DOI: 10.1051/matecconf/201711505004

Modeling the multiphase flows in deformable porous media

Lyudmila Perepechko[1,*], *Evgeniy* Romenski[2], *Galina* Reshetova[3], *Sergey* Kireev[3], and *Yury* Perepechko[1,4]

[1]Kutateladze Institute of Thermophysics, SB RAS, 630090 Novosibirsk, Russia
[2]S.L. Sobolev Institute of Mathematics, SB RAS, 630090 Novosibirsk, Russia
[3]Institute of Computational Mathematics and Mathematical Geophysics, SB RAS, 630090 Novosibirsk, Russia
[4]V.S. Sobolev Institute of Geology and Mineralogy, SB RAS, 630090 Novosibirsk, Russia

Abstract. This work proposes the nonlinear model for the flow of mixture of compressible liquids in a porous medium with consideration of finite deformations and thermal effects. Development of this model is based on the method of thermodynamically consistent systems of conservation laws. Numerical analysis of the model is based on the WENO-Runge-Kutta method of the high accuracy. The model is developed to solve the problems arising when studying the different-scale fluid dynamic processes. Evolution of the wave fields in inhomogeneous saturated porous media is considered.

1 Introduction

This study deals with mathematical and numerical modeling of the deformable porous media saturated with a mixture of compressible liquids. The problem of wave process modeling in the multiphase saturated porous media and filtering a mixture of compressible media is relevant for solving a wide range of applied problems in various industries, as well as the problems arising from the analysis of the natural systems behavior, for example, when studying heat and mass transfer in the fluid-magmatic systems. The solution of this problem is especially relevant when creating new technologies for the development of hydrocarbon fields, analysis of dynamic processes in oil wells and borehole formations. The modern methods of hydrocarbon production are characterized by high pressure gradients in the bed and occurrence of significant rock deformations up to their destruction. The non-linear character of the flows under such conditions requires consideration of the shear stresses. The description of such hydrodynamic systems requires a consistent model of the flow of compressible media and deformation of the porous medium, and reliability of results of mathematical and numerical modeling should be provided both by the methods of deriving the coordinated equations of the model and the methods for their solution. In this study, the model of the flow of a liquid mixture of compressible phases through a porous deformable medium is constructed in the framework of the method of thermodynamically

[*] Corresponding author: ludmila@itp.nsc.ru

consistent system of conservation laws [1]. The equations of motion satisfy the fundamental laws: the laws of conservation of mass, momentum, and energy, invariance relative to Galileo transformations, and fundamentals of thermodynamics. The determining equations are hyperbolic, and this ensures the existence and uniqueness of the solution. An essential property of the derived equations is the possibility of their presentation in the form of conservation laws. These properties of the model equations allow us to apply the effective numerical methods for solving the hyperbolic systems of conservation laws. In this work, we used the algorithm based on the WENO-Runge-Kutta method of high accuracy, successfully used for modeling the two-phase compressible flows and filtering in a saturated porous medium [2-4].

2 Mathematical model

The three-phase medium is a deformable porous medium saturated with a mixture of compressible liquids. The phases are marked with the index “n”: where “1” corresponds to the porous matrix, “2” and “3” correspond to saturating liquids, e.g. water and oil. The unit volume of such a medium is characterized by: volume concentrations α_n ($\alpha_1+\alpha_2+\alpha_3=1$, $\phi=1-\alpha_1$ is porosity), mass concentrations c_n ($c_1+c_2+c_3=1$); mass densities and phase velocities ρ_n and u_i^n, and velocities of liquid phases motion relative to the porous matrix $w_i^n=u_i^1-u_i^n$. The three-phase medium is also characterized by density $\rho=\sum_{n=1}^{3}\alpha_n\rho_n$ and velocity $u_i=\sum_{n=1}^{3}c_n u_i^n$ ($c_n=\alpha_n\rho_n/\rho$), tensor of elastic strain gradient F_{ij}, and mass density of entropy s. Inner energy of the three-phase medium $e=e(\rho,\alpha_n,c_n,w_i^n,F_{ij},s)$ is determined as the internal phase energy e^n, averaged by mass concentration, and supplemented with the kinetic energy of relative phase motion:

$$e\left(\rho,\alpha_n,c_n,w_k^n,F_{ij},s\right)=c_1e^1\left(\rho_1,F_{ij},s\right)+c_2e^2\left(\rho_2,s\right)+c_3e^3\left(\rho_3,s\right)+$$
$$+\frac{1}{2}\left(1-c_2\right)c_2w_k^2w_k^2+\frac{1}{2}\left(1-c_3\right)c_3w_k^3w_k^3-c_2c_3w_k^2w_k^3\,. \tag{1}$$

Generalized internal energy e depends on total entropy s of the three-phase system due to the assumption of temperature phase equilibrium. Dependence of e^1 on F_{ij} is given by the deviator of Finger tensor g_{ij} [1]. The equation of state is determined by relationships for the pressure in phases $p_n=\rho_n^2e_{\rho_n}^n$ (with $p=\rho^2e_\rho$), shear stresses $\sigma_{ik}=\rho_1F_{kj}e_{F_{ij}}^1$ and temperature $T=e_s$.

Thermodynamically consistent system of conservation laws for the flow of mixture of compressible liquids through a deformable matrix takes the form [3,4]:

$$\frac{\partial\alpha_n\rho_n}{\partial t}+\partial_k\left(\alpha_n\rho_n u_k^n\right)=0\,,\qquad n=2,3, \tag{2}$$

$$\frac{\partial\rho\alpha_n}{\partial t}+\partial_k\left(\rho\alpha_n u_k\right)=-\sum_{m=2}^{3}\lambda_{nm}\left(p_1-p_m\right),\qquad n=2,3, \tag{3}$$

$$\frac{\partial w_k^n}{\partial t}+\partial_k\left(h^1-h^n\right)=e_{kij}u_i\omega_j^n-\sum_{m=2}^{3}\chi_{nm}c_m\left(u_k-u_k^m\right),\quad n=2,3 \tag{4}$$

$$\frac{\partial\left(\rho u_i\right)}{\partial t}+\partial_k\left(\sum_{n=1}^{3}\alpha_n\rho_n u_i^n u_k^n+\delta_{ik}\sum_{n=1}^{3}\alpha_n p_n-\alpha_1\sigma_{ik}\right)=0\,, \tag{5}$$

$$\frac{\partial \rho F_{ij}}{\partial t}+\partial_k\left(\rho F_{ij}u_k-\rho F_{kj}u_i\right)=0, \tag{6}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_k}{\partial x_k}=0, \tag{7}$$

$$\frac{\partial \rho s}{\partial t}+\partial_k\left(\rho s u_k\right)=\frac{R}{T}. \tag{8}$$

The considered processes of relaxation of volume phase concentrations to the equilibrium value and relaxation of the phase velocities due to interfacial friction determine the form of dissipative function:

$$R=\sum_{n,m=2}^{3}\chi_{nm}\left(u_i-u_i^n\right)\left(u_i-u_i^m\right)+\frac{1}{\rho^2}\sum_{n,m=2}^{3}\lambda_{nm}\left(p_1-p_n\right)\left(p_1-p_m\right). \tag{9}$$

Here $h^n=e^n+\frac{1}{2}u_i^n u_i^n+p_n/\rho_n$, $e_{ikj}\omega_j^n=\partial_i w_k^n-\partial_k w_i^n$. Variables ω_j^n satisfy the additional conservation laws [2], which are the compatibility conditions for the solutions of system (2) - (8). This allows us to interpret terms $e_{klj}u_l\omega_j^n$ in (5) as the lower terms. Kinetic coefficients λ_{ij} characterize relaxation of pressures in the phases. The coefficients of interfacial friction χ_{ik} are determined by the Darcy formulas with relative permeabilities k_{nm}.

The equations formulated above ensure the fulfillment of the energy conservation law

$$\frac{\partial}{\partial t}\left(\rho e+\frac{1}{2}\rho u_i u_i\right)+\partial_k\left(\sum_{n=1}^{3}\alpha_n\rho_n u_k^n\, h^n-u_i\alpha_1\sigma_{ik}\right)=0. \tag{10}$$

The model describes the flow of a mixture of compressible liquids through a deformable porous medium in the case of finite deformations with consideration of the thermal phenomena.

3 Modeling results

Propagation of nonlinear wave fields near the boundary of the water-oil mixture and fluid-saturated porous formation in the presence of the oil-water contact near the boundary was investigated as an application to the model. The water-oil contact was set in the calculation region ($5\times 4\,\mathrm{m}$) at the distance of $1.5\,\mathrm{m}$ to the left of the boundary between the porous matrix and the water-oil mixture (Fig. 1a). The initial volume concentrations in water-oil mixture were $\alpha_{20}=0.8$ for water and $\alpha_{20}=0.2$ for oil. The parameters of the porous medium layers were taken as follows: for layers (1) and (2) porosity was $\phi=0.1$ and permeabilities were $k_{22}=0.2\cdot 10^{-12}\,\mathrm{m}^2$, $k_{33}=0.2\cdot 10^{-11}\,\mathrm{m}^2$; for layer (2) porosity was $\phi=0.2$ and permeabilities were $k_{22}=k_{33}=0.2\cdot 10^{-10}\,\mathrm{m}^2$. Kinetic coefficients λ_{ij}, characterizing relaxation of pressures in the phases, were taken: $\lambda_{22}=0.11\cdot 10^{-6}\,\mathrm{c/m}^2$, $\lambda_{33}=0.69\cdot 10^{-6}\,\mathrm{c/m}^2$, and $\lambda_{23}=0.4\cdot 10^{-8}\,\mathrm{c/m}^2$. Shear modulus of porous medium were taken $8.9\cdot 10^9\,\mathrm{Pa}$. The initial values of thermodynamic parameters corresponded to the normal conditions. The initial values of velocities were assumed to be zero.

The stress and pressure fields for the moment of acoustic wave propagation in the region of water-oil contact are shown in Fig. 1. The pressure source of the Ricker type is located in the center of the upper part of the region (3). The location of water-oil contact is marked in Fig. 1 by a dash-dotted line. The boundaries of horizontal layers with different values of porosity and permeability are marked by the dashed lines. Sensitivity of the wave fields to the presence of the water-oil contact and isolated layers in a porous medium is observed. The observed effect is more pronounced for the pressure field.

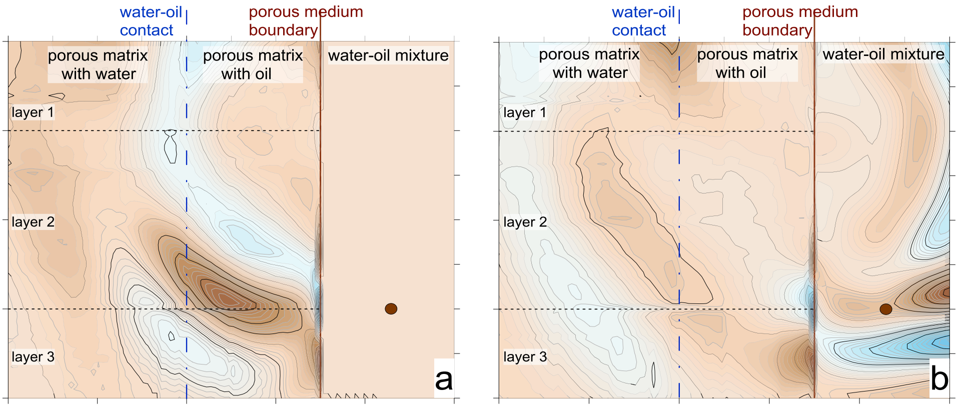

Fig. 1. The fields of stress σ_{xx} (a) and pressure p (b). The pressure source is marked by a circle.

The developed thermodynamically matched mathematical model of multiphase mixtures in the porous media and applied finite-difference algorithm of a high accuracy based on the WENO-Runge-Kutta method make it possible to study the features of the multiphase flow in the deformable porous media in a wide range of model parameters. The model allows us to investigate acoustic wave propagation on a hydrodynamic background, accompanied by dissipative phenomena and temperature variations, and the effect of acoustic action on the character of multiphase flow of fluids in porous media; solve non-stationary problems of non-isothermal multiphase filtration of mixtures of compressible liquids in porous media under the conditions of significant elastoplastic deformations.

The work was supported by the Russian Foundation for Basic Research, grant No. 16-29-15131.

References

1. S. Godunov, E. Romenski, *Elements of Continuum Mechanics and Conservation Laws* (Kluwer Academic/Plenum Publishers, NY, 2003)
2. E. Romenski, D. Drikakis, E.F. Toro, J. Sci. Comput. **42**, 68 (2010)
3. Y.V. Perepechko, E.I. Romenski, G.V. Reshetova, Seismic Technologies **4**, 78 (2014)
4. Yu. Perepechko, E. Romenskiy, G. Reshetova, L. Perepechko, V. Malyshkin, K. Kalgin, S. Kireev, M. Ostapkevich, In book: *Supercomputer Technologies in Science, Education and Industry* 119 (MSU, Moscow, 2013) [in Russian]