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Analysis of influence of the geometric parameters of mast systems on the natural frequency and vibration mode spectrum

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Abstract. Natural frequencies and eigenmodes of oscillations are the determining values in the study of dynamic processes in many constructions, including mast systems when they are affected by time-varying loads: wind, seismic, etc. [7]. The article suggests a method of forming, by means of a purposeful variation of the geometrical parameters of the mast, the spectrum of natural frequencies of bending vibrations. The basis is the necessary optimality conditions for one-dimensional models in the form of the maximum principle [1, 3, 8]. The analysis of the sensitivity of the spectrum [9] to small changes in geometric parameters that inevitably arise during the operation of the masts is carried out. An algorithm for using functional derivatives for finding optimal solutions is constructed.

All calculations in the paper are carried out using the example of a mobile phone mast l = 60m high with six leveled guy-rope attachment (the diagram is shown in Fig. 1).

The sections are joined together by flanges. The booms are represented by tubular elements having a fixed inner and outer diameter within a section. We will assume that the external diameter remains constant from section to section, and the internal diameter can vary according to the data given in Table 1.

Further, we will assume that variable function is the one determining the distributions of the total area of the booms along the height of the mast $F(z)$. In this case, we will replace the discrete character of variation by continuous variation at a certain stage, i.e. we assume that continuous $F(z)$ varies subject to the following constraints

$$F_1 \le F(z) \le F_2 , \tag{1}$$

where F_1, F_2 correspond to grades 1 and 7 in Table 1.

Array for the section shown in Fig. 2 consists of round bars with the diameter ∅1.4 cm. In this case, the array remains the same for the different sections.

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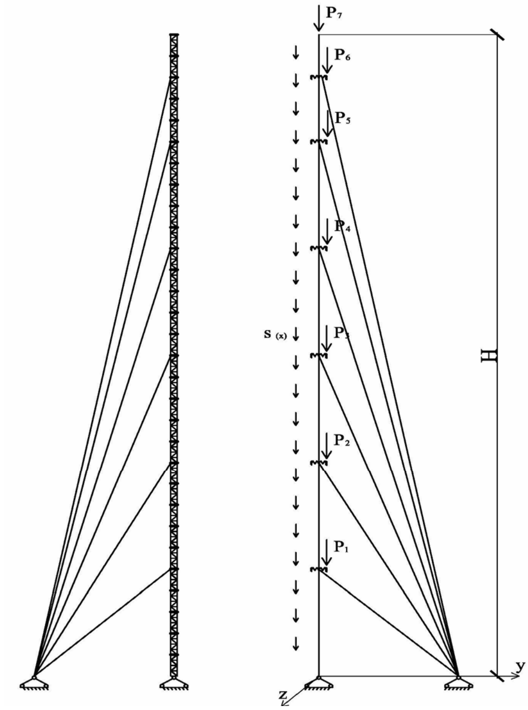


Fig. 1. General view of mast design diagram (beam model).

P1 – P7 – axial concentrated forces arising due to the effect of guy ropes and weight of the equipment on the mast;
s(x) – longitudinal load determined by the mast's own weight.

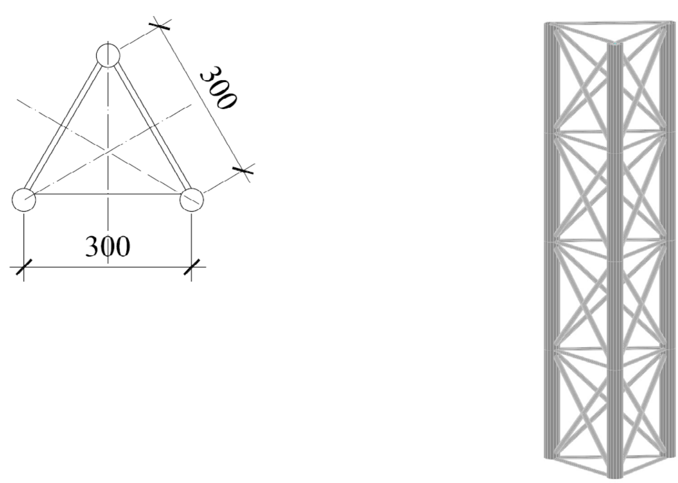


Fig. 2. General view of sections and cross-sectional diagram.

The moment of inertia of cross section during plane bend *zx* will be denoted *I(z)*, while the moment of inertia will be the function of the area within the constraints (1). It can be assumed from the geometry of cross section (Fig. 2) that $I=\alpha F$. Table 1 shows that we can take α=152.9.

Table 1. Possible variants of booms' sections.

№	1	2	3	4	5	6	7
∅, cm	4.8x0.2	4.8x0.22	4.8x0.25	4.8x0.28	4.8x0.3	4.8x0.32	4.8x0.35
F, cm^2	8.670	9.496	10.721	11.928	12.723	13.511	14.679
I_Z, cm^4	1325.6	1452.5	1638.3	1822.4	1943.7	2063.8	2241.7

Modulus of elasticity $E = 2 \cdot 10^6 \dfrac{\text{kg}}{\text{cm}^2}$, specific gravity $\gamma = 7.8 \cdot 10^{-3} \dfrac{\text{kg}}{\text{cm}^3}$, density of the material $\mu = 7.8 \cdot 10^{-6} \dfrac{\text{kg} \cdot \text{sek}}{\text{cm}^2}$ are the same for booms and arrays. Tie-rope (guy rope) is a regular-lay rope of type T with the diameter of ∅ 1.4 cm (GOST 3063-80). Rope pre-stressing force is the same for all levels (200 kg).

Boundary value problem for bending vibrations natural mode will be written in the form [2].

For longitudinal deformation differential equations are used

$$\frac{dw}{dx} = \frac{N}{EF}; \quad \frac{dN}{dx} = -s; \tag{2}$$

with boundary conditions

$$w(0) = 0; \quad N(l) = P_7; \tag{3}$$

and the conditions at the points of application of longitudinal concentrated forces generated by the tension of guy ropes,

$$w(z_i^-) = w(z_i^+); \quad N(z_i^-) + P_i = N(z_i^+); \quad i = \overline{1,6} \tag{4}$$

In these expressions z_i^-, z_i^+ - is the value of coordinate from below and from the top of the point z_i, and functions $w(z)$, $N(z)$ represent movements and axial forces. Distributed axial load has the form

$$s(x) = -\gamma(\mathrm{F} + F_d). \tag{5}$$

The value F_d is obtained by dividing the total volume of the array by the length of the section. In our case it is F_d = 8.3 cm^2. Flexural deformations of the mast can be described by the system of differential equations

$$\frac{du}{dz} = \varphi; \quad \frac{d\varphi}{dz} = -\frac{M}{EI}; \quad \frac{d\varphi}{dz} = -\frac{M}{EI}; \quad \frac{dM}{dz} = Q - N\varphi;$$

$$\frac{dQ}{dz} = \text{-}p^2 \mu (F + F_d) u; \tag{6}$$

with boundary conditions

$$u(0) = M(0) = 0; \quad M(H) = 0; \quad Q(H) = -p^2 m_H u(l), \tag{7}$$

and the conditions at the fixing points of guy ropes

$$u(\mathrm{z}_i^-) = u(\mathrm{z}_i^+); \quad \varphi(\mathrm{z}_i^-) = \varphi(\mathrm{z}_i^+); \quad M(\mathrm{z}_i^-) = M(\mathrm{z}_i^+);$$

$$\mathrm{Q}(\mathrm{z}_i^-) + \mathrm{c}_i u(z_i) = Q(\mathrm{z}_i^+); \quad i = \overline{1,6} \tag{8}$$

In conditions (7) value m_H represents the mass of equipment on the upper platform of the mast, p is the natural frequency. Rigidities c_i (mounting rigidities) are characterized by the parameters of the guy rope unit and were counted according to the ratios of [5].

For any function *F(x)*, subject to constraint (1), the boundary value problem (6), (7), (8) taking into account relations (2), (3), (4) determines the spectrum of natural frequencies $p_1 < p_2 < p_3 < \ldots$ and corresponding eigenfunctions $u_i(z)$ $(i=1,2,3,\ldots)$. Let the oscillation form *u(x)* (or rather the set of functions *u(z), φ(z), M(z), Q(z)*) belong to the set of eigenfunctions corresponding to the natural frequency *p* with a given number. The identification of the natural frequency number in case of simple (non-multiple) frequencies is carried out during their numerical determination. Let us pose the problem of finding the function *F(x)* within the limits of constraints (1) that delivers the extreme value to the natural frequency *p* within the above mentioned variety of natural modes, that is, let consider the problem of minimizing the functional

$$J(F) = kp^2 = \min \tag{9}$$

For $k = +1(-1)$ this corresponds to minimization (maximization) of the natural frequency. The natural frequency can be represented using the Rayleigh formula [3],

$$p^2 = \int_0^{p+} \frac{M^2}{EI} dz \Bigg/ \int_0^H \mu (F + F_d) u^2 dz \tag{10}$$

The necessary optimality conditions in the form of the maximum principle [8] for the functional (9) taking into account the expression (10) are considered in detail in the monograph [3]. In view of the important property of self-adjointness, the necessary conditions will be represented by the boundary value problem (2)–(4), closed on the optimal *F(x)* by the Hamiltonian maximum condition

$$H = k \cdot \left[\frac{M^2}{E\alpha F} + p^2 \mu (F + F_d) u^2 \right]. \tag{11}$$

Taking into account that when maximizing the Hamiltonian (11) values *M, u* are considered as parameters, we can obtain the formulas [3]

for $k=-1$

$$F = \begin{cases} F_2, & F_* \geq F_2; \\ F_*, & F_1 \leq F_* \leq F_2; \\ F_1, & F_* \leq F_1; \end{cases} \tag{12}$$

where $F_* = \dfrac{|M|}{p|u|\sqrt{\alpha \mu E}}$ is found from the factor *H(F)* for $k=-1$ upward-convex function $\left(\dfrac{\partial^2 H}{\partial F^2} \leq 0 \right)$

for $k=1$

$$F = \begin{cases} F_1, & H(F_1) \geq H(F_2); \\ F_2, & H(F_1) < H(F_2). \end{cases} \tag{13}$$

It is the consequence of the fact that Hamiltonian *H(F)* is a downward-convex function $\left(\frac{\partial^2 H}{\partial F^2} \geq 0\right)$ and the optimal solution consists of only sections of the boundaries (1).

The relations (12), (13) which close the original boundary value problem make it nonlinear. The approach described in [3], based on the successive approximation of the optimal solution on the sequence of varying functions, will be used in what follows. An important role is played by sensitivity analysis, which consists of finding the functional derivatives $\frac{\partial J}{\partial F}$. Using the results presented in [9], we can write

$$\frac{\partial J}{\partial F} = \frac{\partial H}{\partial F} = k\left[-\frac{M^2}{\alpha E F^2} + p^2 \mu u^2\right], \tag{14}$$

which determines sensitivity functions for the problem under consideration. Fig. 3 shows the vibration modes and sensitivity functions for the first and second eigenfrequencies in the case $F(x)=F_2$. For function $\delta F(z)$ that determine the necessary changes in the cross-sectional area of the mast within the constraints (1), changes of the natural frequency will be determined by the integral

$$\delta J = k\int_0^l \left(\frac{\partial H}{\partial F}\delta F\right)dz = k\int_0^l \left[-\frac{M^2}{\alpha E F^2} + p^2 \mu u^2\right]\delta F dz \tag{15}$$

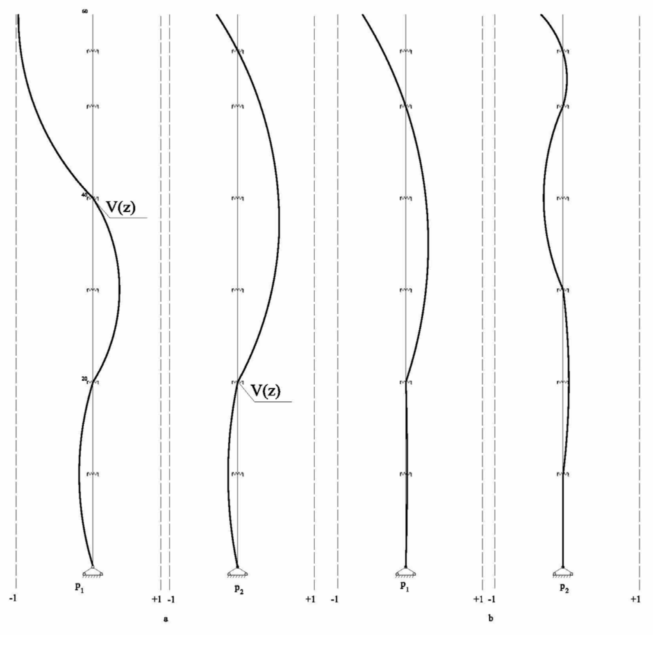


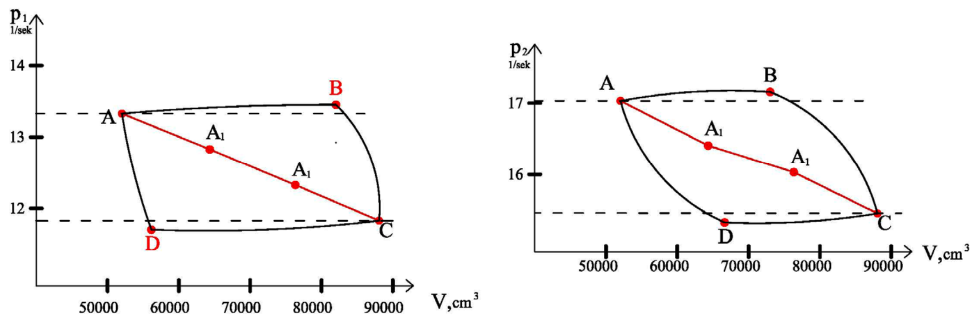
Fig.3 Vibration modes *a* and sensitivity functions *b* for first p_1 and second p_2 natural frequencies.

Variations of the area will be stronger in those sections where the magnitude of derivative is greater and where there is more sensitivity. It should be noted that for the case of maximizing the natural frequency, according to formula (12), the derivative (14) becomes zero in the sections of the optimal solution F^* within the interval $[F_1, F_2]$. For other laws $F(x)$ shows the trend of frequency variation.

The solution of the boundary value problem for the eigenvalue was carried out on the basis of the initial parameter method when using the main program modules from paper [2]. Laws of variation of cross-sectional area $F(z)$ were recalculated in accordance with the dependence

$$F_{i+1}(z) = F_i(z) + k\left[-\frac{M_i^2(z)}{\alpha EF_i^2(z)} + p^i \mu u_i^2(z)\right]\Delta F \tag{16}$$

Moreover, the new approximation $F_{i+1}(z)$ must be subject to constraints (1). If at any point $F_{i+1}(z) > F_2(<F_1)$, $F_{i+1}(z) = F_2(=F_1)$ was accepted. Step size ΔF was constant based on the height of the mast; in particular, in the conducted calculations $\Delta F=(F_2 - F_1)/3$ was adopted. Figure 4 shows the closed curves in the volume coordinates V – natural frequencies p (for second and first natural frequency).



a b

Fig. 4. Zones of possible change of the first (a) and second (b) natural frequencies.

Curves *AA'C* correspond to the frequency dependences on the volume for the masts with constant cross-section along the height. It is characteristic that for cross-sectional area (point A) which is minimal within the limits of constraints (1), there is a larger frequency value than for the maximum area value (point C). Due to the fact that natural frequencies are determined by the ratio of stiffness and inertia characteristics, it can be concluded from this ratio that the increase of the cross-sectional area increases the inertial characteristics to a greater extent. This circumstance occurs both for the first and second natural frequencies.

Figure 5 shows the distributions $F(x)$, corresponding to frequencies p_1^+, p_1^- and p_2^+, p_2^-.

Dashed dotted line indicates the piecewise approximation of the optimal solution in accordance with the discrete variation scheme given in Table 1. For specific z area value within the section is taken equal to the closest of the discrete sets indicated in the table. Such a discretization of optimal distributions $F(z)$ practically does not affect the values of the natural frequencies.

The above relations are generalized for the case of the hill-climbing problem for the natural frequency with a given number provided that the total volume of variable booms is fixed,

$$V = \int_0^l F(z)dz = const \tag{17}$$

For such a task, the quality functional (9) is modified

$$J(F) = kp^2 + k_0 \int_0^l F dz \,, \tag{18}$$

where k_0 is the indefinite Lagrange multiplier to account for the condition of constancy of the volume (17). Hamiltonian in this case takes the form:

$$H = k \cdot \left[\frac{M^2}{E\alpha F} + p^2 \mu (F + F_d) u^2 \right] + k_0 F \tag{19}$$

For a given optimization problem which is designated as suggested in [3], via $V \to p$, the sensitivity function will have the form

$$\frac{\partial J}{\partial F} = k \left[-\frac{M^2}{\alpha E F^2} + p^2 \mu u^2 \right] + k_0 \,, \tag{20}$$

A similar construction is used for the dual problem $V \to p$ designated in accordance with [3] as the problem of finding *F(x)*, at delivers within the limits of constraints (1) the extreme value to the material volume at a fixed value of the corresponding natural frequency. The solution of these problems forms a closed curve in the coordinates *V, p.* The examples of such curves for the first and second natural frequencies of flexural vibrations are shown in Fig. 4. For any fixed volume $V \in [V_1, V_2]$ the upper sections of *ABC* within the framework of the problem $V \to p$ correspond to the maximum natural frequencies, and the lower sections of *ADC* – to the minimum natural frequencies. For a fixed value of the frequency $p \in [p^-, p^+]$ the left *DAB* sections correspond to the minimum volume values and the right *BCD* sections – to the maximum volume values.

The points of these closed curves were constructed by solving problems $V \to p$ based on the formulas analogous to (16):

$$F_{i+1}(z) = F_i(z) + \left[k \left[-\frac{M_i^2(z)}{\alpha E F_i^2(z)} + (p^i)^2 \mu u^2{}_i(z) \right] + \mathrm{k}_0 \right] \Delta F \tag{21}$$

Values k_0, k are normalized as $|k| \le 1$, $|k_0| \le 1$. This is achieved at each step by dividing them by $\max(|k|, |k_0|)$. The issue of fulfilling the conditions (17) at each step is solved by choosing the value k_0 in (21). It should be noted that for $p \to V$ such a problem is difficult to solve.

During normalization the value k in (21) changes and allows us to construct the necessary approximations to the optimal solution.

As can be seen from the nature of the closed curve, in *AB* section for the frequency p_1 the value of the maximum frequency depends little on the volume. The same property is possessed by the minimum frequency for the *DC* section. This takes place because of the peculiar relationship between the stiffness and inertia characteristics of this variant of design.

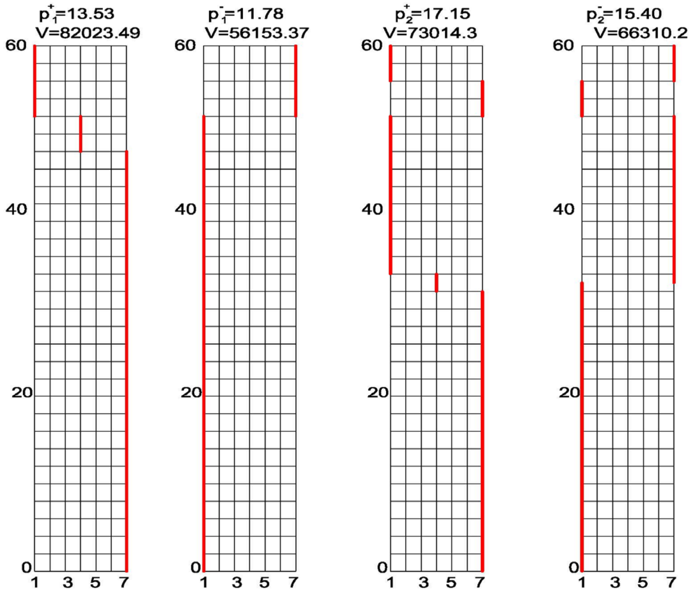


Fig. 5. Distribution of the cross-sectional area of the area for frequencies p_1^+, p_1^- and p_2^+, p_2^-.

Closed *ABCD* curves determine the reserves of this design from the point of view of possible changes in the first and second natural frequencies of bending vibrations and the volume (weight) of booms material.

In order to compare the results of calculations of natural frequencies with other computational models, the first and second natural frequencies of bending vibrations were calculated by the finite element method in the SCAD Office complex. When constructing the computational model, the scheme shown in Fig. 2 was used. The errors for the points of the curves AA_1 A_2C (booms area which is constant along the height of the mast) do not exceed 10%. It should be noted that consideration of the above type optimization problems based on finite element models is very difficult. It is not possible to formulate the necessary optimality conditions analogous to the maximum principle [8]. Optimization of variables based on discrete set is possible, but it entails significant computational difficulties.

Conclusions

The approach to the problem of controlling the natural frequency spectrum of bending vibrations of masts on guy ropes was proposed. The numerical control algorithm is based on the necessary optimality conditions in the form of the maximum principle for beam models. The option of varying the area of the booms along the height of the mast was proposed. The sensitivity analysis for the first and second natural frequencies was carried out and its use for constructing an effective computational process was shown. The possibilities of extending the developed approach to more general problems of spectrum management were shown.

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