# On the stick-slip vibration in the suspension of a freight wagon

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**Abstract.** Damping based on the dry friction is frequently in suspensions of the freight wagons. The paper presents some aspects of vertical vibration of freight wagons. It uses a simple model considering that the vehicle was reduced to a mass suspended on a wheel, moving at constant speed on a rigid track with harmonic irregularity. Stick-slip vibration can occur due to the friction and it is characterized by sudden changes in the wheel acceleration affecting the ride quality. The paper shows the influence of stick-slip vibration on the wheel-rail dynamic force.

## 1 Introduction

Railway vehicle suspension is equipped in many cases with damping elements based on the dry friction [1]. We refer mainly to the suspension of freight wagons which almost all are fitted with such damping elements: leaf springs (two-axle wagons or the bogies HOURS or H), Lenoir devices (Y 25 bogie wagons) and damping with friction wedges (Diamond bogie wagons).

The presence in the suspension systems of the damping with dry friction is likely to cause vibration stick-slip which could affect the ride quality and induce excessive wheel-rail contact forces which result in reducing the fatigue strength of both running gear of the vehicle and track superstructure, as well as increased wear of the wheel/rail rolling surfaces.

Modelling dry friction damping elements is mainly based on the model of Coulomb friction [2]. Friction does not depend on sliding speed, but has its opposite. Coulomb's model can be improved to a certain extent admitting as in reality that static friction is greater than sliding friction.

Mathematical model of the Coulomb friction uses the signum function. Although it is very simple, this model raises serious numerical integration problems when used for modelling the suspension systems of the vehicles. It's mainly the so-called numerical instability whose effect consists of high-frequency vibration that do not has physical correspondent. From the mathematical point of view, this numerical instability is given by nature signum function which is discontinuous at zero crossing. To overcome this, the method is applicable regularization or harmonic balance method. If the first approach, signum function is replaced by another which has a similar shape, but is still in the home. In the case

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of the second approach, it is considered an equivalent to a harmonic series that is modulated both in phase and in amplitude [3]. Another method is the so-called switching method that considers the integration of equations of motion distinct phases with changing the number of degrees of freedom depending on the outcome of checks on the conditions of slipping and adhesion [4].

This paper deals with the forced vibrations of a system with dry friction elastic consists of two rigid bodies arranged vertically connected by two elements, one linear elastic characteristic and the other is a dry friction damping. The system is in permanent contact with a rigid base undulating through the lower body. It is considered that the system moves along the rigid base, and its form is sinusoidal type. Interesting aspects of the stick-slip vibrations and their influence on dynamic contact force between lower body and rigid base. The described model is a simplified representation of a freight wagon suspension friction as dry as, for example, wagon equipped with bogies Y 25.

## 2 Equations of motion

The mechanical model is presented in Figure 1, comprising two rigid bodies connected by a spring element that works together with a dry friction element. This system can simply model 1/8 freight wagon moving at a constant speed $V$ on a track with vertical irregularity $\eta$. Indeed, the mass rigid body $M$ is the suspended mass of the vehicle corresponding to a wheel and the rigid body of mass $m$ is the wheel. Suspension stiffness is $k$ and friction force developed in the damping element is $F_f$. Displacement of the body $M$ is $z$, and $\eta$ is the displacement of the wheel.

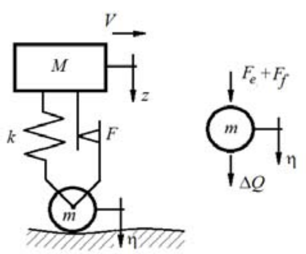


**Fig. 1.** Mechanical model.

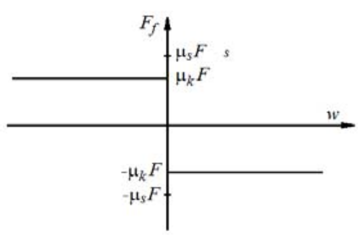


**Fig. 2.** Friction force versus slip velocity.

Friction force has the following form, depending on the slip velocity (see Fig. 2)

$$F_f = -\mu_k N \text{sign}(w) \text{ for } w \neq 0 \tag{1}$$

$$\left|F_f\right| \leq \mu_s N \text{ for } w = 0, \tag{2}$$

where $\mu_k$ is the kinetic friction coefficient, $\mu_s$ - static friction coefficient, $N$ – normal force and $w$ is the slip velocity.

Equations of motions are

$$\begin{aligned} M\ddot{z} + k(z-\eta) + F_f &= 0 \\ m\ddot{\eta} + k(\eta - z) - F_f &= \Delta Q, \end{aligned} \tag{3}$$

where $\Delta Q$ is the dynamic force between wheel and track.

New variable is introduced in order to solve the equations of motion

$$u = z - \eta . \tag{4}$$

Equations of motion become

$$\begin{aligned} M\ddot{u} + ku + F_f &= -M\ddot{\eta} \\ m\ddot{\eta} - ku - F_f &= \Delta Q. \end{aligned} \tag{5}$$

The relative motion between the two bodies exhibits two distinctive alternating sequences: the stick and the slip. During the slip sequence, the friction force is given by the equation (1), where the slip velocity results from Eq. (4).

The following conditions are accomplished during the stick sequence

$$u = \text{constant}, \ \dot{u} = 0, \ \ddot{u} = 0, \tag{6}$$

and the friction force is obtained from the first equation (5)

$$F_f = -ku - M\ddot{\eta} \tag{7}$$

as long as the Eq. (2) is valid.

There is a symmetric lock zone due to the friction, where the motion stops when the stick conditions are accomplished. The lock zone length, $z_l$, can be calculated with

$$z_l = \mu_s N / k . \tag{8}$$

Next, considering the track irregularity of harmonic form

$$\eta = \eta_o \sin \omega t , \tag{9}$$

where $\eta_o$ is the amplitude, ω is the angular frequency and $t$ stands for time, we are looking for the solution to the equations of motion. To this end, the analytical solution for each stick/slip sequence of the motion is established.

For the slip sequence, the following equation has to be solved

$$M\ddot{u}(t^*) + ku(t^*) + \mu_k N \text{sign}[\dot{u}(t^*)] = \omega^2 M \eta_o \sin[\omega(t_k + t^*)] \tag{10}$$

where $t_k$ is the start time moment of the $k$ slip sequence and $t^*$ stands for time during the $k$ slip sequence, and $0 \le t^* \le t^*_k$ with $t^*_k$ the time period of the $k$ slip sequence.

Considering the initial conditions $u(t^* = 0) = u_{ak}$, $\dot{u}(t^* = 0) = 0$, the solution to the equation (10) is

$$\begin{aligned} u(t^*) = &\left\{ u_{ak} + \Delta u_k \text{sign}[\dot{u}(t^*)] - \frac{\Omega^2}{1-\Omega^2}\eta_o \sin(\omega t_k) \right\} \cos(\omega_o t^*) - \Delta u_k \text{sign}[\dot{u}(t^*)] - \\ &- \frac{\Omega^3}{1-\Omega^2}\eta_o \cos(\omega t_k) \sin(\omega_o t^*) + \frac{\Omega^2}{1-\Omega^2}\eta_o \sin[\omega(t_k + t^*)], \end{aligned} \tag{10}$$

where $\Delta u_k = \mu_k N/k$ and $\Omega = \omega/\omega_o$.

The time period of the $k$ slip sequence is obtained from the zero velocity condition

$$\omega_o\left\{u_{ak}+\Delta u_k \mathrm{sign}\left[\dot{u}(t_k^*)\right]-\frac{\Omega^2}{1-\Omega^2}\eta_o \sin(\omega t_k)\right\}\sin(\omega_o t_k^*)+ \\ +\omega_o \frac{\Omega^3}{1-\Omega^2}\eta_o \cos(\omega t_k)\cos(\omega_o t_k^*)-\omega\frac{\Omega^2}{1-\Omega^2}\eta_o \cos\left[\omega(t_k+t_k^*)\right]=0. \quad (13)$$

which can be solved by iteration method.

## 3 Numerical application

This section shows some numerical results derived from the above model, considering the following parameters corresponding to 1/8 empty freight wagon fitted with Y 25 bogie: $M$ = 2325 kg, $m$= 650 kg, $k$ = 830 kN/m, $\mu_k$ = 0,17, $\mu_s$ = 0,204, $N = Mg\mathrm{tg}(\alpha)$ = 9126 N, where $g$ is the gravitational acceleration and $\alpha$ is the Lenoir device angle in respect to vertical axis ($\alpha$ = 21°43′) [1]. It obtains the lock zone length $z_l$ = 2.243 mm and the natural frequency of the vehicle of 3.0 Hz.

Figure 3 shows the free vibration when the initial displacement is 14 $z_b$ and zero velocity condition. It observes 3 periods in the first second of the motion according to the natural frequency of 3 Hz. The wave attenuation is linear and the motion stops after 8 semi-periods.

Figure 4 displays the free vibration in the presence of a high frequency vibration (dither [5]) of 90 Hz and the amplitude of 10 μm. This dither can be originated by the wheel/rail interaction due to the roughness. When the free motion starts, its attenuation is linear, but then it becomes exponential like linear damping case. The motion never stops within the lock zone which has no effect.

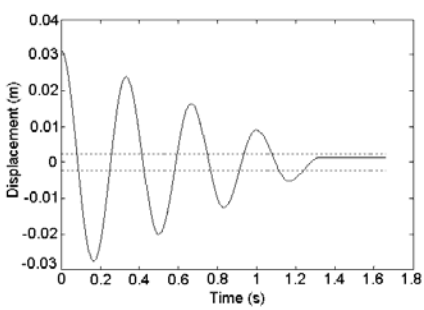


**Fig. 3.** Free vibration:
—, vibration, ---, lock zone.

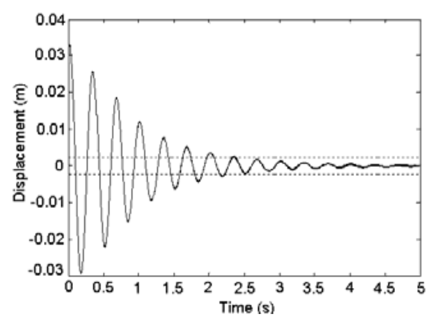


**Fig. 4.** Free vibration in the presence of dither of 90 Hz and 10 μm amplitude:
—, vibration, ---, lock zone.

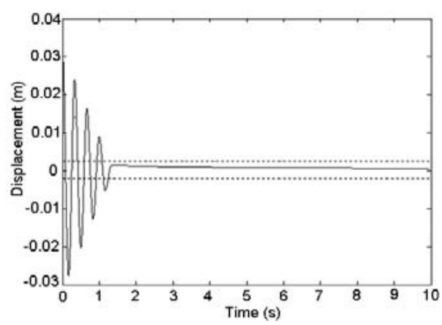


**Fig. 5.** Free vibration in the presence of dither of 90 Hz and 1.75 μm amplitude:
—, vibration, - - -, lock zone.

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It must be underlined that the dither amplitude should be sufficiently high to maintain the motion, alternatively the lock zone stops the motion. For instance, Fig. 5 presents such case, where the amplitude of the dither is only 1.75 μm. Figure 6 shows the forced vibration due to a low-frequency harmonic excitation (6 Hz) with amplitude of 566 μm. Vibration exhibits alternately stick-slip sequences. Two stick/slip sequences occur during a time. Acceleration of the suspended mass and the contact force have sudden variations and high peaks which could affect the ride quality and the wear of the wheel/rail rolling surfaces.

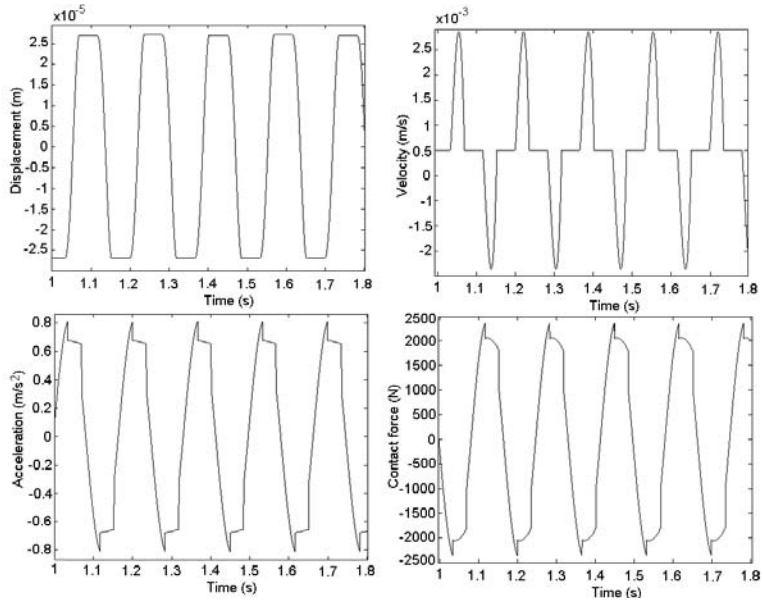


**Fig. 6.** Forced vibration due to a harmonic irregularity ($\eta_o$ = 560 μm, frequency 6Hz).

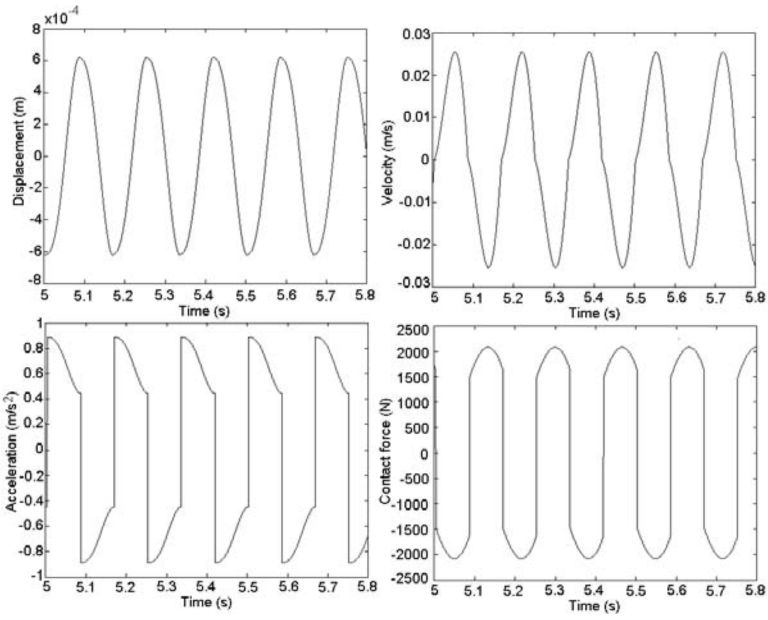


**Fig. 7.** Forced vibration due to a harmonic irregularity ($\eta_o$ = 840 μm, frequency 6Hz).

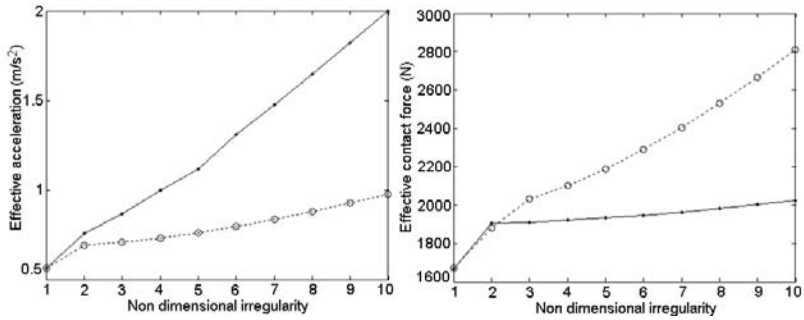


**Fig. 8.** Acceleration of the suspended mass and the contact force:
—•—, excitation 6 Hz; - - -○- - -, excitation frequency 9 Hz.

When the amplitude of the excitation is higher, the vibration is different. For instance, the alternately stick/slip sequences disappear when the excitation amplitude is 841 μm (Fig. 7). Figure 8 shows the acceleration of the suspended mass and the contact force versus the excitation amplitude considering two frequencies, 6 Hz and 9 Hz, respectively. The non-dimensional amplitude equals the ratio between the amplitude and $z_b(\omega_o/\omega)^2$. The suspended mass acceleration is higher at 6 Hz, while the contact force, at 9 Hz.

## 4 Conclusions

Damping based on the principle of dry friction is a technical solution applied to almost all types of freight wagons. To improve the dynamic performance of the freight wagons it is essential to understand the dynamic behaviour in the presence of the dry friction based on the outcome from the theoretical models.

The paper presents a simple model for a freight wagon equipped with bogies Y25 to highlight several important aspects of the free and forced vibration, due to excitation from a track with harmonic irregularity.

Free vibrations are influenced by the existence of high-frequency vibrations such as those induced by wheel-rail interaction in the presence of the rolling surfaces roughness.

If low-frequency excitation amplitude is relatively small, the forced vibration is a jerking motion. At higher amplitudes of track harmonic irregularity, jerky character could disappear.

Vehicle behaviour has nonlinear characteristic which is reflected in the relation between the suspended mass acceleration and contact force, on the one hand, and the excitation amplitude on the other hand.

Outcomes of this paper can be taken as reference to validate more sophisticated models.

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