MATEC Web of Conferences **112**, 06014 (2017) DOI: 10.1051/matecconf/201711206014
IManE&E 2017

On the vertical interaction between a three-axle bogie and track

Traian Mazilu[1,*], and *Marius* Gheţi [2]
[1]University Politehnica of Bucharest, Department of Railway Vehicles, 313 Splaiul Independenţei, Bucharest, Romania
[2]Alstom Transport, 32 Răzoare, Bucureşti, Romania

Abstract. In this paper, the 'moving wheel' model is used aiming to analysis new aspects of the interaction between three-axle bogie and track due to the harmonic roughness of the rail. The three-axle bogie of a locomotive is considered and its model is reduced to three loaded wheels running along a rail supported on a continuous elastic foundation. The track model consists of a Euler-Bernoulli beam resting on elastic foundation including two elastic layers (Winkler model) representing the rail pad and ballast and one inertial layer for sleepers. The rail deflection due to the moving dead loads of the wheels is calculated and the frequency response functions of the wheels, rail in the contact points and contact forces are analysed, including the three resonance frequencies.

1 Introduction

The small-scale undulations (roughness) of both rolling surfaces - the wheel and the rail - are among the common causes of the wheel/rail vertical vibration. The study of the vibration generated by a wheel while rolling on a rail in the presence of the small-scale undulations is critical in predicting the short-pitch rail corrugation [1] and the rolling noise [2].

Usually, the wheel-rail vibration behaviour is studied using a model in which only a single wheel is present [3]. This is not the case in practice, where multiple wheels roll on the rail [4, 5].

The issue of the wheel/rail interaction may be solved in two different ways - using either the model of a 'moving irregularity' between a stationary wheel and rail, or the model of a 'moving wheel' along the rail. The model of the moving irregularity is much easier to use and it is currently utilized for the frequency-domain analysis. However, the 'moving wheel' model is more realistic but requires much more effort to apply [6, 7].

This paper deals with the forced vertical vibrations of a three-axle bogie running along a track with small-scale undulations aiming to point out some of main features of the bogie/track interaction. To this end, the 'moving wheel' approach is applied for a simplified bogie/track model due to the symmetry. Equations of motion are solved using the Green's function method and the rail deflection under moving dead loads and frequency response

*Corresponding author: trmazilu@yahoo.com

functions are calculated and analysed. It is shown that the three-axle bogie/track system exhibits three resonance frequencies, but these emerge in one due to the damping.

2 Equations of motion

The mechanical model applied in this section is presented in Figure 1, where it can be seen three loaded wheels of mass M uniform moving with the velocity V along a continuous elastically supported rail. The three loaded wheels represent the model of a three-axle bogie, while the continuous elastically supported rail is idealised representation of a track, both considered as symmetric structure. Each wheel is considered rigid body and is loaded with the force Q_0. The distance between the front wheel and the second wheel is a_{12} and the distance between the second wheel and the rear one is a_{23}, respectively. Wheels are only coupled through the rail and the coupling via suspension can be neglected when the frequency is higher than the natural frequency of the bogie.

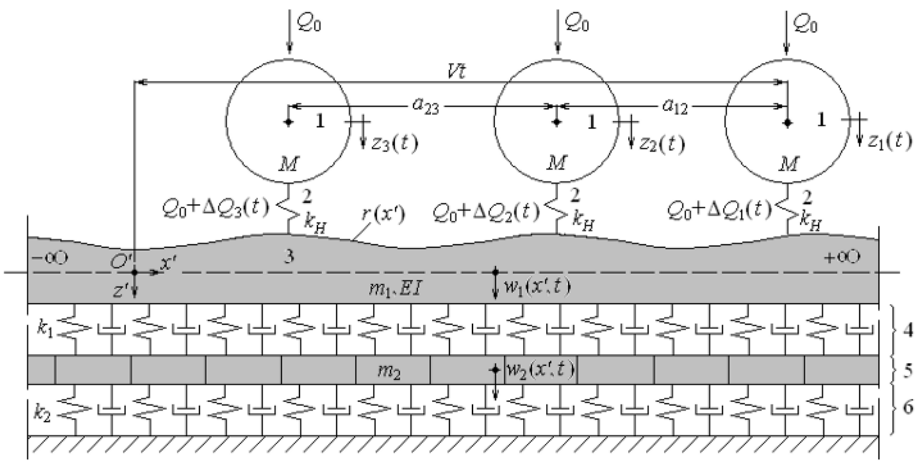

Fig. 1. Mechanical model: 1. wheel; 2. wheel-rail contact; 3. rail; 4. railpad; 5. sleeper; 6. ballast.

The rail is modelled as an infinite uniform Euler-Bernoulli which is continuous supported on two elastic layers, representing the viscoelastic characteristics of both railpad and ballast. The sleeper's inertia is modelled using an intermediate inertial layer between the railpad layer and the ballast layer. Practically, following this manner, the influence of the equidistant support given by sleepers is neglected; this approach can be applied up to 6-700 Hz [3].

The parameters for the track model are: EI – the bending stiffness of the rail, m_1 – the rail mass per unit length, k_1 – the railpad stiffness per unit length, c_1 – the damping constant per unit length of the railpad, m_2 – the sleeper mass per unit length, k_2 – the ballast stiffness per unit length and c_2 – the damping constant per unit length of the ballast.

Vertical displacement of the i wheel is $z_i(t)$ with i = 1 to 3, the rail deflection is $w_1(x',t)$ and the displacement of the sleeper layer is $w_2(x',t)$. The interaction between the wheels and the rail is the result of the presence of the roughness $r(x)$ on the rail surface. Considering that the dynamic component of the contact force ΔQ_i is much smaller than the static load Q_0, the dynamic of the wheels/rail contact is in the limits of the linear model.

Applying the mechanical laws, we can write the equations of motion:

- for wheels

$$M\ddot{z}_i = -\Delta Q_i \tag{1}$$

- for the track

$$EI\frac{\partial^4 w_1}{\partial x'^4}+m_1\frac{\partial^2 w_1}{\partial t^2}+c_1\left(\frac{\partial w_1}{\partial t}-\frac{\partial w_2}{\partial t}\right)+k_1(w_1-w_2)=\sum_{i=1}^{3}(Q_0+\Delta Q_i)\delta(x'-Vt-a_i)$$
$$m_2\frac{\partial^2 w_2}{\partial t^2}+c_1\left(\frac{\partial w_2}{\partial t}-\frac{\partial w_1}{\partial t}\right)+c_2\frac{\partial w_2}{\partial t}+k_1(w_1-w_2)+k_2 w_2=0 \qquad (2)$$

- for the wheels/rail contact

$$z_i-w_1(Vt+a_i)-r(Vt+a_i)=\frac{\Delta Q_i}{k_H},\ \text{i=1 to 3}, \qquad (3)$$

where $a_1 = 0$, $a_2 = a_{12}$, $a_3 = a_{12} + a_{23}$ and k_H – the stiffness of the wheel/rail contact.
New coordinate is introduced

$$x'=Vt+x, \qquad (4)$$

to solve the equations of motion. Subsequently, the equations of motion of for the track are transformed by applying the following transformations

$$\frac{\partial^n}{\partial x'^n}\to\frac{\partial^n}{\partial x^n},\ \frac{\partial^n}{\partial t^n}\to\left(\frac{\partial}{\partial t}-V\frac{\partial}{\partial x}\right)^n. \qquad (5)$$

It obtains

$$EI\frac{\partial^4 w_1}{\partial x^4}+m_1\left(\frac{\partial^2 w_1}{\partial t^2}-2V\frac{\partial^2 w_1}{\partial t\partial x}+V^2\frac{\partial^2 w_1}{\partial x^2}\right)+c_1\left(\frac{\partial w_1}{\partial t}-V\frac{\partial w_1}{\partial x}-\frac{\partial w_2}{\partial t}+V\frac{\partial w_2}{\partial x}\right)+$$
$$+k_1(w_1-w_2)=\sum_{i=1}^{3}(Q_0+\Delta Q_i)\delta(x-a_i)$$
$$m_2\left(\frac{\partial^2 w_2}{\partial t^2}-2V\frac{\partial^2 w_2}{\partial t\partial x}+V^2\frac{\partial^2 w_2}{\partial x^2}\right)+c_1\left(\frac{\partial w_2}{\partial t}-V\frac{\partial w_2}{\partial x}-\frac{\partial w_1}{\partial t}+V\frac{\partial w_1}{\partial x}\right)+$$
$$+c_2\left(\frac{\partial w_2}{\partial t}-V\frac{\partial w_2}{\partial t}\right)+k_1(w_1-w_2)+k_2 w_2=0. \qquad (6)$$

Also, the contact equation becomes

$$z_i-w(a_i)-r(a_i)=\frac{\Delta Q_i}{k_H}, \qquad (7)$$

Considering the linearity of the mechanical model, the issue of the wheels/rail interaction can be divided in two: the stationary response and steady-state harmonic behaviour. To solve these issues the Green's function method is applied [6].

First, we occupy by the steady-state harmonic behaviour if all variables are harmonic

$$\overline{w}_{1,2}(x,t)=\overline{W}_{1,2}(x)e^{\mathrm{i}\omega t},\ \overline{z}_i(t)=\overline{Z}_i e^{\mathrm{i}\omega t},\ \Delta\overline{Q}_i(t)=\Delta\overline{Q}_i e^{\mathrm{i}\omega t}\ \ \overline{r}_i(t)=r_o e^{\mathrm{i}\omega(t-a_i/V)}. \qquad (8)$$

Inserting these variables in equations of motion for the track (6), it reads

$$\mathbf{L}_{x,\omega}\mathbf{p} = \mathbf{q}\,, \tag{9}$$

where

$$\mathbf{L}_{x,\omega} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix},\ \mathbf{p} = \begin{bmatrix} \overline{W}_1 & \overline{W}_2 \end{bmatrix}^{\mathrm{T}},\ \mathbf{q} = \begin{bmatrix} \sum_{i=1}^{3} \Delta Q_i \delta(x - a_i) & 0 \end{bmatrix}^{\mathrm{T}}$$

with

$$L_{11} = EI\frac{\mathrm{d}^4}{\mathrm{d}x^4} + m_1 V^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} - V(c_1 + 2\omega m_1 \mathrm{i})\frac{\mathrm{d}}{\mathrm{d}x} + k_1 - \omega^2 m_1 + c\omega\mathrm{i}$$

$$L_{12} = L_{21} = c_1 V\frac{\mathrm{d}}{\mathrm{d}x} - k_1 - c\omega\mathrm{i}$$

$$L_{22} = m_2 V^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} - V(c_1 + c_2 + 2\omega m_2 \mathrm{i})\frac{\mathrm{d}}{\mathrm{d}x} + k_1 + k_2 - \omega^2 m_2 + \omega(c_1 + c_2)\mathrm{i}.$$

The complex amplitude of the rail can be written as

$$\overline{W}_1(x) = \int_{-\infty}^{\infty} G(x,\xi)\sum_{i=1}^{3}\Delta\overline{Q}_i\delta(\xi - a_i) = \sum_{i=1}^{3} G(x,a_i)\Delta\overline{Q}_i\,, \tag{10}$$

where

$$G(x,\xi) = m_2 V^2 G^o_{\xi\xi} - V(c_1 + c_2 + 2\omega m_2 \mathrm{i})G^o_{\xi} + \left(k_1 + k_2 - \omega^2 m_2 + \omega(c_1 + c_2)\mathrm{i}\right)G^o\,,$$

with G^o=$G^o(x,\xi)$ is the Green's function of the differential operator

$$L_{x,\omega} = L_{11}L_{22}\,.$$

The contact forces are obtained from the following set of equations

$$\begin{aligned} &(G_w + G_{11} + G_H)\Delta\overline{Q}_1 + G_{12}\Delta\overline{Q}_2 + G_{13}\Delta\overline{Q}_3 = -r_0 e^{-\mathrm{i}\omega a_1/V} \\ &G_{21}\Delta\overline{Q}_1 + (G_w + G_{22} + G_H)\Delta\overline{Q}_2 + G_{23}\Delta\overline{Q}_3 = -r_0 e^{-\mathrm{i}\omega a_2/V} \\ &G_{31}\Delta\overline{Q}_1 + G_{32}\Delta\overline{Q}_2 + (G_w + G_{33} + G_H)\Delta\overline{Q}_3 = -r_0 e^{-\mathrm{i}\omega a_3/V} \end{aligned} \tag{11}$$

where $Gij = G(a_i,a_j)$ and

$$G_w = -\frac{1}{\omega^2 M}\,,\ G_H = \frac{1}{k_H}$$

are the wheel receptance and the contact elasticity, respectively.

The complex amplitude of the wheels and the rail in the contact points are

$$\overline{w}_i = \sum_{j=1}^{3} G_{ij}\Delta\overline{Q}_j\,,\ \overline{z}_i = G_w\Delta\overline{Q}_i\,. \tag{12}$$

Finally, the response function for contact forces, wheel and rail displacements in the contact points can be derived

$$\overline{H}_i^{\Delta Q} = \frac{\Delta \overline{Q}_i}{r_0}, \ \overline{H}_i^w = \frac{\overline{z}_i}{r_0}, \overline{H}_i^r = \frac{\overline{w}_i}{r_0}. \tag{13}$$

The stationary behaviour can be solved following the same way with $\omega = 0$ and Q_0 instead of ΔQ_i.

3 Numerical application

This section shows some numerical results derived from the above model, considering the following parameters: $M = 1000$ kg, $a_{12} = 2.10$ m, $a_{23} = 2.25$ m, $EI = 6.42$ MNm^2, $m_1= 60$ kg, $c_1 = 92.95$ kNs/m^2 (damping ratio = 0.3), $k_1 = 400$ MN/m^2, $m_2= 129$ kg, $c_2 = 92.95$ kNs/m^2 (damping ratio = 0.3), $k_2 = 400$ MN/m^2, $Q_0 = 100$ kN and $k_H = 1.5$ GN/m.

Figure 2 shows the rail deflection under dead load of the wheels for $V = 60$ m/s. At that velocity, the rail deflection is like the case of the stationary loaded wheels. However, the deflection is smaller than the one of a single loaded wheel due to relative long distances between wheels.

The frequency response function of the wheel's exhibit three peaks corresponding to the resonance frequencies at 43.1, 44.2 and 44.9 Hz as can be seen in Fig. 3, where the result from the undamped case is shown.

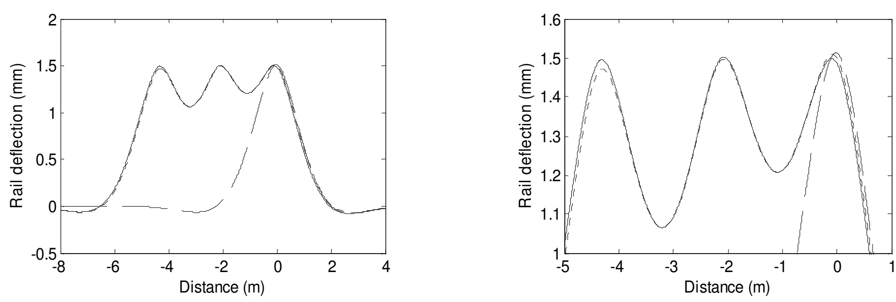

Fig. 2. Rail deflection: —, $V = 60$ m/s; - - -, $V = 0$; – – –, wheel alone at $V= 60$ m/s.

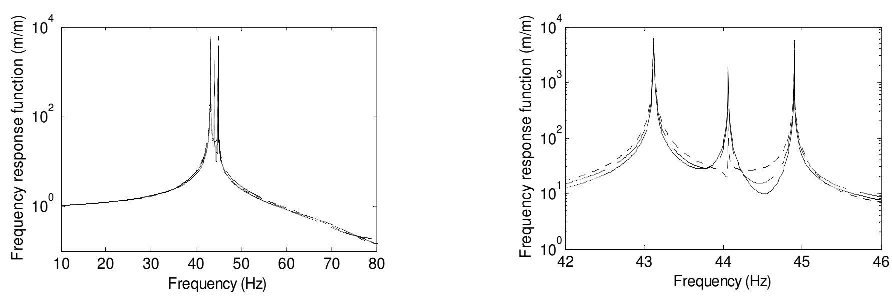

Fig. 3. Frequency response function of the wheels: —, front wheel; - -, second wheel; – –, rear wheel.

Figure 4 displays the frequency response functions of the wheels, rail in the contact point and contact forces. The three resonance frequencies are included into a single peak due to the damping of the track. At high frequencies, the response of the three-axle bogie/track system exhibits alternant peaks and deeps due to the rail bending waves reflected between wheels.

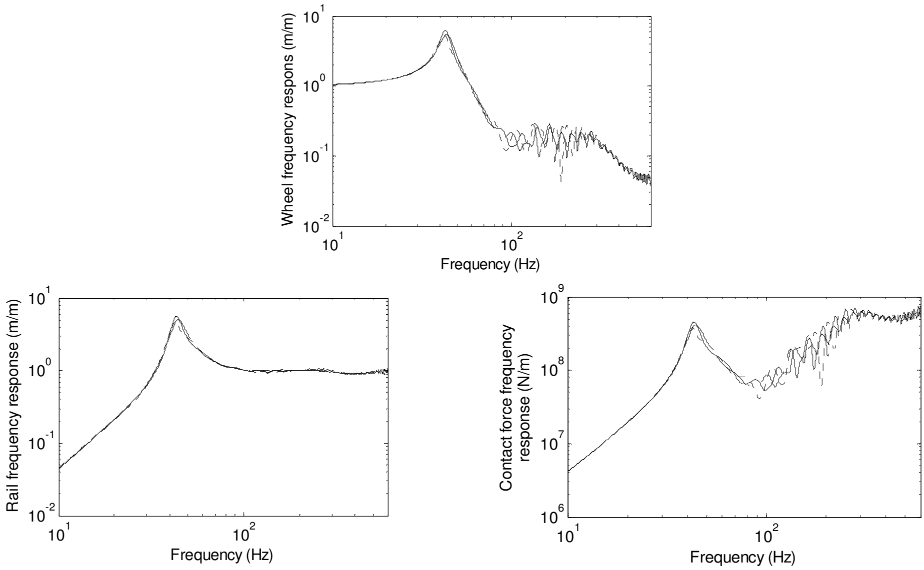

Fig. 4. Frequency response functions of the bogie/track at 60 m/s: —, front wheel; - -, second wheel; – –, rear wheel.

4 Conclusions

Wheel/rail vibration generated by the roughness of the rolling surfaces represents an interesting topic from many reasons, particularly the rolling noise and wear of the rolling surfaces. This paper is focused on the interaction between a moving three-axle bogie and track due to harmonic roughness. Mix 'moving wheel' model and the Green's function method is applied to calculate the rail deflection under dead loads of the wheels and the frequency response functions of the wheels, rail in the contact points and contact forces.

Three-axle bogie/track exhibits three resonance peaks (undamped track) which shift in one due the track damping. As the knowledge authors, this aspect never has been revelled in the past. Contact forces have alternate peaks/dips which can contribute or not to the growing of the undulations on the rolling surfaces. However, at the damped resonant frequency, all three wheels have the similar behaviour, meaning the all three exhibit the same dynamic stress.

This work has been funded by UEFISCDI, project number 76PED/2017, code PN-III-P2-2.1-PED-2016-0748.

References

1. Y. Soeta, R. Shimokura, Applied Acoustics **74**, 1160-1166 (2013)
2. N. Correa, E. G. Vadillo, J. Santamaria, J. Herreros, Wear **352-353**, 196-208 (2016)
3. S. L. Grassie, R. W. Gregory, D. Harrison, J. Mech. Eng. Sci. **24**, 77-90 (1982)
4. T. X. Wu, D. J. Thompson, J. Sound Vib. **239**, 69-97 (2001)
5. T. X. Wu, D. J. Thompson, J. Acoust. Soc. Am. **108**, 1046-1053 (2000)
6. T. Mazilu, J. Sound Vib. **239**, 4597-4619 (2013)
7. X. Sheng, T. Zhong, Y. Li, J. Sound Vib. **395**, 160-186 (2017)