Dynamics and Heat and Mass Transfer of Liquid-Droplet Cloud in the Emergency Discharge of Aviation Fuel into the Atmosphere

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Abstract. A mathematical model for the propagation of liquid-droplet cloud in the atmosphere during the emergency discharge of aviation fuel is presented. The results of a numerical analysis of the dynamics and heat and mass transfer of a liquid-droplet cloud in the emergency discharge of aviation fuel into the atmosphere, depending on the season-climatic and meteorological conditions of specific regions of Russia are given.

1 Introduction

To reduce the possible risks in the event of emergency situations during flight it is the practice of dumping fuel into the atmosphere from filling tanks before an urgent landing (Fig. 1) [1].

Fig. 1. Emergency discharge of fuel from the aircraft Northwest Airlines Boeing 747.

Since the mass of jet fuel to be discharged is tens of tons, it is necessary to provide emergency discharge conditions in which the fuel droplets deposition on the earth's surface will be minimized. Currently, the conditions for the dumping of aviation fuel in emergency situations are regulated by the rules of the International Civil Aviation Organization (ICAO). The minimum height of fuel discharge is 1850 m for flights over Western Europe territory in accordance with ICAO rules. These conditions ensure that the bulk of the fuel evaporates in the atmosphere, and only \sim 8\% of the mass of the fuel reaches the ground surface. It should be noted that when flying over the territories of other regions whose climatic conditions are significantly different from those in Western Europe, these regimes

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of discharging do not always guarantee that there is no significant amount of fuel on the earth's surface.

The aim of the presented paper is to analyze the dynamics and heat and mass transfer of a liquid-droplet cloud in the emergency discharge of aviation fuel into the atmosphere, depending on the season-climatic and meteorological conditions of specific regions of Russia.

2 Mathematical modeling of dynamics and heat and mass transfer of liquid-droplet cloud in the emergency discharge of aviation fuel into the atmosphere

Numerical analysis was carried out under the following assumptions. At the initial time, the volume of fuel injected into the atmosphere was considered as a drop of spherical shape (a liquid-droplet cloud) with a diameter equal to the characteristic width of the impact jet (Fig. 1). The initial velocity of the liquid-droplet spherical cloud was equal to the speed of the aircraft. It was believed that the liquid-droplet cloud propagates in the atmosphere under the influence of gravity, the force of aerodynamic drag and wind force. The processes of repeated crushing of drops, evaporation and heat exchange with the ambient air were taken into account. Within the framework of the assumptions made, the mathematically considered process is described by a system of equations [2–4]. The motion of a drop in a Cartesian coordinate system was described by the equations of motion in the velocity field of the steady gas flow with allowance for the forces of viscous drag and gravity:

\[
\begin{aligned}
\frac{dV_{px}}{dt} &= \frac{3\rho_f}{4\rho_d d_p(t)} C_D \left[ U_f - V_p \right] \left( U_{fx} - V_{px} \right), \\
\frac{dX_p}{dt} &= V_{px}, \\
\frac{dV_{py}}{dt} &= \frac{3\rho_f}{4\rho_d d_p(t)} C_D \left[ U_f - V_p \right] \left( U_{fy} - V_{py} \right) - g, \\
\frac{dY_p}{dt} &= V_{py},
\end{aligned}
\] (1)

where \( t \) is time; \( d_p \) is the droplet diameter; \( \rho_f, \rho_d \) are the density of air and droplet substances, respectively; \( U_{px}, U_{py}, V_{px}, V_{py} \) are projections of the velocity of air movement and the velocity of the droplet movement on the axis of the Cartesian coordinate system; \( X_p, Y_p \) are the current coordinate of the drop movement along the \( x \) and \( y \) axes, respectively; \( C_D \) is coefficient of aerodynamic drag, \( g = 9.81 \text{ m/s}^2 \) is acceleration of gravity.

The thermal state of a drop was described by the thermal conductivity equation in a spherical coordinate system [3–5]:

\[
\rho_p(T_p(r,t)) \cdot c_p(T_p(r,t)) \cdot \frac{\partial T_p(r,t)}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \lambda(T_p(r,t)) \cdot r^2 \cdot \frac{\partial T_p(r,t)}{\partial r} \right),
\] (2)

where \( r \) is the spherical coordinate; \( r_p \) is the radius of a drop; \( T_p \) is drop temperature; \( c_p \) and \( \lambda \) are the specific isobaric heat capacity and the coefficient of thermal conductivity of the droplet substance, respectively.

Taking into account the processes of evaporation and convective heat exchange with air, equation (2) was supplemented by the following initial and boundary conditions [3, 6]:
\[ t = 0, \ 0 \leq r \leq r_p : \ T_p (r) = T_{p0} , \]  
\[ t > 0, \ r = 0 : \ \frac{\partial T_p (t)}{\partial r} = 0 , \]  
\[ t > 0, \ r = r_p : \ \lambda (T_p (t)) \cdot \frac{\partial T_p (t)}{\partial r} = \alpha_f \cdot (T_f - T_p (t)) - q_{vap} \cdot w_{vap} (t) , \]  
where \( T_{p0} \) is the initial drop temperature; \( T_f \) is ambient temperature (air); \( q_{vap} \) is the heat of evaporation of drop substance; \( w_{vap}(t) \) is the rate of drop evaporation; \( \alpha_f \) is the coefficient of heat exchange with air. The coefficient of heat transfer \( \alpha_f \) in (5) is determined by the formula for flow past a spherical particle:

\[ \alpha_f (t) = \text{Nu} \cdot \frac{\lambda_f}{d_p (t)} , \]  
where \( \lambda_f \) is the thermal conductivity coefficient of the air; \( \text{Nu} \) is the Nusselt number.

The Nusselt number in (6) is determined by the Rantz-Marshall relationship [5] under the conditions considered:

\[ \text{Nu} = (2 + 0.6 \cdot \text{Re}^{1/2} \cdot \text{Pr}^{1/3}) , \]  
where \( \text{Re} \) is Reynolds number; \( \text{Pr} \) is Prandtl number.

The Reynolds and Prandtl numbers for the flow around a spherical drop by air are determined by the relations

\[ \text{Re}_f = \frac{\rho_f \cdot |U_f - V_p| \cdot d_p}{\mu_f} , \quad \text{Pr}_f = \frac{\mu_f c_{pf}}{\lambda_f} , \]  
where \( \mu_f \), \( c_{pf} \), and \( \lambda_f \) are the dynamic viscosity coefficient, the specific isobaric heat capacity and air thermal conductivity coefficient.

The change of a drop diameter due to evaporation was determined by the formula [2]:

\[ \frac{d_p (t)}{dt} = \frac{2 \cdot G_p (t)}{\rho_p (t) \cdot S_p (t)} , \]  
where \( d_p (t) \) is a drop diameter; \( G_p (t) = w_{vap} (t) \cdot dt \) is the mass of the evaporated substance in a time \( dt \); \( S_p (t) \) is the surface area of a spherical drop.

The crushing of drops under the action of inertia forces (Rayleigh-Taylor instability) was taken into account by calculating the value of the Bond’s number

\[ \text{Bo} = \Delta \rho \cdot |\omega| \cdot d_p^2 (t) , \]  
where \( \Delta \rho = \rho_p - \rho_i \); \( \Delta \rho \) is the surface tension coefficient of a drop; \( |\omega| \) is the modulus of acceleration vector of a drop.

The crushing of drops under the action of aerodynamic drag (Kelvin-Helmholtz instability) was taken into account by calculating the value of the Weber’s number
We = \frac{\rho_f \cdot |U_f - V_p|^2 \cdot d_p^2(t)}{\sigma_p} \tag{11}

It was assumed that when the critical value of the Bond number is \( Bo = 22.5 \) or the critical Weber number \( We = 17 \), the drop is split into two spherical drops of equal mass with an initial velocity equal to the velocity of drop initial [2].

When the mathematical model (1)–(5) was parametrized, it was taken into account that in the conditions of emergency fuel discharge from the filling tanks, the liquid-droplet cloud initially falls into the “trace” – into dragged by co-current aircraft wing flow. Thus, at the initial moment of time, the liquid-droplet cloud has a velocity equal to the airflow velocity in the “trace”. The air velocity profile in the “trace” is described by the equation [7] in case of an axisymmetric co-current:

\[ \frac{\frac{u}{U}}{1 - \eta^3/2}, \tag{12} \]

where \( u \) is the airflow velocity in the “trace”; \( U \) is the speed of the aircraft; \( \eta = y / \delta \), \( y \) is the distance from the “trace” axis, \( \delta \) is the half-width of the “trace”.

The half-width of the “trace” \( \delta \) was estimated according to [7]:

\[ \delta \approx \left( \frac{F_x}{\rho_f \cdot U^2} \right)^{1/3}, \tag{13} \]

where \( F_x \) is the lift force of the wing, \( U \) is the aircraft speed at the moment of fuel discharge.

The mathematical model (1)–(5) is implemented as a software package.

### 3 Analysis of the results of numerical modelling

A numerical study was made of the effect of the most significant factors limiting the crushing of droplets during the propagation of a liquid-drop cloud in the atmosphere on the basis of the mathematical model (1)–(6). Based on the results of the analysis of the preliminary calculations, the size of the drops formed during crushing during gravity deposition was estimated, depending on the initial size of the liquid-drop cloud. The initial size of the liquid-drop cloud was varied in the range \( d_p = (0.01 \div 0.5) \) m. The speed of the aircraft varied in the range \( U_f = (600 \div 900) \) km/h at the time of emergency fuel discharge.

It is found that drops form of diameter \( d_p \approx 10^{-3} \) m during the time of ~ 1 s in the whole range of speed of the aircraft, regardless of the initial size of the liquid-drop cloud. At \( U_f = 600 \) km/h, the minimum drop size was \( d_p \approx 1.25 \cdot 10^{-3} \) m, at \( U_f = 900 \) km/h – \( d_p \approx 0.98 \cdot 10^{-3} \) m. The aerodynamic crushing is completed when the drop is vertically moved to a distance of < 10 m, depending on the initial size of the liquid-droplet cloud. The drop size decrease is determined by the evaporation process in the process of further propagation in the atmosphere. The minimum height \( h_{\min} \) of the emergency fuel discharge, which ensures complete evaporation of fuel droplets in the atmosphere were determined, depending on the season-climatic conditions. Analysis is performed for the regions of Russia located in different climatic zones: Kaliningrad and Novosibirsk Regions, Yakutia (Sakha Republic). The atmospheric parameters averaged over time are taken from [8]. The results of the minimum height calculating \( h_{\min} \) are showed in Fig. 2–4.
Fig. 2. Minimum heights of emergency fuel discharge for different regions of Russia, $U_{fr}=600$ km/h.

From the data ones shown in Fig. 2 it follows that when speed of aircraft is 600 km/h, the minimum fuel discharge heights, ensuring no precipitation of aviation fuel to the earth's surface, do not meet the requirements of the ICAO in the cold months of the year. This period is from January to April and from October to December for the territory of the Novosibirsk region, from January to May and from September to December for Yakutia, from January to March and December for the Kaliningrad region. There is a risk of precipitation of unevaporated fuel to the surface of the earth in these periods.

Emergency discharge of aviation fuel with an increase in aircraft speed to 900 km/h allows to reduce ecologically dangerous periods of the year (Figure 3) for the regions of Yakutia and the Novosibirsk region. The minimum heights of the emergency discharge above the territory of the Kaliningrad region ensure the evaporation of fuel droplets in the atmosphere throughout the year at this speed. The obtained results are in satisfactory agreement with ICAO rules developed for flights over the territory of Western Europe (Fig. 4). This is explained by the compliance of the seasonal and climatic conditions of the Kaliningrad region with the conditions of the European regions. Seasonal and climatic
conditions of Yakutia and the Novosibirsk region are significantly different from European by continuance of cold periods and by air temperature ranges during these periods.

Based on the mathematical model (1)–(5), the values of the mass fraction of aviation fuel, falling on the earth's surface in ecologically dangerous periods depending on the speed of the aircraft at the time of discharge were determined. In particular, it was obtained that ~ 0.5 mass of fuel is deposited on the earth's surface during emergency discharge from an airplane over the territory of Yakutia in the coldest period of the year (from December to February).

4 Conclusion

Analysis of the results of mathematical modeling of dynamics and heat and mass transfer of a liquid-droplet cloud in the emergency discharge of aviation fuel, depending on the season-climatic and meteorological conditions of specific regions of Russia, showed that the speed of the aircraft, at which an emergency fuel discharge occurs, is one of the most significant factors affecting the processes of crushing and evaporation of drops in the atmosphere. It seems advisable to develop regional rules, similar to ICAO rules, regulating the organization of fuel dumping in accidents, taking into account the seasonal and climatic conditions of specific regions. The development of regional rules can be carried out on the basis of mathematical modeling using the proposed model.

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