The simulation solution of nonlinear problems of ergodic electric-power processes transformation

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Abstract. The algorithm for the simulation solution of the stochastic electric-power processes problems (quadratic inertial smoothing (QIS) and quadratic cumulative averaging (QCA)) has been stated. It includes simulation of initial process realizations, transformation of realizations and ensemble statistical analysis of these processes, as well as simulation quality check by the test problems. Peculiarities of skewness, kurtosis and rated maxima (quantiles) calculations based on the ensemble of the stochastic ergodic process realizations have been considered. Density and cumulative distribution function of the quadratic stochastic processes with normal distribution have been analyzed. Ensemble and single realization errors for rated maxima have been evaluated. It has been proposed to use test check with distribution quantiles when searching for simulation solution of QIS on the stage of quadratic transformation. Approximation of rated maxima using the gamma distribution law is given.

1 Introduction

The current standard for electromagnetic compatibility (EMC) [1] involves the principle of quadratic cumulative averaging [2] to take into account the inertia of objects. The rated maximum of half-hour load [3] also implements a cumulative principle. A simulation solution for a nonlinear problem has been proposed in [4] through the example of quadratic inertial smoothing (QIS) of stochastic electric-power processes. The inertial principle [5] is free from the disadvantages inherent in the cumulative principle of EMC evaluation. The standard [6] is based on the inertial principle of EMC interference evaluation. Some approximate approaches for solution of the problem have been considered in [7].

The detailed definition of numerical characteristics, cumulative function and density distribution for a random variable is given in [8-9] and for stochastic processes in [9-10]. In practice, as a rule, single realizations of stochastic ergodic processes are considered. The simulation solution implies the statistical manipulation of characteristics in the ensemble of stochastic process realizations. The presented material is based on the example of heating an object with the constant of inertia *T*.

2 A heat model

The superheat temperature of the object and the current are related by the linear differential equation

$$T\vartheta'_T + \vartheta_T = c_\vartheta I^2 = c_\vartheta z, \tag{1}$$

where $z = \vartheta_0 = I^2$, and the coefficient c_ϑ is equal to the ratio of the continuous permissible (symbol $\cap$) temperature $\hat{\vartheta}$ in ºC to the square admissible continuous current $\hat{I}$ in amperes.

The mean temperature

$$\vartheta_c = c_\vartheta I_e^2 = c_\vartheta z_e \tag{2}$$

is proportional to the squared effective value I_e (in short − effective, rms) or rms z_e of quadratic process *z*(*t*). They don't depend on *T*.

In addition to the continuous permissible the maximum permissible temperature is standardized $\hat{\vartheta}_\text{M}$.

The rated maximum $\vartheta_{T\text{M}}$ for stochastic process $\vartheta_{T\text{M}}(t)$ is determined by a cumulative distribution function $F_{\vartheta T}(\vartheta_T)$ from the equation

$$F_{\vartheta T}(\vartheta_{T\text{M}}) = 1 - E_x. \tag{3}$$

As a rule, a marginal probability E_x is taken in the range from 0.05 to 0.001 (for example, in [1] it is taken equal to 0.05 for the voltage quality indicators). If the rated maximum of the temperature $\vartheta_{T\text{M}}$ exceeds the $\hat{\vartheta}_\text{M}$, then the object must be turned off.

Let us consider a probabilistic model of load variation [11-12]. The normal distribution law of the process *I*(*t*) is characterized by the mean I_c and the standard deviation σ_I. An exponential autocorrelation function (CF) for the electric-power processes is proposed in [2]:

$$K_I(\tau) = \sigma_I^2 \exp\{-\alpha|\tau|\}. \tag{4}$$

The parameter α is inverse to the correlation time τ_k. An exponential-cosine CF is used in [12-13]:

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$$K_I(\tau) = \sigma_I^2 \exp\{-\alpha|\tau|\}\cos(\beta\tau). \qquad (5)$$

The parameter β characterizes the periodic component of the process. The effective current is $I_e = \left(I_c^2 + \sigma_I^2\right)^{0.5}$.

A per-unit system (pu, symbol $*$) is introduced for generalization. Base values are the correlation time τ_k and standard deviation σ_I In per-unit system $I_{c*} = I_c / \sigma_I$, $\sigma_{I*} = 1$, $T_* = \alpha T$, $\tau_* = \alpha\tau$, $\alpha_* = 1$. After replaceing in (1) I^2 with I_{c*}^2 / σ_{I*}^2 we obtain

$$\vartheta_{T*} = \vartheta_T / c_\vartheta \sigma_I^2 .$$

Taking into account that $dt_* = \alpha dt$ and $T_* = \alpha T$ the equation (1) can be expressed as

$$T_* \vartheta'_{T*} + \vartheta_{T*} = I_*^2 . \qquad (6)$$

The per-unit system allows us to simulate the current for the values $\alpha_* = 1$, $\sigma_{I*} = 1$ and I_{c*} only. The resulting T_*-characteristics are universal. They cover any T, α, I_c and σ_I values in actual values.

The block diagram of a heat model is shown in Figure 1. A stationary stochastic process $I_*(t)$ is an input to the model. QIS transformation is implemented using a squarer (block 1) and the 1-st order inertial link (block 2). A quadratic inertial process $\vartheta_{T*}(t)$ is an output of block 2. In case of need square-rooting is performed to obtain a reduced inertial process $I_{eT*}(t)$.

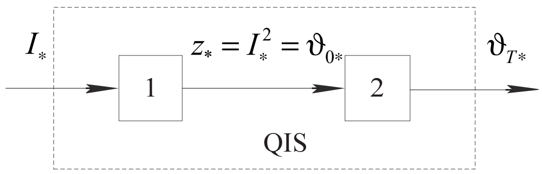

Fig. 1. A heat model.

The exact solution for the distribution density of the process is unknown. Only some numerical characteristics can be determined precisely. For CF of the form (4) mean value is

$$\vartheta_{Tc} = I_e^2 = I_c^2 + \sigma_I^2, \qquad (7)$$

the standard deviation is

$$\sigma_{\vartheta T} = \sqrt{2}\sigma_I \sqrt{\frac{\sigma_I^2}{1+2\alpha T} + \frac{2I_c^2}{1+\alpha T}}, \qquad (8)$$

in pu

$$\vartheta_{Tc*} = I_{e*}^2 = I_{c*}^2 + \sigma_{I*}^2 = I_{c*}^2 + 1, \qquad (9)$$

$$\sigma_{\vartheta T*} = \sqrt{2}\sigma_{I*} \sqrt{\frac{\sigma_{I*}^2}{1+2T_*} + \frac{2I_{c*}^2}{1+T_*}} = \sqrt{2}\sqrt{\frac{1}{1+2T_*} + \frac{2I_{c*}^2}{1+T_*}}. \qquad (10)$$

3 Test problems

Subject to the existence of an inertia there can only be a simulation solution of the problem, so it is advisable to set high requirements for the quality of initial stochastic process realizations, as well as to verify the simulation solution on basis of test problems.

A test problem (a check) is the special case, which an exact analytic solution can be found for. In regard to QIS transformation of stochastic electric-power processes the two test problems are given in [4]: a quadratic inertialess transformation and QIS transformation when the mean value of an initial process is equal to 0. In the first case an exact analytic solution is known after transformation (the process distribution law), in the second case – solution is given for moments of distribution.

3.1 Quadratic inertialess transformation

A quadratic process $z(t)$ occurs at the output of block 1 (Fig.1). It is possible to verify the simulation quality at this stage. The calculation is performed for the ensemble simulation density and distribution function to compare them with the exact solution. It is also possible to evaluate errors of distribution moments though it is not necessary in this case.

The distribution density of a quadratic stationary stochastic process $z = I^2$ is given by the exact formula in [10]:

$$f_z(z) = \frac{1}{\sqrt{2\pi}\sigma_I 2\sqrt{z}} \times \left[\exp\left\{-\frac{\left(\sqrt{z} - I_c\right)^2}{2\sigma_I^2}\right\} + \exp\left\{-\frac{\left(-\sqrt{z} - I_c\right)^2}{2\sigma_I^2}\right\}\right]. \qquad (11)$$

The distribution density can be expressed through a hyperbolic function

$$f_z(z) = \frac{1}{\sqrt{2\pi}\sigma_I \sqrt{z}} \exp\left\{-\frac{z + I_c^2}{2\sigma_I^2}\right\} \mathrm{ch}\left\{\frac{I_c\sqrt{z}}{\sigma_I^2}\right\}. \qquad (12)$$

Considering quadratic transformation $z \geq 0$.

Kolmogorov-Smirnov test gives evaluation in *Y*-direction of the distribution function $F_z(z)$, but evaluation of maximum rated values in accordance with practical confidence principle [14] is performed in *X*-direction. The distribution can have a considerable length (Fig.2) in the area of the rated values (long «tails»). This can introduce a significant error. It is proposed to increase the accuracy in addition to the evaluation with Kolmogorov-Smirnov test further to test simulation solutions for quantiles of inertialess quadratic transformation.

3.2 QIS of zero-mean initial process

The second test check is run for $T > 0$. In this case the exact solution can only be found for the moments of a process distribution $\vartheta_{T*}(t)$. In pu the dispersion

$$D_{\vartheta T*} = D_{\vartheta T} / \sigma_I^4 = \mu_{\vartheta T 2*} = \frac{2}{1+2T_*} + \frac{4I_{c*}^2}{1+T_*}, \qquad (13)$$

where $\mu_{\vartheta T2*}$ – is the second central moment in pu. The mean value is determined by the formula (9).

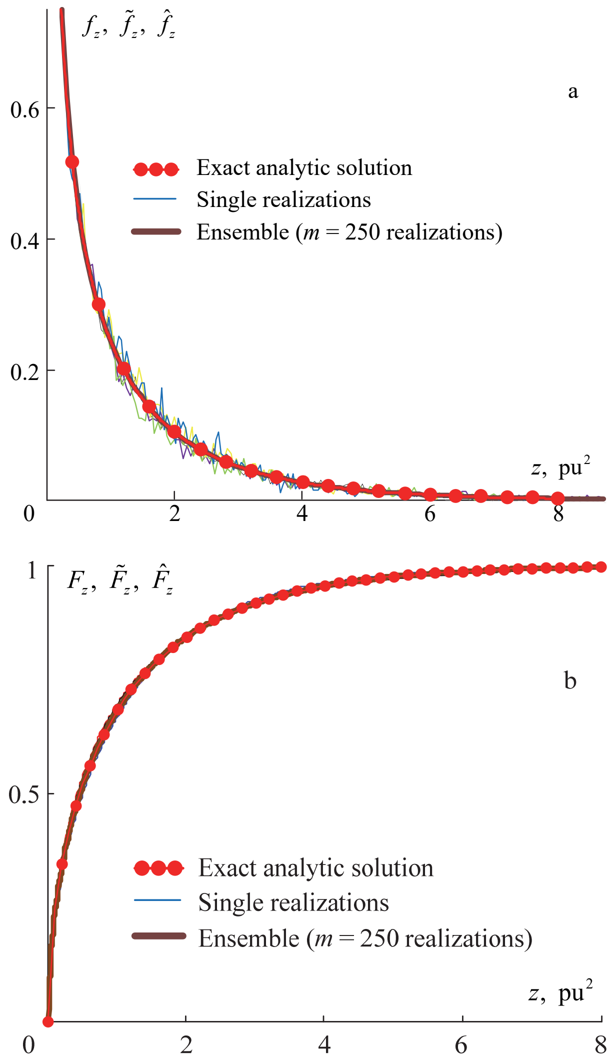

Fig. 2. Distribution law of a quadratic stationary stochastic process ($I_{c*} = 0$ pu): density (a) and cumulative function (b).

In theory the general formulae derived from [10] allow us to define the third and fourth central moments with any mean values of I_c, but the result acceptable for practical application can be received only when $I_c = 0$.

Leaving out simple but cumbersome calculations according to the formulae (34.33) and (34.27), we give the finite expressions:

$$D_{\vartheta T*} = \frac{2}{1+2T_*}, \tag{14}$$

$$\mu_{\vartheta T3*} = \mu_{\vartheta T3}/\sigma_I^6 = \frac{8}{(1+T_*)(1+2T_*)}, \tag{15}$$

$$\mu_{\vartheta T4*} = \mu_{\vartheta T4}/\sigma_I^8 = \frac{12(15+25T_*+2T_*^2)}{(1+2T_*)^2(3+5T_*+2T_*^2)}. \tag{16}$$

The skewness and kurtosis are

$$\mathrm{Sk}_{\vartheta T} = \frac{2\sqrt{2}}{1+T_*}\sqrt{1+2T_*}, \qquad \mathrm{Ex}_{\vartheta T} = \frac{36+50T_*}{3+5T_*+2T_*^2}. \tag{17}$$

4 Algorithm for the simulation solution

The main statements of the method to obtain the simulation solution of the QIS problem have been given in [4]. Let us state the algorithm taking into account that it is mostly common to use reduced values which are equidimensional with the initial process, in instances, when power quality factors are evaluated (a generalized model for EMC estimation is given in [15]).

1. The simulation of ensemble initial realizations of a stochastic process.
2. The verification and correction of mean values and standard deviations in the simulated initial realizations.
3. The verification of the correlation function (CF) reproduction precision.
4. The verification of the initial process distribution law.
5. The quadratic transformation of the process realizations.
6. The verification of the ensemble quadratic process distribution law and ensemble quantiles.
7. The inertial smoothing of the quadratic process realizations.
8. Obtaining a reduced inertial process.
9. The calculation of the process realizations numerical characteristics (mean value, standard deviation, variance, 3-d and 4-th order distribution moments).
10. Averaging to obtain the ensemble numerical characteristics.
11. The ensemble skewness and kurtosis calculation.
12. The calculation of the simulation density and cumulative distribution function of process realizations.
13. Averaging to obtain the ensemble simulation density and cumulative distribution function.
14. The quantiles calculation (rated maxima) of quadratic inertial or reduced inertial processes (using the ensemble cumulative distribution function).

5 Ensemble characteristics

The proposed algorithm contains the calculation of process numerical characteristics. The single realizations calculation (symbol ~) and ensemble averaging (symbol ^) are used. Such characteristics as skewness and kurtosis are useful for the test problem solutions, while they are not the ultimate goal of calculations. The calculation of the skewness and kurtosis is performed with ensemble averaged values of standard deviation, 3rd and 4th central moments:

$$\hat{\mathrm{Sk}} = \frac{\hat{\mu}_3}{\hat{\sigma}_z^{1.5}} = \frac{\mathrm{M}\{\tilde{\mu}_3\}}{\mathrm{M}\{\tilde{\sigma}_z\}^{1.5}}, \qquad \hat{\mathrm{Ex}} = \frac{\hat{\mu}_4}{\hat{\sigma}_z^2} - 3 = \frac{\mathrm{M}\{\tilde{\mu}_4\}}{\mathrm{M}\{\tilde{\sigma}_z\}^2} - 3. \tag{18}$$

Required rated maxima are determined with an ensemble cumulative distribution function

$$\hat{F}_z(z) = \mathrm{M}\{\tilde{F}_z(z)\} \tag{19}$$

6 Approximation by the gamma distribution law

The density can be approximated by dint of gamma distribution when $T = 0$:

$$f_{\vartheta 0}(\vartheta_0)=\frac{\lambda^{\eta}}{\Gamma(\eta)}\vartheta_0^{\eta-1}\exp\{-\lambda\vartheta_0\}, \qquad (20)$$

where $\Gamma(\eta)$ – is complete gamma function determined by a dimensionless shape parameter η. The scale parameter λ has a dimension of $(\text{pu})^{-2}$. Both parameters depend on the effective value I_e of the initial process $I(t)$ and standard deviation σ_z of the quadratic process $z(t)$. In case of the quadratic inertialess transformation:

$$\lambda=\frac{I_e^2}{\sigma_{\vartheta 0}^2}=\frac{I_c^2+\sigma_I^2}{2\sigma_I^2\left(\sigma_I^2+2I_c^2\right)},$$
$$\eta=\frac{I_e^4}{\sigma_{\vartheta 0}^2}=\frac{\left(I_c^2+\sigma_I^2\right)^2}{2\sigma_I^2\left(\sigma_I^2+2I_c^2\right)}. \qquad (21)$$

Let us define numerical characteristics of the gamma distribution when approximating. The mean value is

$$\vartheta_{0c}=\frac{\eta}{\lambda}=\frac{I_e^4/\sigma_{\vartheta 0}^2}{I_e^2/\sigma_{\vartheta 0}^2}=I_e^2. \qquad (22)$$

The standard deviation is

$$\sigma_{\vartheta 0}=\frac{\sqrt{\eta}}{\lambda}=\sqrt{2}\sigma_I\sqrt{\sigma_I^2+2I_c^2}. \qquad (23)$$

The skewness and kurtosis are

$$\text{Sk}=\frac{2}{\sqrt{\eta}}=\frac{2}{\sqrt{I_e^4/\sigma_{\vartheta 0}^2}}=\frac{2}{I_e^2/\sigma_{\vartheta 0}}=$$
$$=\frac{2\sqrt{2}\sigma_I\sqrt{\sigma_I^2+2I_c^2}}{\sigma_I^2+I_c^2}, \qquad (24)$$

$$\text{Ex}=\frac{6}{\eta}=\frac{6}{I_e^4/\sigma_{\vartheta 0}^2}=\frac{12\sigma_I^2\left(\sigma_I^2+2I_c^2\right)}{\left(I_c^2+\sigma_I^2\right)^2}. \qquad (25)$$

If the mean value of the initial process is zero, the densities calculted according to the formulae (12) and (20) are exactly the same. But when $I_c > 0$, the distribution types are different (Fig.3).

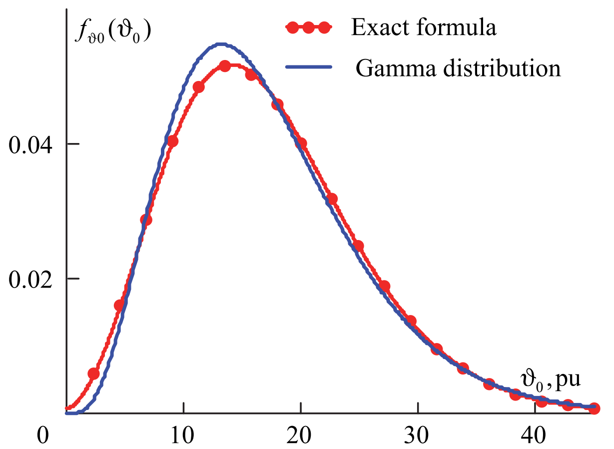

Fig. 3. The distribution density of a quadratic stationary stochastic process and gamma distribution law ($I_{c*} = 4$ pu).

For the problem under consideration the quality of the approximation is evaluated based on how accurately the inverse distribution of a quadratic process is reproduced when $E_x = 0.05$. The percentage error is calculated using a formula :

$$\delta=\left(\frac{\vartheta_{0\text{M}\Gamma}}{\vartheta_{0\text{M}}}-1\right)100\%,$$

where $\vartheta_{0\text{M}}$ is an exact solution according to (12), $\vartheta_{0\text{M}\Gamma}$ is an approximate solution according to (21). The calculation results for different mean values are given in Table 1.

Table 1. Errors of approximation: $T_* = 0$, $E_x = 0.05$.

I_{c*}, o.e.	$\vartheta_{0\text{M}*}$	$\vartheta_{0\text{M}\Gamma*}$	δ, %
0	3.841	3.841	0
1	7.002	6.928	−1.057
2	13.285	13.366	0.61
3	21.575	21.804	1.061
4	31.867	32.183	0.992
5	44.154	44.528	0.847

While approximating the error when defining the rated maxima can be both negative and positive.

Let us consider the gamma distribution characteristics in case of a nonzero inertia. The distribution density when $T > 0$ is

$$f_{\vartheta T}(\vartheta_T)=\frac{\lambda^{\eta}}{\Gamma(\eta)}\vartheta_T^{\eta-1}\exp\{-\lambda\vartheta_T\}. \qquad (26)$$

The parameters are defined according to the expressions:

$$\lambda=\frac{I_e^2}{\sigma_{\vartheta T}^2},\quad \eta=\frac{I_e^4}{\sigma_{\vartheta T}^2}. \qquad (27)$$

Taking into account (10), in pu

$$\lambda_*=\frac{I_{c*}^2+1}{\dfrac{2}{1+2T_*}+\dfrac{4I_{c*}^2}{1+T_*}},$$
$$\eta_*=\frac{\left(I_{c*}^2+1\right)^2}{\dfrac{2}{1+2T_*}+\dfrac{4I_{c*}^2}{1+T_*}}. \qquad (28)$$

7 Simulation

The initial simulation process $I(t)$ consists of $m = 250$ realizations with $I_{c*}=0$ and exponential CF. Single realization minimum and maximum values of z_M (Fig.4, large bright circles) were 3.613 and 4.044 pu^2.

Ensemble value $\hat{z}_M = 3.837$ pu^2 (a large dark circle). The evaluation by formulas (12) gives the exact value equal to 3.841 pu^2.

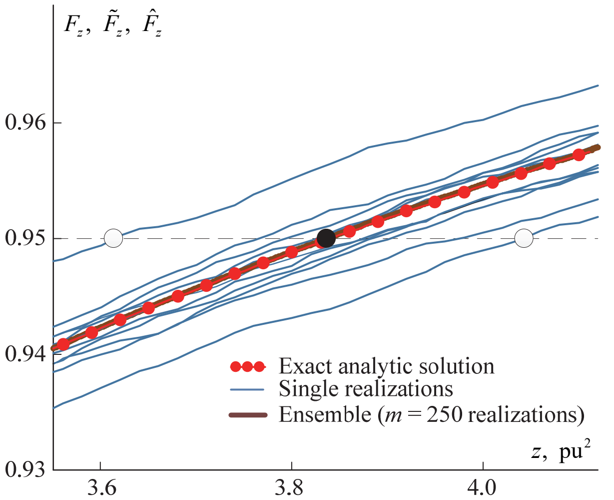

Fig. 4. Cumulative distribution functions of a quadratic stochastic process and rated maxima.

The maximum and minimum ratio errors for the single realization rated maxima z_M were −5.94% and 5.29%. The ratio error for the ensemble was 0.104%. It confirms the necessity to use ensemble of realizations of a stochastic process in the simulation solution of nonlinear transformations of the stochastic power processes, because even inertialess nonlinear transformation of the estimated maxima for individual realizations are reproduced with considerable error.

Simulation skewness and kurtosis of a quadratic inertial process $\vartheta_{T*}(t)$ taken over ensemble and realizations are given in Fig.5. The exact solution is calculated according to (17).

The distribution moments based on the single realizations differ greatly from the exact values. As it can be expected the dispersion for kurtosis is greater than the dispersion for skewness. The ensemble-averaged values of these characteristics are close to the exact values.

The simulation solution of the QIS problem is given in Fig.6 in the form of T_*-characteristic. The simulation is performed for every value of I_{c*}. The results of the approximation with the help of the gamma distribution based on the formulae (26-28) are marked with circles.

The quantitative meanings of maxima and approximation errors are partially shown in Tables 2 and 3. It is necessary to point out that the approximation errors of the process rated maxima $\vartheta_{T*}(t)$ when E_x = 0.05 do not exceed 1.2% as per their absolute value. At the same time when E_x = 0.001 the approximation quality essentially deteriorates. According to the experiment data there is no steady error reduction.

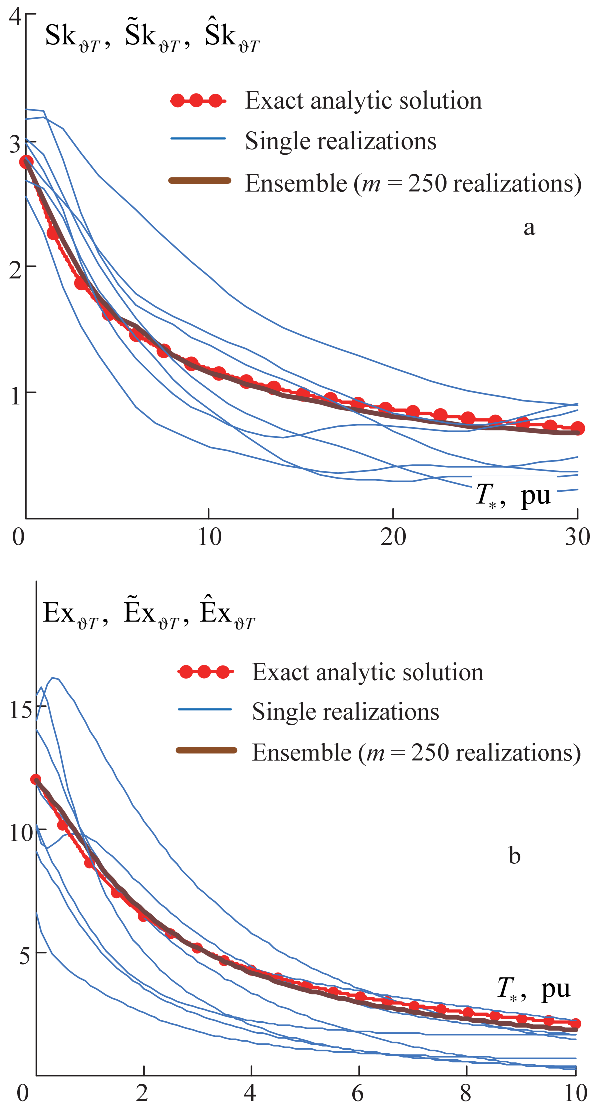

Fig. 5. Numerical characteristics of quadratic inertial process: skewness (a) and kurtosis (b).

The maximum error is 1.176% when E_x = 0.05, and −14.562% when E_x = 0.001.

Table 2. Errors of approximation: I_{c*} = 1, E_x = 0.05.

T_*	$\hat{\vartheta}_{TM*}$	$\vartheta_{TM\Gamma*}$	δ, %
1	5.272	5.21	−1.176
3	4.195	4.154	−0.977
9	3.309	3.294	−0.453
15	3.001	3.003	0.067
21	2.833	2.845	0.424
30	2.681	2.704	0.698

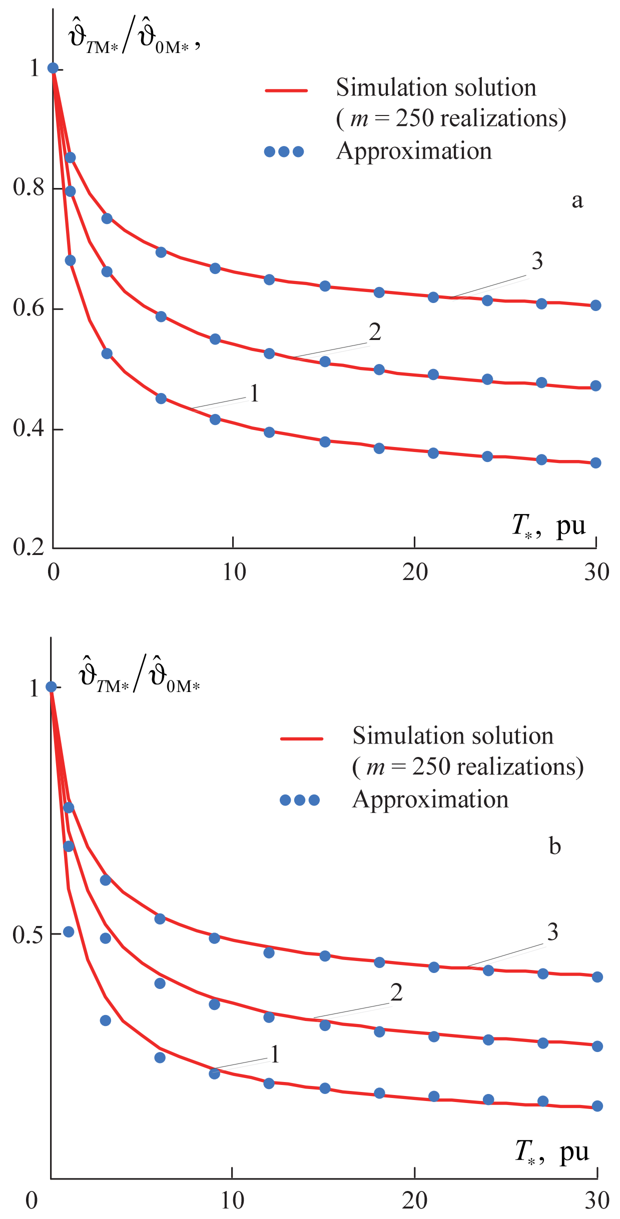

Fig. 6. T_* -characteristics of quadratic inertial process and their approximation when E_x = 0.05 (a) and E_x = 0.001 (b): 1 – $I_{c*} = 0$ pu; 2 – $I_{c*} = 2$ pu; 3 – $I_{c*} = 4$ pu.

Table 3. Errors of approximation: $I_{c*} = 0$, $E_x = 0.001$.

T_*	$\hat{\vartheta}_{TM*}$	$\vartheta_{TM\Gamma*}$	δ, %
1	6.353	5.442	−14.652
3	3.983	3.475	−12.749
9	2.403	2.306	−4.039
15	1.962	1.971	0.446
21	1.747	1.8	1.891
30	1.568	1.611	4.768

8 Conclusion

To increase the simulation solution accuracy of the nonlinear problems of ergodic electric-power process transformation it is reasonable to fulfil the statistical analysis of ensemble realizations rather than single realizations.

The rated maxima of the quadratic inertial process can be approximated by gamma distribution with a relatively low error when $E_x = 0.05$.

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