Hob mill for spline shaft, profiled by generating trajectories method. Graphical application in CATIA

Florin Susac, Virgil Gabriel Teodor^{}, Nicușor Baroiu, and Nicolae Oancea*

“Dunărea de Jos” University of Galați, Department of Manufacturing Engineering, Domnească 111, Galați, Romania

Abstract. The hob mill is a cutting tool with a wide application for generation in machining of splined shafts. A method for profiling the hob mill for generation of square splines is presented in this paper. The analytical development was done based on a complementary theorem of surface enwrapping — the method of generating trajectories. A numerical application for a standardized spline shaft is presented. Based on the same theorem is presented a graphical application, developed in the CATIA design environment, for profiling the hob mill which generated standardised square splines.

1 Introduction

The squared splines are ordered curls of surfaces with parallels flanks and rectilinear generation. The generation of these complexes surfaces is made by milling with hob tools. This imposes the profiling of the primary worm of this tool's type.

They were created graphical methods [1, 2], which allow approximating the profile of hob mill for generating such composed surfaces.

The cylindrical surface of the splines and the helical surface of the worm, approached as reciprocally enwrapping surfaces, represent issues of surfaces enveloping of second degree, according to the Olivier theorems [3, 4].

In this way, it is possible to analytically determine a rigorous form of the enwrapping worm of the splined shaft, as base for the hob mill. The analytical method, based on the second theorem of Olivier, assume handling of complicated equations, with obvious disadvantages.

There were developed complementary theorems for the surfaces enveloping, which, for real generating cases, has sentences which simplify the analytical effort for surface determining. Such complementary theorems are: the “minimum distance” method; the method of “substitutive circles family”, the method of “generating trajectories” [5].

Also, there were presented graphical methods, developed in graphical design environments as AutoCAD, [6], which allows, by 3D modelling the generating of the surfaces in order to determine the solid model of the hob mill.

^{*}Corresponding author: virgil.teodor@ugal.ro

2 Hob mill for splined shaft

They are defined the reference systems and shaft's crossing section presented in Figure 1.

The reference systems are: xyz is the global reference system, with z axis as the main axis of the shaft; XYZ — mobile reference system, joined with the C_1 centre of the shaft. Initially, this system is overlapped to the global system; $\xi\eta\zeta$ — mobile reference system, joined with the C_2 centre of the rack-gear tool.

Fig. 1. Splined shaft. Reference system.

2.1 Generating rack-gear

In Figure 1, the crossing profile of the splined shaft and the reference systems are presented. The process kinematics assumes the movements: rotation of the C_1 centre with φ_1 angular parameter and translation of the generating tool, with parameter λ . The link:

$$\lambda = R_r \cdot \varphi_1, \quad (1)$$

is defined as rolling condition. R_r represents the rolling radius of the splined shaft.

The analytical equations of the splined shaft are:

$$\begin{cases} X = -u; \\ \Sigma Y = b; \\ Z = t, \end{cases} \quad (2)$$

with u and t independent variables. Usually, the rolling radius is $R_r = \sqrt{R_c^2 - b^2}$. The generating kinematics assumes the motions:

$$\mathbf{x} = \omega_3^T(\varphi_1) \mathbf{X}; \quad \xi = \mathbf{x} - \mathbf{a}; \quad \mathbf{a} = \begin{pmatrix} -R_r \\ -R_r \cdot \varphi_1 \\ 0 \end{pmatrix}. \quad (3)$$

The relative motion is:

$$\xi = \omega_3^T(\varphi_1) \cdot \mathbf{X} - \mathbf{a}. \quad (4)$$

The Σ flanks family, in the rack-gear's reference system is:

$$(\Sigma)_{\varphi_1} \begin{cases} \xi = -u \cdot \cos \varphi_1 - b \cdot \sin \varphi_1 + R_r; \\ \eta = -u \cdot \sin \varphi_1 + b \cdot \cos \varphi_1 + R_r \cdot \varphi_1; \\ \zeta = t. \end{cases} \quad (5)$$

The enwrapping of this surfaces' family is the generating rack-gear flank. The enwrapping condition is determined based on the "generating trajectories" theorem. The direct parameters of the normal to the spline's flank are calculated. By partial derivative, from (2), results:

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{j}, \quad (6)$$

so, the direction of the $\vec{N}_\Sigma$ normal on to the current point of the flank (2) has equation:

$$\vec{N}_\Sigma = -u \cdot \vec{i} + (b+q) \cdot \vec{j} + t \cdot \vec{k}, \quad (7)$$

where, q is a variable scalar value. The generating trajectory of the normal (7) is:

$$(\vec{N}_\Sigma)_{\varphi_1} \begin{cases} \xi = \omega_3^T(\mathbf{j}) \times \mathbf{N}_\Sigma + \mathbf{a}. \end{cases} \quad (8)$$

It is imposed the condition that the $(\vec{N}_\Sigma)_{\varphi_1}$ trajectories pass through the gearing pole:

$$P \begin{cases} \xi = 0; \\ \eta = R_r \cdot \varphi_1; \\ \zeta = t. \end{cases} \quad (9)$$

From (8) and (9) result the conditions:

$$-u \cdot \cos \varphi_1 - (b+q) \cdot \sin \varphi_1 + R_r = 0; \quad -u \cdot \sin \varphi_1 + (b+q) \cdot \cos \varphi_1 + R_r \cdot \varphi_1 = R_r \cdot \varphi_1, \quad (10)$$

from which, eliminating the q parameter, the specific enwrapping condition results:

$$u = R_r \cdot \cos \varphi_1. \quad (11)$$

The assembly of equations (5) and (11) is the enwrapping of the rack-gear's family flanks (12). The rack-gear is a cylindrical surface with generation parallel with ζ axis.

$$C_{\varphi,t} \begin{cases} \xi = -R_r \cdot \cos^2 \varphi_1 - b \cdot \sin \varphi_1 + R_r; \\ \eta = -R_r \cdot \sin \varphi_1 \cdot \cos \varphi_1 + b \cdot \cos \varphi_1 + R_r \cdot \varphi_1; \\ \zeta = t. \end{cases} \quad (13)$$

2.2 Helical surface conjugated with generating rack-gear

In Figure 2, it is presented the axis position of the future hob mill, the axis $\vec{V}$, inclined regarding the front plane of the spline with angle ω . Let assume that the helical motion with

$\vec{V}$ axis and p helical parameter is decomposed in two motions: one translation along the rack-gear's cylindrical surface's generation, denoted with $T(\vec{t}, v)$ and one rotation around an axis parallel with $\vec{V}$ and at distance a_0 from this, $(\vec{A}, \varphi)$.

In this way, the decomposition of the helical movements becomes:

$$(\vec{V}, p) \square T(\vec{t}, v) + (\vec{A}, \varphi). \quad (14)$$

Fig. 2. Hob mill's axis and reference systems.

We have to notice that the $C_{\varphi, t}$ surface, in the motion $T(\vec{t}, v)$ it's moving in the direction of its own generation. So, the characteristic curve of the $C_{\varphi, t}$ surface does not depend by this motion. The characteristic curve will depend only by the rotation around the $\vec{A}$ axis. The a_0 distance is defined as $a_0 = p \cdot \tan \theta$, where θ is the angle between the $\vec{V}$ axis and $\vec{t}$ generation.

2.3 Worm generation kinematics

There are defined the reference systems: $x_0 y_0 z_0$ is global reference system, with y_0 axis overlapped to the worm's axis; $X_1 Y_1 Z_1$ — mobile reference system, with Y_1 axis overlapped to the $\vec{V}$ axis and $\xi_1 \eta_1 \zeta_1$ — mobile reference system, with η_1 axis overlapped to the $\vec{A}$ axis.

The generating process kinematics assume: the motion I is the rotation of the shaft around its own axis; II — translation of rack gear along the η axis and III rotation of the hob mill. The relative position of fixed reference systems is given by:

$$\mathbf{x}_0 = \beta \cdot (\mathbf{x} - \mathbf{A}); \mathbf{A} = \begin{pmatrix} -R_r - a_0 \\ 0 \\ 0 \end{pmatrix}; \beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \omega & -\sin \omega \\ 0 & \sin \omega & \cos \omega \end{pmatrix}. \quad (15)$$

The rotation and translation of the rack-gear around and along the $\vec{V}$ axis is given by:

$$\mathbf{X}_1 = \omega_2(\varphi_1) \cdot \beta \cdot (\mathbf{x} - \mathbf{A}). \quad (16)$$

The rotation and translation of the rack-gear around and along the $\vec{V}$ axis is given by:

$$\mathbf{X}_1 = \omega_2(\varphi_1) \cdot \boldsymbol{\beta} \cdot (\mathbf{x} - \mathbf{A}). \quad (17)$$

By developing equation (18), see also (19), the following is obtained:

$$\left[C(\varphi, t) \right]_{\varphi_2} \begin{cases} X_1 = (\xi + a_0) \cos \varphi_2 + (\eta - p \varphi_2 \cos \omega) \sin \varphi_2 - \zeta \sin \varphi_2 \cos \omega; \\ Y_1 = (\eta - p \varphi_2 \cos \omega) \cos \omega - \zeta \sin \omega; \\ Z_1 = (\xi + a_0) \sin \varphi_2 + (\eta - p \varphi_2 \cos \omega) \cos \varphi_2 \sin \omega + \zeta \cos \varphi_2 \sin \omega. \end{cases} \quad (20)$$

The enwrapping of the (20) surfaces' family is obtained using the specific theorem of the generating trajectories. The direct parameters of the normal to the helical surface are given by (21). In the generating movement (4), the normal (22) describe a family. Considering that the normal family intersect the $\vec{A}$ axis, results the enwrapping condition.

$$\vec{n}_c = \dot{\eta}_\varphi \cdot \vec{i} - \dot{\xi}_\varphi \cdot \vec{j}. \quad (22)$$

3 Numerical application

A numerical application is presented for a hob mill to generate a splined shaft with: $b = 9$ mm; $R_e = 62.5$ mm; $R_i = 56$ mm; $R_r = 62.338$ mm. The graphical method presumes determination of the profile for the rack-gear cutting tool, using the method of "generating trajectories".

Fig. 3. a). Characteristic curve; **b).** Axial section.

Consequently, the following algorithm is applied. The flank surface of the rack-gear is created extruding the profile of the rack-gear tool along the direction of ζ axis. This is the I surface. The point which represents the origin of the $X_I Y_I Z_I$ reference system is created.

A line which passes through the origin of the $\xi \eta \zeta$ reference system and is inclined with angle ω regarding the η axis is draw. This line represents the $\vec{A}$ axis. The support plane of this axis is the $\eta \zeta$ plane. The new $X_I Y_I Z_I$ reference system is created. The axis of this new system are $X_I \equiv X$ and $Y_I \equiv \vec{A}$. Parallel with axis $\vec{A}$ is created the axis of future hob mill, the $\vec{V}$ axis. This will pass through the origin of the $X_I Y_I Z_I$ reference system. Onto the axis $\vec{A}$, a current point is generated. From this point, a line normal to the surface I is drawn.

According to the "generating trajectories" theorem, the intersection point between the normal and the surface I belongs to the characteristic curve too. This point is kept using the

ISOLATE command and the last two steps are resumed in order to find others points onto the characteristic curve.

In Figure 3.a is presented the form of the characteristic curve and in Figure 3.b is presented the form of the axial section for the hob mill. In Table 1, there are presented the coordinates of points form for the characteristic curve and axial section of hob mill.

Table 1. Coordinates of points form characteristic curve and axial section.

Crt. no.	Characteristic curve			Axial section	
	X [mm]	Y [mm]	Z [mm]	X [mm]	Y [mm]
1	0.000	4.500	-0.234	0.000	4.500
2	1.149	4.666	-0.587	1.192	4.674
3	2.325	4.905	-0.774	2.373	4.918
4	3.493	5.210	-0.905	3.538	5.225
5	4.649	5.568	-1.021	4.688	5.584
6	5.791	5.966	-1.133	5.825	5.983
7	6.922	6.399	-1.237	6.951	6.415
8	8.041	6.862	-1.329	8.063	6.877
9	9.148	7.357	-1.409	9.163	7.370
10	10.242	7.882	-1.481	10.249	7.894

4 Conclusions

The CATIA design environment allows, by software capabilities, to obtain a faster solution, very rigorous and, at the same time, very intuitive, for the helical surface which represents the primary worm of the hob mill.

An application of the “generating trajectories” method, developed for enwrapping surfaces with single point contact is proposed in this paper. An algorithm, developed in analytical form, is presented for a numerical application of the same square splined shaft.

The paper details a numerical example for which the results are presented in Table 1 and Figure 3. There is presented the form of characteristic curve at the contact with rack gear tool and, at the same time, the axial section of the peripheral surface of the worm.

The method presented in this paper can be also applied to other ordered curves of profiles (toothed parts), both for noninvolute profiles and involute tooth. This method can be developed and integrated in a software product.

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