A polytropic model of a critical two-phase flow in a bed of spherical particles

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Abstract. The paper is concerned with a model of isenthalpic flow of vapor-water mixture in a fixed bed of solid particles. The mixture expansion process is considered to be polytropic. Similarly to the known problem of gas dynamics of a granular bed we obtained the relationships for calculation of a critical mass velocity. The results of the calculation based on a theoretical model are compared with the experimental data obtained in the packed beds of steel balls, 2 mm and 4 mm in diameter.

1 Introduction

The first experimental results of the research into a critical flow of a vapor-liquid mixture through the densely-packed beds of spherical particles were presented in a recent paper [1]. Here an important conclusion was drawn on linear dependence between the critical mass velocity and ratio of a particle diameter to bed height $(d/H)^{0.5}$. This dependence, revealed experimentally, coincides with the result obtained with the use of theoretical model of the homogeneous compressible medium motion in a granular bed [2]. An overview study [3] emphasizes that the widely known theoretical models for the determination of a critical flow rate of the two-phase mixture differ from each other in terms of choosing mixture density in a critical cross-section. The analogy between the flow of a one-phase compressible medium and the flow of a two-phase mixture is used to study a critical flow through the nozzles, diaphragms and short pipes in [4]. The authors of [5, 6] applied this analogy to the vapor-liquid mixture flow in the packed beds of solid particles to determine pressure drop. In the present research the isenthalpic expansion of the two-phase mixture is approximated by the equation of polytropic process. The corresponding polytropic coefficient is determined taking account of pressure P, flow quality $x,$ and a phase slip ratio s.

2 Theoretical model

The equations of gas dynamics of a granular bed [2] were used to describe a critical flow of the vapor-liquid mixture through the fixed bed of spherical particles. The governing equations include:
 the equation of motion

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$$\left[1+\frac{4}{3}(1-m)\right]\rho w\frac{dw}{dy}=-\frac{dP}{dy}-\frac{3}{2}\frac{m(1-m)}{\psi d}\rho w^2 \ , \tag{1}$$

where w – is the velocity of mixture in porous space, m – is the medium porosity, d – is the particle diameter, $\psi = 0.508 + 0.56(1-m)$ – is the relative minimum flow section; the continuity equation

$$\rho w = \rho_1 w_1 \ , \tag{2}$$

the polytropic equation

$$P/P_1 = (\rho/\rho_1)^n \ . \tag{3}$$

The integration of the equations with respect to the bed height from 0 to H under the assumption that $d << H$ results in the following relationship for the maximum mass velocity:

$$(\rho w)_{cr} = \left[\frac{2n}{3(n+1)}\frac{d}{H}\frac{\psi}{m(1-m)}P_1\rho_1\right]^{0.5} \ . \tag{4}$$

To use equation (4) we need to know the polytropic coefficient n and mixture density ρ_1 at the granular bed inlet. Pressure P_1 and flow quality x_1 at the inlet are considered to be given.

For equilibrium vapor-liquid flow we have

$$\varphi = \left[1+s\frac{\rho''}{\rho'}\frac{1-x}{x}\right]^{-1} \ , \tag{5}$$

$$\rho = \rho' - \varphi(\rho' - \rho'') = \rho''\varphi + \rho'(1-\varphi), \tag{6}$$

where φ – is the void fraction, and $s = w''/w'$ – is the slip ratio. The variables ρ', w' and h' refer to the properties of the saturated liquid, and the variables ρ'', w'' and h'' refer to the properties of the saturated vapor.

The absence of correlation dependences for s as applied to the two-phase flow in granular media creates a certain problem. This makes some authors use in such a case the known relationships obtained for two-phase flows, which, according to these authors, are similar in nature. For example in [6] the slip ratio calculation is based on the recommendation for an adiabatic flow of vapor- water mixture in vertical pipes of a small diameter and ring ducts

$$s = (P_{cr}/P)^{0.38} \ , \tag{7}$$

where P_{cr} – is the critical pressure. Equation (7) can be applied at the following conditions: $1 < P < 22$ MPa, $400 < \rho w < 3340$ kg/(m^2 s).

In the present research, to close the system of governing equations we obtained an approximating relationship for the slip ratio in critical conditions. The nonlinear regression method based on the experimental data on critical mass velocity $(\rho w)_{cr}$ [1], and equations (4-6) were used to obtain the following expression:

$$s = 1 + \frac{\left(7.0 - 8.0P + 4.0P^2\right)}{\exp\left(\left(0.058 - 0.13P + 0.075P^2\right)/x + \left(2.8 - 3.0P + 1.8P^2\right)x\right)}. \tag{8}$$

Equation (8) was obtained for the critical flow in the granular media under following conditions: $0.6 < P < 1.2$ MPa, $200 < \rho w < 1200$ kg/(m^2 s), $0.02 < x < 0.2$.

Comparison of the slip ratios calculated by equations (7), and (8), is presented in Fig.1. It is seen that for $x > 0.02$ both equations give the values of the slip ratio between 3 and 4. Moreover, equation (8) takes into account the effect of the flow quality.

This research suggests approximating the isenthalpic process by the polytropic one defined by equation (3). The flow quality for isenthalpic process was calculated as follows:

$$x(P) = \frac{h'(P_1)(1 - x_1) + h''(P_1)x_1 - h'(P)}{h''(P) - h'(P)}. \tag{9}$$

Considering the obtained relationship (8), together with equations (3), (5), (6) and (9) we found n for all combinations of the input data, and approximated it as follows (Fig. 2):

$$n(x_1) = 0.42 + 0.45\left(1 - \exp\left(-x_1/0.078\right)\right), \quad 0.02 < x_1 < 0.20. \tag{10}$$
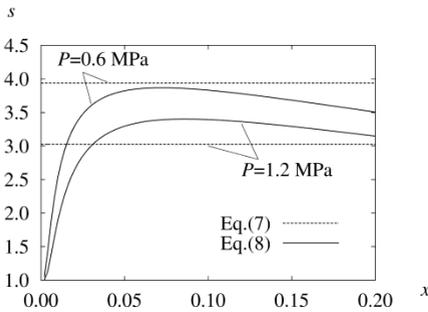

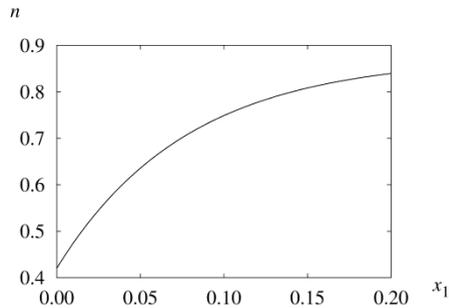**Fig.1.** Approximation of the slip ratio.

Fig. 2. Approximation of the polytropic coefficient, x1 – is the inlet flow quality.

3 The results of experiments and calculation

A description of an experimental equipment and a technique of the experiments are presented in detail in [1]. The same paper presents a table of results associated with a critical flow of vapor-water mixture through the random packed beds, 250 and 355 mm high. The experiments were carried out at the plant that contained a working segment represented by a vertical section in the form of a pipe with an internal diameter of 39 mm. The pipe had an insert of spherical particles which were steel balls, 2 and 4 mm in diameter. Before the working segment, water passed through the pump, heater and orifice. The high pressure water was throttled through orifice and the thus obtained vapor-liquid mixture was supplied to the working segment inlet. The flow quality x_1 after the orifice was determined by measuring the water temperature and pressure before the orifice, and the pressure of the saturated mixture after the orifice. The pressure applied at the working segment inlet was 0.6, 0.9 and 1.2 MPa. The outlet pressure was near atmospheric one. The flow rate was calculated using the volume method by measuring the time of filling the control balloon

placed after the working segment and condenser, where the mixture was changed to liquid and cooled to the room temperature.

In addition to the experiments [1] we obtained new experimental data for shorter packed beds, 50 and 100 mm high, consisting of steel balls, 2 mm in diameter, at a bed inlet pressure $P_1 = 0.6$ MPa (the data for $\left(d/H\right)^{0.5} = 0.14$ and 0.20 in Fig. 3(a)).

Figure 3 presents the calculation results for a critical mass velocity and the experimental data. Figure 3(a) shows the effect of the ratio of a particle diameter to bed height, while Fig. 3(b) illustrates the effect of pressure before the packed bed on $(\rho w)_{cr}$. The difference between calculations and experimental results does not exceed 7% throughout the entire array of available data.
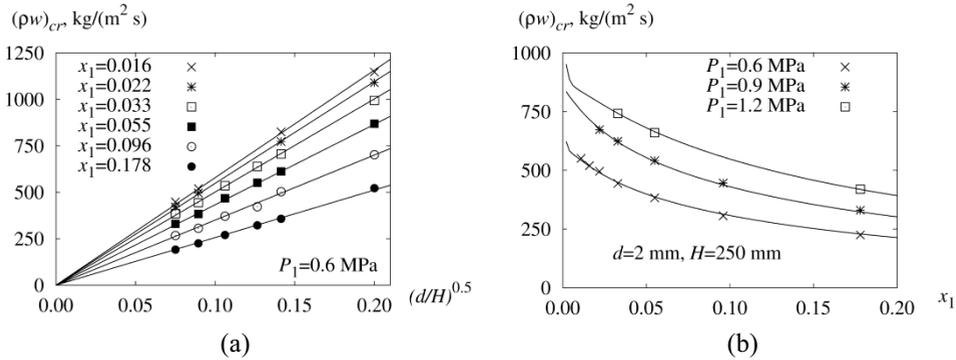

Fig.3. Effect of the ratio of a particle diameter to bed height (a), and the flow quality (b) on the critical mass velocity. Dots are experiment, lines show calculation.

The obtained results make it possible to conclude that the presented model has sufficiently good predictive capabilities for random packed beds of spherical particles within the considered range of operating parameters P_1 and x_1, as well as geometrical parameters d and H.

The proposed method can find a wider application with the emergence of new data which can be used to specify relationships (8) and (9). The considered approach to the calculation of critical two-phase flows in granular beds can be applied to packed beds of the particles of another geometrical shape different from the spherical one.

References

1. E.A. Tairov, B.G. Pokusaev, S.M. Bykova, High Temp. **54**, 261 (2016)

2. M.A. Goldstick, *Transfer processes in granular layer* (Novosibirsk, 2005) [in Russian]

3. S. Kim, I. Mudawar, Int. J. Heat Mass Transfer **87**, 497 (2015)

4. R.E. Henry, H.K. Fauske, Trans. ASME J. Heat Transfer **93**, 179 (1971)

5. A.A. Avdeev, R.I. Soziev, High Temp. **46**, 223 (2008)

6. A.A. Avdeev, B.F. Balunov, R.A. Rybin, R.I. Soziev, G.A. Filippov, High Temp. **41**, 377 (2003)