

# **Estimation of flexures of the reinforced concrete elements according to the National Ukrainian & European standards**

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**Abstract.** The objective of this work is analysing of the calculating results of theoretical values of the beam flexure by the Ukrainian National Standard, European Standard and by the experimental data of concrete beams testing. The results of the experimental investigations of flexures of the reinforced concrete beams are shown. The theoretical comparison of the flexures of the reinforced concrete beams according to the Ukrainian National Standard and European Standard is done. It was proved that the flexures by these methods have an identical convergence with experimental data. The algorithm for determining deformations of materials at the known bending moments by using the method of simple iteration or method of successive approximations is proposed. The solution of this problem allows practical use of the National Standard and European Standard for calculating flexures. Analyzing of the experimental flexure and the calculated value is showed; the significant difference can be explained by the empirical coefficient.

## **1 Introduction**

The design of concrete structures in Ukraine is carried out according to National Standards [1, 2], which is harmonized with European Standard Eurocode 2 [3], but their use causes some difficulties in estimation of structures.

The basis of the National and European Standards contains calculations by the limit states, but the algorithms have some differences. Use of the algorithm of these standards should be compared with experimental results. However, this is a major problem in the design of concrete structures, namely the difference between the calculation results obtained by the National Standards [1, 2] and the European Standard Eurocode 2 [3].

Practical calculations to determine the limit states for the carrying capacity (the first group uls) of concrete bending elements by the Standards [1, 2] are partly covered in [4, 5, 6, 7, 8], but the results of calculations by the limit states of the availability for operation (the second group sls) are very scarce, except for a few works [9, 10, 11, 12, 13].

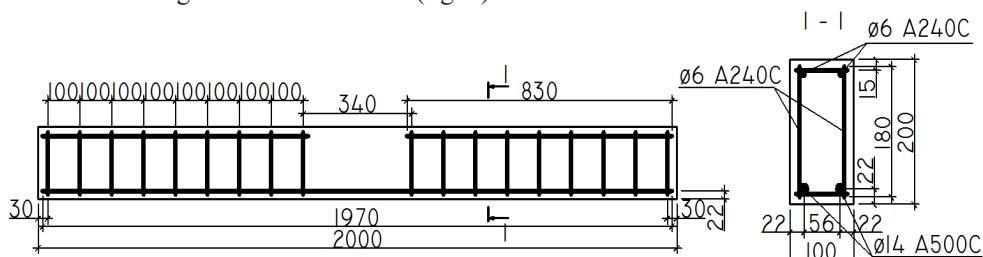
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The objective of this work is to continue analyzing of the calculating results of theoretical values of beam flexure by the Standards [1, 2, 3] and by the experimental data of concrete beams testing.

## 2 Methods and techniques

Experimental study of the reinforced concrete elements were conducted at the beams, twin – samples, the cross-section size is 100×200 mm, the total length 2000 mm. Heavy concrete of class C25/30 was used to manufacture samples. Reinforcing was made of the welded frame with a longitudinal reinforcement  $\varnothing 14$  mm of class A500 and transverse  $\varnothing 6$  mm of class A240 fitting increments 100 mm (fig. 1).



**Fig. 1.** The reinforcement scheme of the experimental beams.

Sample 150×150 mm cubes, 150×150×600 mm prisms and crescent reinforcing samples of profile  $\varnothing 14$  mm, class A500, length 400 mm were tested to determine the strength and deformation characteristics of materials, along with the study of concrete beams.

Experimental studies of beams were conducted using the bench transferring the load from a hydraulic jack to a traverse. So the beams were tested by the free lying beam scheme, which was established on two supports. The test bench for beams is shown on fig. 2.

A load on the beams was increasing in increments of 0.05 to 0.1 of the maximum to the destruction. During testing at each stage of loading there was a prolonged exposure for 10 to 15 minutes, during which the parameters of devices were recorded and the visual inspection of the samples was performed. We also determined the nature of crack width and crack opening.



**Fig. 2.** The install for researching of experimental beams

### 3 Results and Discussion

Before testing the beams, we determined the mechanical properties of materials experimentally. The strength of concrete cubes was  $f_{ck,cube} = 30.2 \text{ MPa}$ , of prisms -  $f_{ck,prism} = 27.8 \text{ MPa}$ , the initial modulus of elasticity of concrete  $E_{ck} = 20.9 \text{ GPa}$ , the relative deformation of concrete  $\varepsilon_c = 1.78\%$ .

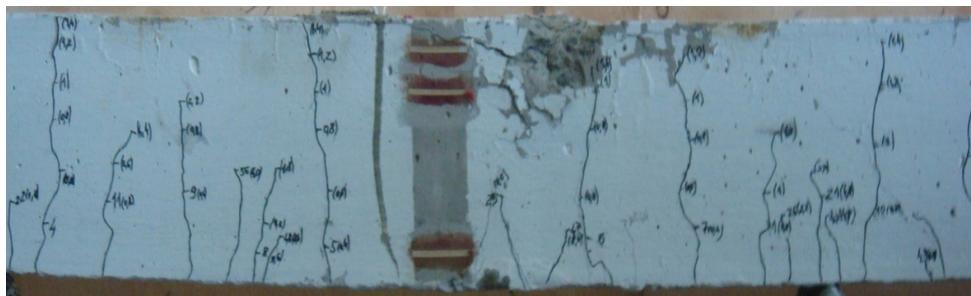
The test of reinforcement class A500 Ø14 mm has showed the following characteristics:

- strength of the fluidity limit  $f_{yk} = 61.7 \text{ MPa}$ ;
- tensile strength  $f_{ik} = 727.7 \text{ MPa}$ ;  $E_s = 196.509 \text{ MPa}$ ;
- reinforcement deformations, which fit  $\sigma_y$ :  $\varepsilon_s = 3.12\%$ .

The destruction of the beams was a result of puncturing compressed zone of concrete. By the results of three twin - beams the average destruction load is  $2F_{ULS} = 79.5 \text{ kN}$  (destruction moment  $M_{ULS} = 23.9 \text{ kN}\cdot\text{m}$ ). The results of experimental investigations are noted in Table. 1, and the fracture of beams is shown in fig. 3.

**Table 1.** Results of the test of experimental beams.

beam mark	destruction load		deformations		moment of the first cracks $M_w, \text{ kN}\cdot\text{m}$	the maximum width of the crack opening $w_{max}, \text{ mm}$	the maximum flexure $f_{max}, \text{ mm}$
	$M_{ULS}, \text{ kN}\cdot\text{m}$	$2F_{ULS}, \text{ kN}$	concrete $\varepsilon_c, \%$	armature $\varepsilon_s, \%$			
b-1	23.7	79.1	2.11	2.93	2.4	0.59	17.2
b-2	24.5	81.7	2.47	2.93	2.4	0.31	16.9
b-3	23.3	77.8	2.13	2.58	2.4	0.28	18.8
aver.	23.9	79.5	2.24	2.81	2.4	0.39	17.6



**Fig. 3.** The fracture of beams.

Flexures of reinforced concrete beams are determined by the general rules of structural mechanics based on the bending deformation characteristics of the element sections by its length. Value of flexures of reinforced concrete beams is determined by the curvature of the element, which depends on the deformation of materials and determines by formula [2]:

$$\kappa = \frac{1}{r} = \frac{(\varepsilon_{cm} + \varepsilon_{sm})}{d}, \quad (1)$$

$\varepsilon_{cm}$  - deformation of compressed concrete extreme fiber;

$\varepsilon_{sm}$  - averaged strain stretched reinforcement.

$d$  - height of the section of the beam.

Curvature of an element is determined by the formula (1) according to National Standard [2], but the algorithm for determining the values of deformation of materials at known bending moments are not created.

For solving this problem we use a simple iteration method or the method of successive approximations to the condition of equality efforts at the compressed and stretched zones of the cross section:

$$S_{cc} = S_{ct} + S_s, \quad (2)$$

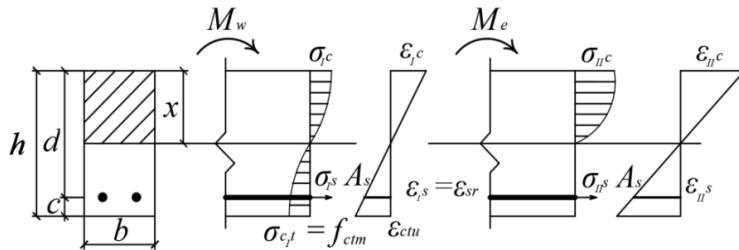
$S_{cc}$  - internal efforts into concrete of the compressed zone;

$S_{ct}$  - internal efforts into concrete of the stretched zone;

$S_s$  - efforts into stretched fixture.

Using the deformation model as a fifth-degree polynomial [1], the equilibrium condition can be written as (fig. 4):

$$\begin{aligned} f_{ck} b x \sum_{k=1}^5 \frac{a_k}{k+1} \left( \frac{\varepsilon_{ci}}{\varepsilon_{c1}} \right)^k &= \\ = f_{ctm} b (h-x) \sum_{k=1}^5 \frac{a_k}{k+1} \left( \frac{\varepsilon_{sr}}{\varepsilon_{ctu}} \right)^k + E_s A_s \frac{\varepsilon_{ctu}}{h-x} (h-x-c), & \end{aligned} \quad (3)$$



**Fig. 4.** Stress-strained state of reinforced concrete elements in the limit state: "no cracks" (a); "with cracks" (b).

Given the stress-strained state of reinforced concrete elements of rectangular section and using the hypothesis of flat sections, the ratio between the strains can be written as:

$$\varepsilon_{sr} = \varepsilon_{si} = \frac{\varepsilon_{ctu}}{h-x} (h-x-c), \quad (4)$$

$$\varepsilon_{ci} = \frac{\varepsilon_{ctu}}{h-x} x, \quad (5)$$

To determine  $\varepsilon_{ci}$  and  $\varepsilon_{si}$  it needs to find the height of the compressed zone of concrete, which is determined based on the equilibrium conditions of internal efforts into a cross-section.

With the help of dependencies (4) and (5) pre having area's compressed height, by the method of successive approximations, we reach the condition of equilibrium of internal forces in section (2) to within 2%. As a first approximation we have  $x = 0,5d$  taken in the future to change the condition of equilibrium within  $\pm 2\%$ .

Flexure is determined by a known formula from structural mechanics:

$$f = k_m \chi l_d^2, \quad (6)$$

$k_m$  - coefficient that depends on the design of a beams scheme (in the case of experimental conditions  $k_m = 0.10648$ ) (Table. 5.5 [2]);

$\chi$  - curvature of the element

$l_d$  - estimating step of beams (experiment  $l_d = 1800$  mm).

Determination of flexures refers to the calculation of limit states for availability to operation, therefore flexures comparing better to perform in the stage of operation. Conditionally assumed operational load  $M_{SLS} = 0.6 \cdot M_{ULS} = 0.6 \cdot 23.9 = 14.4 \text{ kN}\cdot\text{m}$ .

Estimated flexure for beams at the operating moment  $M_{SLS}=14.4 \text{ kN}$  is  $f_{DSTU}=6.3 \text{ mm}$ . The discrepancy between the experimental value of the flexure of reinforced concrete beams ( $f_{exp}=9.3 \text{ mm}$ ) with  $M_{SLS}=14.4 \text{ kN}\cdot\text{m}$  and calculated ( $f_{DSTU}=5.62 \text{ mm}$ ) is 39.6%. Such a significant discrepancy between experiment and calculation can be explained by the fact that into the formula of the definition of the flexure there is an empirical coefficient, which may give some error.

Algorithm for estimation of the flexure shown in Chapter 7 (par. 7.4.3) of the European Standard [3] is to determine the curvature of the elements of the formula:

$$\kappa = \xi \kappa_H + (1 - \xi) \kappa_I, \quad (7)$$

$\kappa$  - total curvature of the beam;

$\xi$  - coefficient of distribution is determined by the formula:

$$\xi = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)^2, \quad (8)$$

$\beta$  - coefficient of the impact of the term of load or stress repetition of the load on the mean deformation:

1.0 (for a short single load);

0.5 (for frequent or many cycles of repeated loads);

$\sigma_{sr}$  - stress in the tension reinforcement, which is calculated for the section "no cracks" if the load at the moment of formation of the first crack;

$\sigma_s$  - stress in the tension reinforcement, which is calculated for the section "with crack";

$\kappa_I$  - "no cracks" beam curvature;

$\kappa_H$  - "cracked" beam curvature.

There is the same situation as into the National Standard [2], a clear sequence of the determination of "no crack" and "cracked" curvatures isn't created. Using the already known method of simple iteration (successive approximation), we find the values  $\kappa_I$ ,  $\kappa_H$ .

According to the formula (7) we find the common curvature. By the formula (8) a theoretical flexure of control beams at  $M_{SLS}=14.4 \text{ kN}\cdot\text{m}$  is  $f_{EN}=5.57 \text{ mm}$ , the disagreement with experiment is 40.1%.

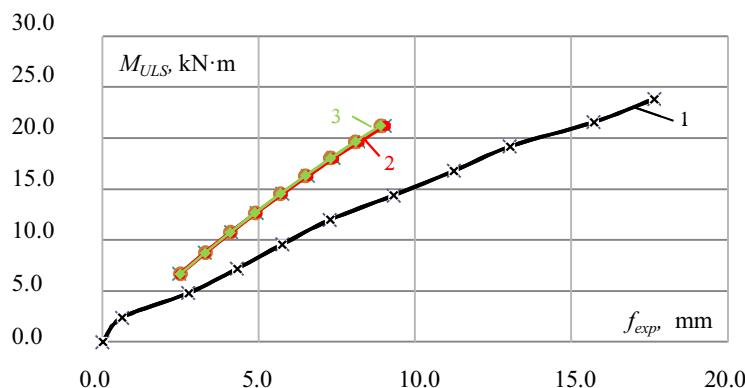
The findings of experimental and calculated flexures during the operating bending moment ( $M_{SLS}=14.4 \text{ kN}\cdot\text{m}$ ) are shown in Table 2.

**Table 2.** Comparison of experimental and calculation flexures.

$M_{SLS}, \text{kN}\cdot\text{m}$	$f_{exp.}, \text{mm}$	$f_{DSTU}, \text{mm}$	$\Delta, \%$	$f_{EN}, \text{mm}$	$\Delta, \%$
14.4	9.3	5.62	39.6	5.57	40.1

Thus, the Table 2 demonstrates convergence of estimated flexure, which was determined by the National [2] and European [3] Standards is almost the same. Particular attention is attracted the significant difference between the experimental and estimated flexure, which is about 40%.

Comparative diagram of flexures is shown in Fig. 5. Such significant difference between experiment and estimation can be explained by the fact that the formula (6) contains the empirical coefficient  $k_m$  for determining flexure, which can give an error.



**Fig. 5.** Diagram of flexures: 1 - experimentally; 2 - estimated by DSTU; 3 - estimated by Eurocode 2.

A similar problem is considered in articles [9-12], where there is almost the same as the convergence estimation of flexures by the National [2] and European [3] Standards.

## 4 Conclusions

The algorithm for determining deformations of materials at the known bending moments by using the method of simple iteration or method of successive approximations is proposed. The solution of this problem allows practical use of the National [2] and European [3] Standards for calculating flexures.

Analyzing of the experimental flexure and the calculated value showed, that they have a significant difference 39.6 to 40.1%. This difference can be explained by the empirical coefficient.

The flexures, which are theoretically determined by the National [2] and European [3] Standards are the same.

In the following publications, the authors plan to continue comparing experimental and estimated data on other test items to determine the limit states.

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