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Testing the assumption of linear dependence between the rolling friction torque and normal force

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Abstract. Rolling friction is present in all nonconforming bodies in contact. A permanent topic is the characterization of the moment of rolling friction. A number of authors accept the hypothesis of linear dependency between the rolling torque and the normal force while other researchers disagree with this assumption. The present paper proposes a method for testing the hypothesis of linear relationship between rolling moment and normal pressing force. A doubly supported cycloidal pendulum is used in two situations: symmetrically and asymmetrically supported, respectively. Under the hypothesis of a linear relationship, the motions of the pendulum should be identical.

1 Introduction

The interaction between two solid bodies can be accomplished in two ways: by direct contact or via a field. In mechanical engineering applications, the first manner of interface, by direct contact, is met in the majority of cases. Most recently, according to the requirements imposed by diminution of energy utilization and reduction of wear, modern solutions were adopted, specifically the direct contact between the elements of a mechanical system was eliminated. Thus there can be mentioned the magnetic bearings, pneumatic actuators and so on. Even with all these advancements, a mechanical system within which there is no direct interaction (mechanical contact) between its elements is implausible. The scheme of a contact between two solid bodies is presented in Figure 1. It is assumed that the boundary surfaces are coming into contact in point P, where at least one of the two surfaces admits a unique normal $\boldsymbol{n}$. One of the bodies is considered as reference body, therefore its action upon the other body is materialized by the reaction force $\boldsymbol{R}$ and reaction moment $\boldsymbol{M}$. The relative motion between the two bodies is characterized

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by the sliding velocity $\boldsymbol{v}_{al}$ situated in the common tangent plane and by the angular velocity ω, which in its turn decomposes into the spin angular velocity ω_s, parallel to the normal $\boldsymbol{n}$ and the rolling angular velocity ω_r, placed in the tangent plane. Sequentially, the components of the reaction torsor are projected as follows: the reaction $\boldsymbol{R}$ into normal reaction $\boldsymbol{N}$ and tangential (friction) reaction $\boldsymbol{T}$ which is parallel but in opposite sense to the sliding velocity; the reaction moment having as components the spin torque $\boldsymbol{M}_s$, parallel and opposite to the spin velocity, and the rolling torque $\boldsymbol{M}_r$ parallel and opposite to rolling angular velocity ω_r. The characterization of the components specific to friction, $\boldsymbol{T}$, $\boldsymbol{M}_r$ and $\boldsymbol{M}_s$ is currently an open subject, still.

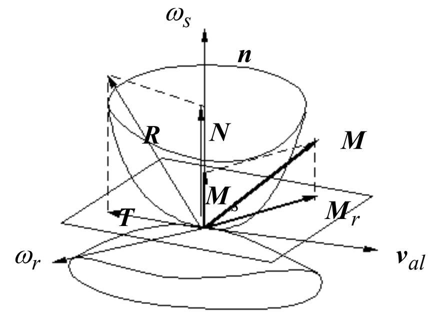

Fig. 1. Reaction forces and reaction torques from a mechanical contact.

Another remark to be considered concerns the dimensions of the contact region. Two situations are observed: first, when the contact zone has dimensions comparable to those of the bodies and therefore the contact point cannot be established - the case is stated as *conform contact*; second, when the contact between two nonconforming bodies is theoretically made in a point - the situation of *nonconform contact* or *Hertzian contact*. It is to be noted that while the friction force $\boldsymbol{T}$ and spin torque $\boldsymbol{M}_s$ can be met in both types of contact, the rolling torque $\boldsymbol{M}_r$ is met only in the contacts of nonconforming bodies. For an actual contact situation, two or even all three components can co-exist. If the aim is to obtain pure rolling motion between two bodies, this assumes that sliding velocity and spin angular velocity are simultaneously vanished ($v_{al} = 0$; $\omega_{sp} = 0$).

In the technical literature there are presented expressions of the three components of friction torsor depending on the normal pressing force $\boldsymbol{N}$ and on the characteristics of the relative motion between the two bodies. It is unanimously accepted that in dry friction conditions the friction force is proportional to the normal pressing force. The spin torque depends on normal pressing force at a power law, the exponent depending on the pressure distribution on contact area. Regarding the rolling friction torque, there are works that accept the hypothesis of proportionality to normal pressing force [1-3] while other papers disagree with this assumption, and in most of them there are proposed dependencies of power law type between rolling friction moment and normal force [4]. Cherepanov [5] reaches the conclusion that for the contact between a ball and an elastic half-space the exponent is $4/3$. In the present paper an answer is sought concerning the linear dependency between the rolling friction torque and normal force. An affirmative answer to this question would substantially simplify the model of dynamical behaviour of a mechanical system where rolling friction is encountered since the linear dependency should conduct to linear differential equations.

2 Principle of method

The employment of a mobile body that makes two identical contacts with an immobile body is proposed in order to verify the principle of linearity between rolling friction torque and normal force. All the components of reaction forces torsor $\boldsymbol{N}_{1,2}$, $\boldsymbol{T}_{1,2}$ $\boldsymbol{M}_{r_{1,2}}$ occur in the contact points. Assuming the pure rolling motion hypothesis, the velocities of the contact points from the mobile body are zero (or equal, in this particular case) and therefore the angular spin velocity ω_{sp} is identical to zero. Under these conditions, the motion of the body is a parallel motion. A mobile system in translational motion is considered, as presented in Figure 2.

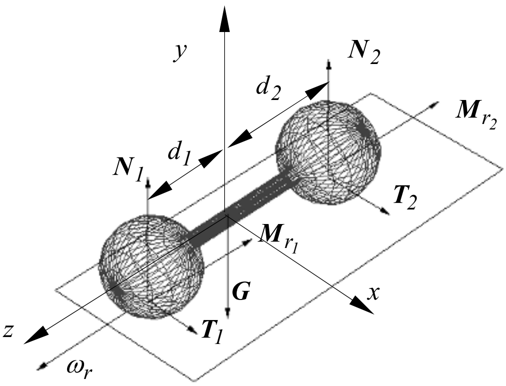

Fig. 2. Scheme of a body with two contact points in pure rolling motion.

The motion of the centre of mass projected on the vertical axis has the equation:

$$N_1 + N_2 - G = 0 \tag{1}$$

The body does not have rotation motion with respect to x axis, so it results:

$$N_1 d_1 = N_2 d_2 \tag{2}$$

where $d_{1,2}$ is the distance from the contact point to the weight's support. With the notation:

$$d = d_1 + d_2\,; \tag{3}$$

it results:

$$N_1 = (1-\xi)G, \quad N_2 = \xi G \text{ where } \xi = d_1 / d \tag{4}$$

If the rolling friction torque depends on the normal pressing force and obeys the law:

$$M_r = s_{r\beta} N^{\beta} \tag{5}$$

where $s_{r\beta}$ is a proportionality factor, the total rolling friction torque is:

$$M_r = M_{r1} + M_{r2} = s_{r\beta}[(1-\xi)^{\beta} + \xi^{\beta}] = s_{r\beta} f(\xi,\beta) \tag{6}$$

where $s_{r\beta}$ is a constant and $f(\xi,\beta)$ is:

$$f(\xi,\beta)=(1-\xi)^{\beta}+\xi^{\beta} \tag{7}$$

In Figure 3 the variation of the function (7) that appears in the structure of the rolling friction torque is represented for four values of parameter β . For pure rolling hypothesis, assuming that an external torque M_a , parallel to the support axis, acts upon the body, the moment of momentum theorem written with respect to the instantaneous rotation axis is:

$$J_G + mr^2 = M_a - M_r = M_a - s_{r\beta}[(1-\xi)^{\beta}+\xi^{\beta}] \tag{8}$$

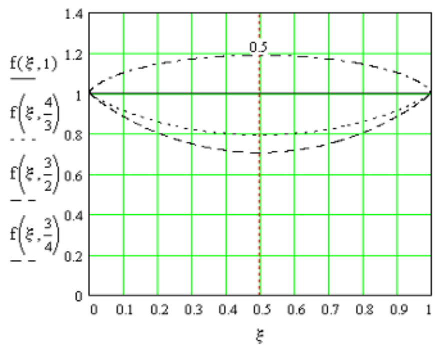

Fig. 3. Friction torque variation with respect to the position of the centre of mass.

Figure 3 and relation (8) show that the motion of the body is independent on the position of the centre of mass of the body only for $\beta = 1$ when linear dependency between the rolling torque and normal force exists.

3 Experimental set-up

The experimental device presented in Figure 4 was used for experimental validation. On a cylindrical rod 1 a vertical rod 2 is fixed.

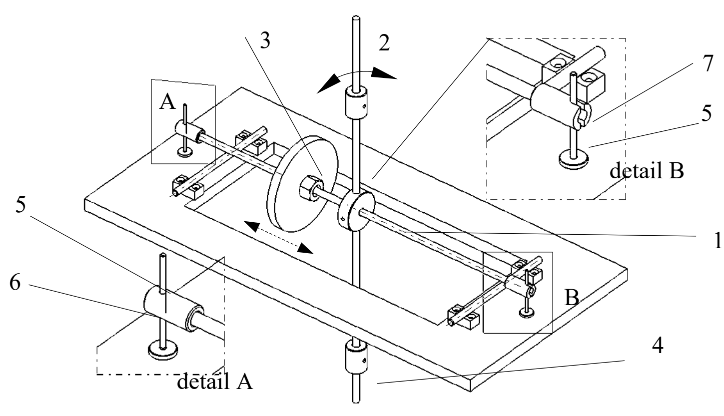

Fig. 4. Principle of experimental device.

A disc 3 can slide along the horizontal rod. The rod 1 has two points of contact with two identical parallel rods, placed in a horizontal plane. The system configuration materializes a cycloidal pendulum. When the pendulum is brought out of the equilibrium position and

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then set free, it will oscillate, and the oscillation motion is damped due to rolling friction torque. The oscillation period of the pendulum can be varied using the mass 4. A laser stick device is mounted at the end of the rod and thus the pendulum motion can be registered via a non-contact technique. To ensure the same launching position of the pendulum, when the position of the disc is changed, the horizontal shaft is blocked using the stud 5 and bushings 6 and 7, fixed on the shaft. The experimental test consists in the study of pendulum motion for different positions of the centre of mass (accomplished by different positions of the disc 3). In the case when the torque depends linearly on normal pressing force, according to Figure 3, the rolling friction torque is the same, irrespective of the position of the disc 3, and therefore the motion of the pendulum should be the same, regardless of the position of the mobile disc. In an opposite situation, the rolling friction torque depends on the position of the centre of mass, attesting nonlinear dependence between the rolling friction torque and the normal force. Based on the design presented in Figure 4, the experimental test-rig was built and it is presented in Figure 5.

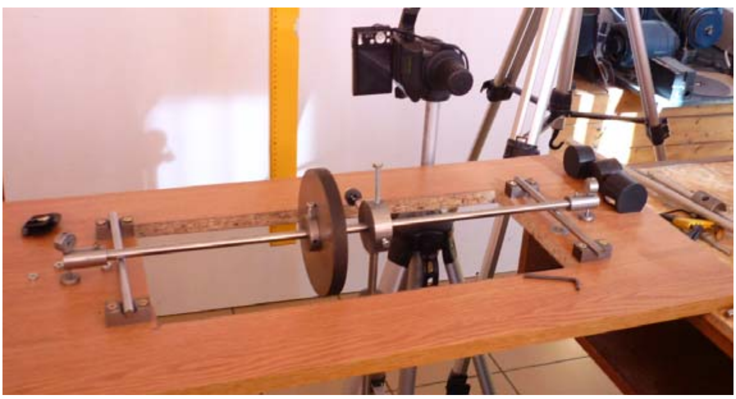

Fig. 5. Actual experimental device.

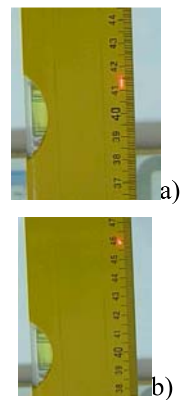

Fig. 6. Spot image.

A series of tests was carried out on the experimental device for certain pairs of materials, namely steel-aluminium, steel-rubber and steel-steel. An experimental analysis consists of establishing the pair of tested materials, positioning the disc in the lateral or central position on the rod, launching the oscillatory motion, and recording the motion of the spot along the scale with a digital camera.

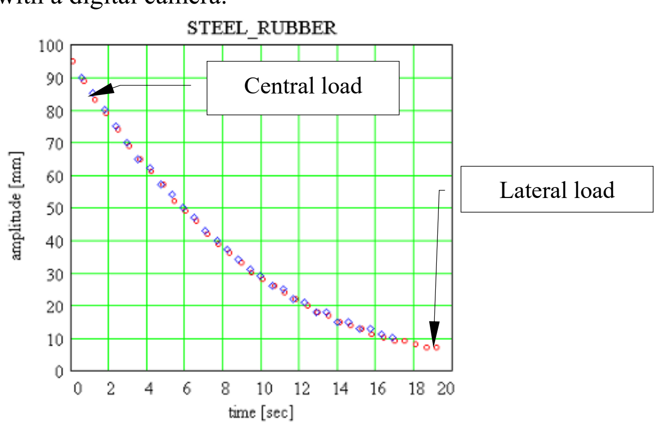

Fig. 7. Steel-rubber amplitude decrease.

The video is then split into frames and the amplitude of the motion and corresponding instants are found. The amplitude can be determined accurately, as shown in Figure 6, since at maximum elongation the spot is immobile (Figure 6b). The experimental data are plotted as follows: on the same graph, the amplitude decrease is compared for the two positions of the disc on the rod, for the same pair of materials. In Figure 7 the behaviour of damping from steel-rubber contacts is presented, and it can be noticed there is a coincidence between the points from the two plots. In Figure 8 there are plotted the data obtained for the pair steel- aluminium, and the decrease in amplitude differs - in the case of the central disc there is a stronger damping of amplitude.

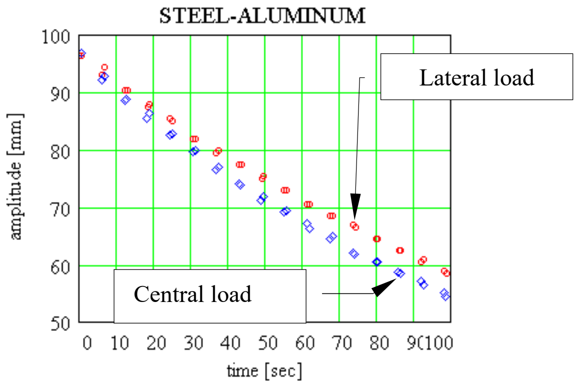

Fig. 8. Amplitude decrease for steel-aluminium pair.

4 Conclusions

The paper presents a proposed method and device for the study of linear dependence between the rolling friction torque and normal load of a rolling contact. The principle of operation of the test rig consists in damped motion of a cycloidal pendulum with double contact. Testing dissimilar materials like rubber-steel and aluminium-steel, one can state that there is not a general tendency for the pairs of materials that is a linear dependency between rolling friction torque and normal load. As a general conclusion, for any pair of materials it is necessary to find the particular law expressing the dependence between rolling torque and normal load. The paper regards only incipient research in applying the proposed method.

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