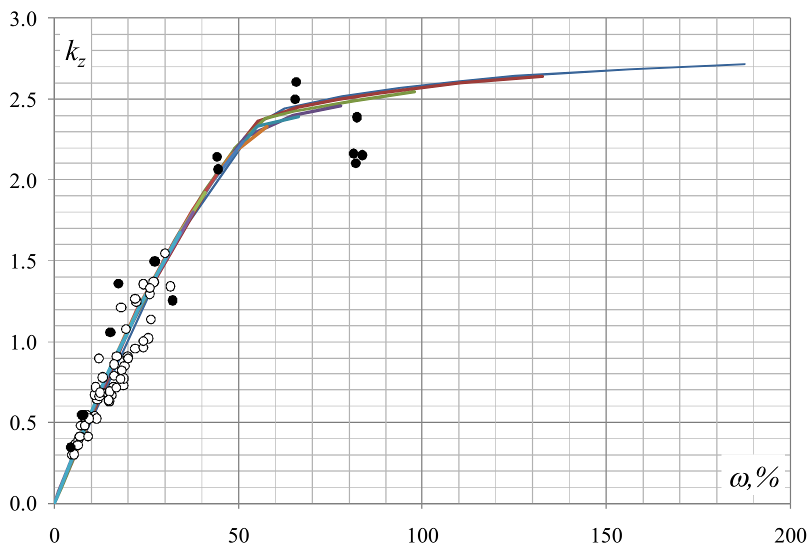

Fig. 2. Experimental verification of dependence $\varpi = f(k_z)$

Table 1. Dependence of $\varpi - k_z$ by the flat bend for a rectangular section with single reinforcement.

ϖ	0	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.6	0.7	1	2	3
k_z	0	0.568	0.828	1.071	1.299	1.511	1.706	1.885	2.028	2.07	2.14	2.195	2.31	2.476	2.542

Table 2. Dependence of $\varpi - k_z$ by the flat bend for a rectangular section with symmetrical reinforcement.

ϖ	0	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.6	0.7	1	2	3
k_z	0	0.292	0.429	0.565	0.700	0.834	0.967	1.100	1.232	1.36	1.63	1.888	2.66	4.85	6.671

Table 3. Dependence of $\varpi - k_z$ by the band for round section with symmetrical reinforcement.

ϖ	0	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.6	0.7	1	2	3
k_z	0	0.368	0.523	0.673	0.821	0.967	1.11	1.253	1.394	1.53	1.81	2.072	2.78	5.06	7.32

Table 4. Parameter values k_z for non-central compressed elements of rectangular cross-section with symmetrical reinforcement, $\lambda \le 4$ ($\lambda \le 0$).

Mechanical reinforcement coefficient ϖ	Relative initial eccentricity of longitudinal force application e_0/d									
	0.01	0.15	0.30	0.65	1.00	1.50	2.00	3.00	4.00	5.00
0.10	1.159	0.817	0.575	0.206	0.090	0.047	0.032	0.019	0.014	0.011
0.15	1.209	0.875	0.640	0.272	0.131	0.070	0.047	0.029	0.020	0.016
0.20	1.259	0.918	0.682	0.326	0.169	0.092	0.062	0.038	0.027	0.021
0.25	1.309	0.959	0.721	0.375	0.204	0.114	0.077	0.047	0.034	0.026
0.30	1.359	0.999	0.758	0.420	0.238	0.136	0.092	0.056	0.040	0.031
0.35	1.409	1.040	0.794	0.461	0.270	0.157	0.107	0.065	0.047	0.037
0.40	1.458	1.077	0.829	0.499	0.301	0.177	0.123	0.075	0.054	0.042
0.45	1.509	1.113	0.864	0.535	0.330	0.197	0.137	0.084	0.060	0.047
0.50	1.558	1.148	0.897	0.568	0.359	0.217	0.152	0.093	0.067	0.052
0.60	1.658	1.216	0.964	0.619	0.413	0.256	0.180	0.111	0.080	0.062
0.70	1.758	1.284	1.029	0.667	0.464	0.293	0.209	0.130	0.093	0.073
1.00	2.058	1.544	1.221	0.804	0.597	0.400	0.290	0.183	0.133	0.104
2.00	3.058	2.297	1.340	1.222	0.916	0.676	0.535	0.354	0.261	0.205
3.00	4.057	3.068	1.462	1.665	1.256	0.930	0.738	0.515	0.384	0.305

3 Examples of application of the proposed calculation method

3.1 Example 1

Reinforced concrete beams has a working section $b \times d = 20 \times 55$ cm. Determine reinforcement for the perception estimated moment $M_{Ed} = 242$ kNm. Reinforcement grade is accepted A500C ($f_{yd} = 435$ MPa), concrete grade C20/25 ($f_{cd} = 14.5$ MPa).

Solution. The estimated resistance of reinforced concrete that provides beam's strength and parameter k_z are calculated

$$f_{zM} = \frac{6M_{Ed}}{bd^2} = \frac{6 \times 242 \times 10^3}{20 \times 55^2} = 24.0 \text{ MPa}; k_z = \frac{f_{zM}}{f_{cd}} = \frac{24.0}{14.5} = 1.655.$$

According to Table 1 $\varpi = 0.337$ depending on $k_z = 1.655$.

The required contents of reinforcement

$$\rho_f = \frac{f_{cd}}{f_{yd}} \omega = \frac{14.5}{435} 0.337 = 0.011.$$

The required sectional area of reinforcement

$$A_s = \rho_f \times b \times d = 0.011 \times 20 \times 55 = 12.25 \text{ cm}^2.$$

3.2 Example 2

The longitudinal strength $N_{Ed} = 2000\,\text{kN}$ and bending moment $M_{Ed} = 120\,\text{kNm}$ on the reinforced concrete elements with section $b \times d = 30 \times 40$ cm length $l_0 = 200\,\text{cm}$, made of kind of concrete C20/25 ($f_{cd} = 14.5$ MPa). Determine the required sectional area of steel reinforcement of grade A400C ($f_{yd} = 365$ MPa) with symmetrical reinforcement.

Solution. The relative initial eccentricity and element's flexibility are determined

$$\frac{e_0}{d} = \frac{M_{Ed}}{N_{Ed} d} = \frac{120 \times 10^2}{2000 \times 40} = 0.15; \lambda = l_0/d = 200/40 = 5.$$

The element will be considered as rigid according to flexibility $\lambda = 5$. The required estimated resistance of reinforced concrete in compression and parameter k_z are calculated

$$f_{zN} = \frac{N_{Ed}}{bd} = \frac{2000}{30 \times 40} \times 10 = 16.67 \text{ MPa}; k_z = \frac{f_{zN}}{f_{cd}} = \frac{16.67}{14.5} = 1.150.$$

According to Table 4, using interpolation on k_z and relative initial eccentricity e_0 / d, $\varpi = 0.503$ is defined.

Reinforcement coefficient is calculated

$$\rho_f = \frac{f_{cd}}{f_{yd}} \omega = \frac{14.5}{365} 0.503 = 0.02.$$

The required sectional area of reinforcement

$$A_s + A_{sc} = \rho_f \times b \times d = 0.02 \times 30 \times 40 = 24.0 \text{ cm}^2.$$

4 Conclusions

The methodology of practical calculation of strength of reinforced concrete elements based on deformation model is proposed. The tables are compiled based on this methodology allowing to quickly and easily perform calculations of strength of overreinforced and non-overreinforced beams by deformation methodology without using computer programs. To simplify the work of designers, engineers-practitioners and students of construction specialties it is appropriate to include the table into design standards of concrete elements.

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