The energetic load smoothing along the rack tool cutting edge in toothing process

Mihai Bordeanu, *Gabriel* Frumuşanu *, and *Nicolae* Oancea

Dunărea de Jos University of Galaţi, Department of Manufacturing Engineering, Domnească Street 111, 800201 - Galaţi, Romania

Abstract. The method of generating the gear tooth with a rack tool is applied on MAAG machine tools. Their specific cutting scheme leads to an irregular energetic loading of the tool cutting edge, during the rolling process. In this paper, we present a new method for analyzing the energetic load along the cutting edges of the rack shaped cutter, based on graphical modelling of the cutting process performed in CATIA. The energetic load smoothing is proposed to be obtained by finding an appropriate variation law for the circular feed communicated to the workpiece by toothing machine kinematical chain, during the rolling motion that permanently respects the rolling condition established between the centrodes attached to both tool and workpiece. A numerical solution in a concrete case of analysis is also included.

1 Introduction

Generating the tooth of cylindrical gear having involute or non-involute profile is a machining process that can be performed on different machine tools by using a multitude of tool types: slotting with rack shaped tool or with pinion cutter, hobbing with worm tool, milling with profiled end mill or profiled disc tool.

The process of generating the tooth space of machined gear takes place with high non-uniformity due to the rolling process between the centrodes to which they are associated the tool cutting edges (usually straight lines in the case of rack with trapezoidal teeth), on one side, and the enwrapped flanks of workpiece teeth (with involute profile, in the mentioned case), on the other side. At the same time, one should not neglect that rack shaped cutter teeth must cut from full material during the process of tool engagement in workpiece [1, 2].

Analysis of tooth space generating process have been already realized and presented in dedicated literature [3-5]. Most of them address to the case of a mono-tooth rack shaped cutter and they all prove that, during the toothing process, the area of the chip detached by tool cutting edge shows high variation when determined for successive rolling positions considered for the couple of centrodes attached to both tool and workpiece. This lack of uniformity leads to occurrence of highly variable cutting specific stress, depending on the tool position relative to the workpiece, with direct impact on the magnitude of main cutting force [6].

* Corresponding author: gabriel.frumusanu@ugal.ro

There are also presented effective solutions suggesting constructive modifications of the generating rack gear in order to diminish the mentioned lack of uniformity [7].

In this paper, we present an analysis of the tooth space generating process, based on graphical modelling in CATIA and approaching the case of cylindrical gear with involute profile machined by mono-tooth tool. This is specific mainly to the gears with high module manufactured on MAAG-type machine tools. The tooth space generating process is tried to be uniformed by adopting a non-uniform process of rolling between the conjugated centrodes attached to both tool and workpiece, achieved by discrete variation of workpiece circular feed and aiming to uniform the area of the chip detached at each double stroke of the rack shaped cutter.

Next section is about modelling the variation of detached chip area. Third section presents the procedure to find the variation law of workpiece circular feed, if targeting to uniform the chip area during gear machining, sampled in a concrete case. The last section is for conclusion.

2 Detached chip area

A solution for measuring the detached chip area when machining a gear with mono-tooth rack tool, based on graphical modelling in CATIA, is further proposed.

2.1 Kinematics of tooth generating by rolling

In Figure 1, there is depicted the kinematical scheme of generating the gear tooth with a rack shaped cutter [8, 9].

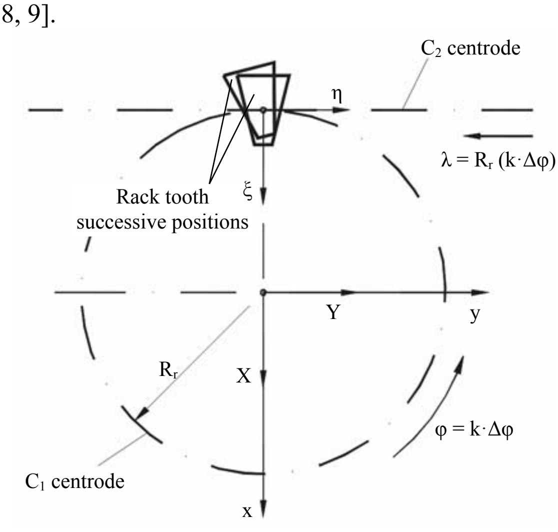


Fig. 1. Reference systems and rolling centrodes

By C_1 is denoted the centrode associated to workpiece, which is a circle having rotation motion of φ angular parameter, while C_2 is the tool centrode – a straight line in translation motion of λ linear parameter.

The following reference systems have to be considered in order to find the equation of the relative motion between workpiece and tool:

- xy, which is the global system, having the origin in workpiece centre
- XY – relative system, attached to the workpiece and initially overlapped to xy
- $\xi\eta$ – relative system, attached to the tool.

The equation of workpiece absolute motion is:

$$x = \omega_3{}^T \left(k \cdot \Delta\varphi\right) \cdot X \ , \tag{1}$$

while the equation of tool absolute motion is:

$$x = \xi + a \text{ , with } a = \begin{pmatrix} -R_r \\ -R_r \cdot \left(k \cdot \Delta\varphi\right) \end{pmatrix}. \tag{2}$$

The equation of relative motion between workpiece and tool results from (1) and (2), after eliminating *x*:

$$X = \omega_3\left(k \cdot \Delta\varphi\right)\left[\xi + a\right]. \tag{3}$$

In relations from above, $\Delta\varphi$ means the increment in workpiece's rotation motion and *k* is the order number of tool double stroke. In up to now approach, the increment remains constant all along the toothing process. In our approach, smoothing of detached chip area should be obtained by transforming $\Delta\varphi$ in a variable parameter, its variation law following to be determined. We also must observe that to $\Delta\varphi$ increment in workpiece's rotation motion corresponds $\Delta\lambda$ increment in tool rectilinear motion,

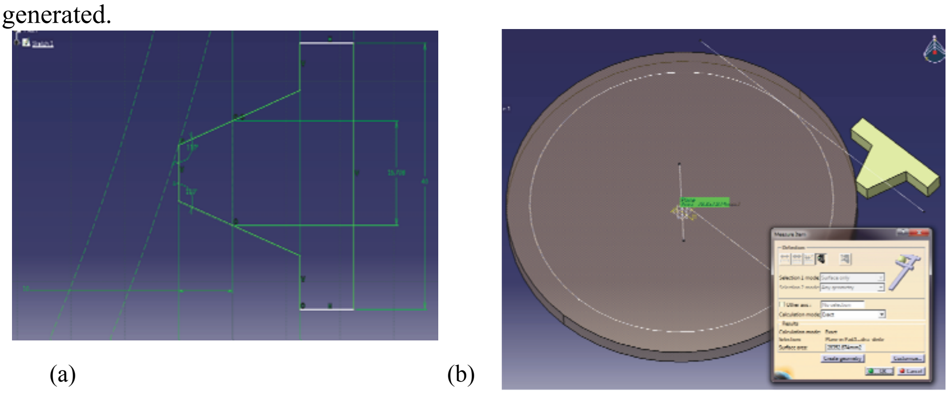
$$\Delta\lambda = R_r \cdot \Delta\varphi \ . \tag{4}$$

<u>Note</u> Obviously, kinematics of the rolling process taking part on MAAG machine tool needs to be modified by introducing a stepper motor and an adequate numerical control system in order to actually obtain the aimed rolling motion with variable speed.

2.2 CATIA modelling of tooth generating process

The area of the chip detached at each active stroke of the rack shaped cutter can be found by performing a graphical modelling in CATIA of the toothing process. In this purpose, two stages need to be covered.

In first stage, the solid models of both workpiece and tooth representing the active part of the generating tool are built by using *Sketch* module of CATIA. First, the workpiece is sketched and geometrical / dimensional constrains are applied to it. Workpiece model is a cylindrical disc having the exterior diameter equal to head diameter of the gear to be generated.



(a) (b)

Fig. 2. Tool tooth sketching – a) & positioning – b)

Then, the tool tooth is sketched in a separate file (Figure 2-a) and submitted to geometrical / dimensional constrains, being positioned in contact with the projection of gear head circle. The condition of tangency between rack shaped cutter rolling line and workpiece rolling circle is also imposed (Figure 2-b). After sketching, the profiles are extruded at the required dimensions with the help of *Pad* modelling tool (available in *Sketch Based Features* toolbar), hereby two solids being generated.

In second stage, the motions following to be imposed to each of solids from above are established such as reproducing the process kinematics (Figure 3). The motions are supposed to be discrete, having $\Delta\varphi$ and $\Delta\lambda$ increments, which respects the rolling condition (4).

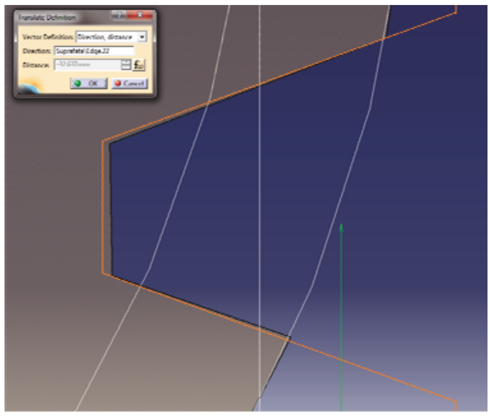


Fig. 3. Imposition of the relative kinematics

2.3 Measurement of detached chip area

For finding the areas of successively detached chips, the tooth model is imported in the file where workpiece disk was sketched. After import, the workpiece is rotated with $\Delta\varphi$ angle, while the tooth is translated along the rolling line with $\Delta\lambda$ distance. By using *Remove* command of *Operations* modelling tool, the common region of the two elements is extracted. The workpiece area is measured before and after extraction (with *Measure Item* tool from *Measure* toolbar) and the removed chip area results by subtraction (Figure 4). This procedure requires importing tooth model before each new intersection.

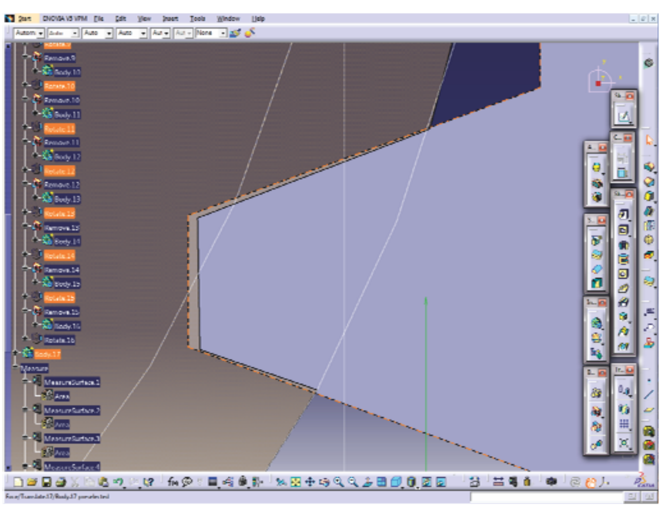


Fig. 4. Measurement of removed chip area

3 Algorithm to find the variation law of workpiece circular feed aiming detached chip area equalisation

The purpose of research here presented was to find the variation law of workpiece circular feed (obviously, in strict correlation with rack shaped cutter linear feed) such as the energetic load along tool cutting edge is smoothened. The smoothening can be achieved by equalising the areas of successively detached chips. A dedicated two-steps algorithm has been developed in order to reach the mentioned target. For better understanding the way it works, we further present algorithm implementation in a concrete case.

3.1 Variation law of detached chip area

Before finding the feed variation law, the variation law of detached chip area need to be determined. The method for measurement of detached chip area A_c presented in previous section is applied when machining a gear with following actual parameters: module m = 10 mm and number of teeth z = 60, see also Figure 5.

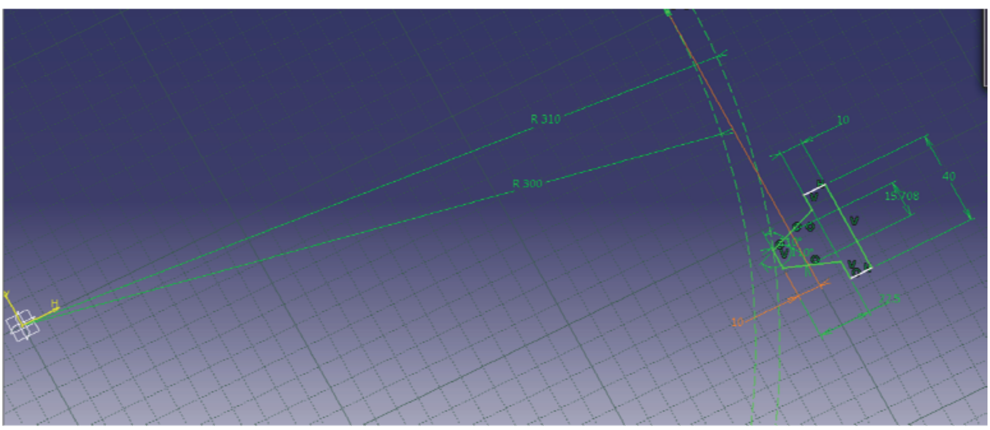


Fig. 5. Machined gear geometry

Table 1. Removed chips area

Crt. no.	Angle, φ [°]	Displacement of tool, λ [mm]	Workpiece area, [mm^2]	Removed chip area, A_c [mm^2]
1	0	0	301907.054	0.000
2	0.394	2.063	301906.131	0.923
3	0.694	3.634	301904.211	1.920
4	0.994	5.205	301901.264	2.947
5	1.294	6.775	301897.316	3.948
		....		
25	5.494	28.767	301813.531	6.394
26	5.794	30.337	301807.128	6.403
27	6.094	31.908	301800.716	6.412
28	6.394	33.479	301794.306	6.410
29	6.694	35.050	301787.906	6.400
		....		
92	27.394	143.435	301585.398	0.021
93	27.694	145.006	301585.385	0.013
94	27.994	146.577	301585.378	0.007
95	28.294	148.147	301585.375	0.003
96	28.594	149.718	301585.375	0.000

The circular feed value was considered as constant, $\Delta\varphi_0$ = 0.3 deg/dbl stroke (starting from the second effective double stroke), corresponding to a rack tool displacement $\Delta\lambda_0$ = 1.571 mm. Excerpts from the table containing the results obtained for all 96 dbl strokes needed for generating the entire tooth space are presented in Table 1 for sampling. By graphical representing the variation of removed chip area during tooth space machining, the curve from Figure 6 resulted.

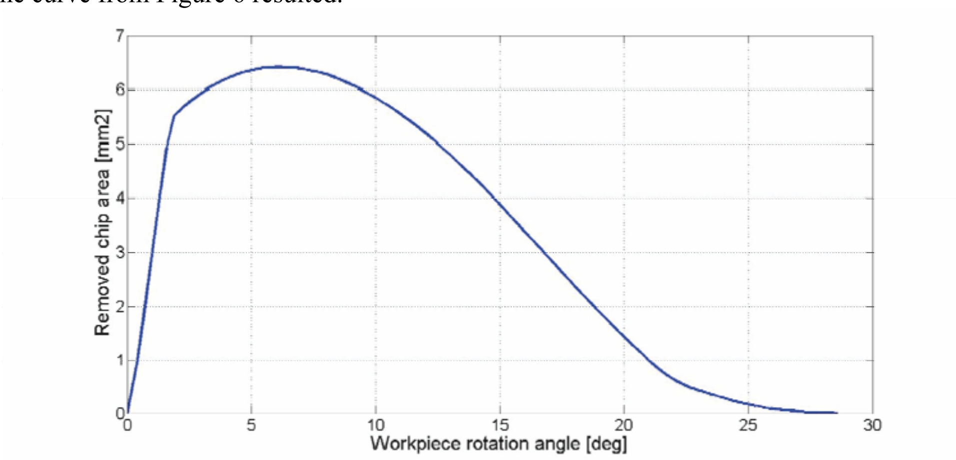


Fig. 6. Variation of detached chip area

As it can be easily noticed, detached chip area presents a high variability during toothing process, this leading to an irregular energetic loading of the tool cutting edge. In order to find the analytical expression of the chip variation law, an approximation curve fitted to the points (φ, A_c) from Table 1 can be found with the help of *Curve Fitting Tool* toolbox from MatLab soft. If choosing to approximate by a 9th degree polynomial function:

$$A(\varphi) = p_1 \cdot \varphi^9 + p_2 \cdot \varphi^8 + \ldots + p_9 \cdot \varphi + p_{10}, \tag{5}$$

then fitting session results are: p_1 = 2.997e-10, p_2 = -4.473e-8, p_3 = 2.87e-6, p_4 = -0.0001038, p_5 = 0.002322, p_6 = -0.03313, p_7 = 0.2987, p_8 = -1.659, p_9 = 5.18, and p_{10} = -0.4928, with a satisfactory goodness of fit (the root mean square error being 0.1238).

3.2 Variation law of workpiece circular feed

By accepting a local proportionality (when workpiece position is defined by angle φ) between A_c and $\Delta\varphi$, one can write:

$$A_c = \frac{\Delta\varphi}{\Delta\varphi_0} \cdot A(\varphi). \tag{6}$$

The knowledge of chip area A variation law in analytical form (5) enabled the development of a MatLab application aiming to find in any moment the actual value of $\Delta\varphi$ (this time considered as variable), corresponding to which the area A_c of next chip detached will have an imposed value, A_{c0}. Hereby, the application determines the variation law of workpiece circular feed leading to a constant value of chip area. The input data are the limits of angle φ variation domain (φ_{min} and φ_{max}), the imposed value of chip area (A_{c0}), the expression of area approximation function (A – imported from *Curve Fitting Tool* toolbox), the value of angular increment ($\Delta\varphi_0$) based on which the chip area variation law was determined. There are also two auxiliary variables involved in numerically solving the

 DOI: 10.1051/matecconf/201711201008

equation (6), namely an incremental parameter $d\varphi$ ($\Delta\varphi = i \cdot d\varphi$, $i = 1, 2, ...$) and the maximum admissible value of the error, ε.

In the implementation example here addressed, $\varphi_{min} = 0°$, $\varphi_{max} = 28.594°$, $A_{c0} = 2$ mm^2, the values of A function coefficients are the ones from above, $\Delta\varphi_0 = 0.3°$, $d\varphi = 0.0001°$, $\varepsilon = 0.01$. By plotting the calculated values of $\Delta\varphi$ relative to double stroke current number and by joining the points, the variation law of workpiece circular feed results in graphical form (Figure 7).

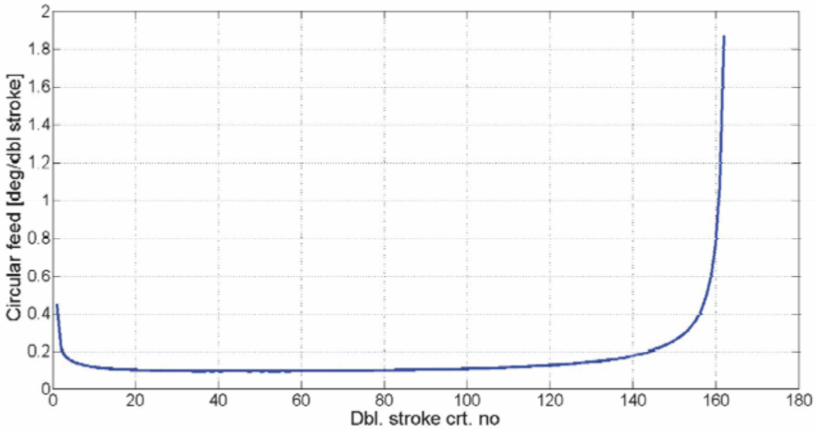


Fig. 7. Circular feed variation law

After examining the diagram from Figure 7, two remarks should be made:

- The number of tool double strokes required for machining the tooth space by removing chips with constant area is different from the one resulted if machining with constant feed (in the addressed case, higher)
- At both beginning and end of machining process, the circular feed values resulted very high, unacceptable in practice because by applying them, the generated surface geometry could be affected. This can be solved by limiting the maximum value of the circular feed, at a preset value (e.g. 0.4°/dbl stroke), see Figure 8. The only disadvantage is, in this case, a higher number of required double strokes.

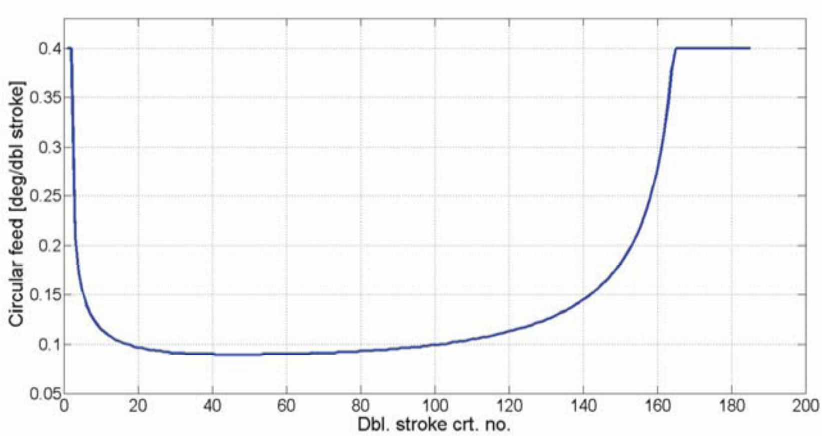


Fig. 8. Circular feed variation law (modified)

The variation of removed chip area resulted if applying the determined variation law of the circular feed is presented in Figure 9.

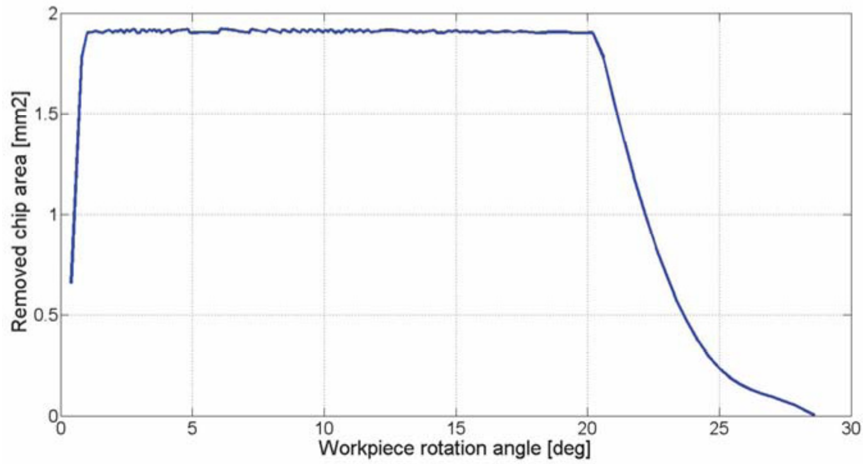


Fig. 9. Smoothened variation of removed chip area

4 Conclusion

This paper analyses the possibility of smoothing the energetic load along the rack tool cutting edge, when machining gear teeth. In this purpose, a graphical modelling of the toothing process performed with a mono-tooth rack shaped cutter has been realized in CATIA. The variation law of removed chip area has been deducted on this base, as the analytical expression of a polynomial function fitted to the points representing the areas of successively detached chips, with the help of MatLab soft facilities. A MatLab application was also written in order to determine the variation law of the workpiece circular feed, if aiming to maintain a constant area of the removed chips, hence to smooth the energetic load along the rack tool cutting edge. The results obtained by solving the approached problem in a particular case prove that, from theoretical point of view, the proposed methodology works fast and precisely and removed chip area may be considered constant for a significant number of tool double strokes. The remaining challenges are to extend the application domain to other types of tools & generated surfaces and to develop a control system enabling to actually implement in practice the imagined solution.

References

1. I. Lăzărescu, *Calculation and construction of cutting tools*, (Tehnică Publishing House, Bucureşti, 1961).
2. I.I. Semencenko, V.M. Matiushin and G.N. Saharov, *Cutting tools design*, (Science and technology for machine building, Moscow, 1962).
3. M. Dima, N. Oancea and V. Teodor, *Modelling the cutting schemes of the toothing processes*, (Cermi, Iaşi, 2007).
4. A. Antoniadis, N. Vidakis and N. Bilalis, J Manuf Sci Eng, **124**, pp.784-791, (2002).
5. V. Georgescu, N. Oancea and E. Constantin, Buletinul Universităţii din Galaţi, Fasc. V, pp.33-42, (1982).
6. J. Ma, X. Ge, S.I. Chang and S. Lei, Int J Adv Manuf Technol, **74**, pp.1701–1708, (2014).
7. M Dima and N. Oancea, Romanian Journal of Technical Sciences, Applied Sciences, **49**, pp.237-241, (2004).

8. S.P. Radzevich, *Kinematics Geometry of Surface Machining* (CRC Press, London, 2008).
9. N. Oancea, *Surface Generation by Enwrapping, Vol. I, Fundamental theorems* („Dunărea de Jos” Publishing House, Galaţi 2004).