MATEC Web of Conferences **108**, 11002 (2017)
ICMAA 2017
DOI: 10.1051/matecconf/201710811002

Sensible Performance Analysis of Multi-Pass Cross Flow Heat Exchangers

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Abstract. In this paper, a steady state sensible performance analysis of a multi-pass cross flow exchanger exhibiting various flow circuiting is considered. Counter cross flow, parallel cross flow and pure cross flow (where the flow circuiting is neither in parallel nor in counter flow) are considered in this paper. A previously developed matrix approach is used to study the heat exchanger performance at each individual pass. The equations required for modeling a cross flow heat exchanger for each flow arrangement are presented. Thereafter, a baseline heat exchanger geometry was selected and performance of the heat exchanger for each flow circuiting was described. As expected, the best thermal performance was seen in a counter cross flow heat exchanger and the performance of pure cross flow was intermediate between that of a parallel and a counter cross flow heat exchanger.

1 Introduction

This paper presents results from a steady state sensible performance investigation of a multi-pass cross flow exchanger possessing various flow circuiting. The analysis is based on the matrix approach previously reported in Silaipillayarputhur and Idem [1]. Silaipillayarputhur and Idem [2] considered practical validation of the matrix performance model. In that study the governing equations required to model a multi-pass counter cross flow heat exchanger with continuous wavy fins were presented. The heat exchanger selected for validation was a chilled water coil used at a chemical facility in Chattanooga, TN, USA. The predictions obtained from the performance model were compared with actual data from the chilled water coil and the theoretical performance data from the manufacturer. Based on the comparisons, it was concluded that the matrix heat exchanger performance model predicted the performance of a counter cross flow heat exchanger with at least 95% accuracy.

In process industries, several heat exchanger flow configurations occur, due to existing piping connections and space constraints. However, the thermal advantage of using counter flow heat exchangers is well known. In the present paper a baseline, finned six-pass cross flow heat exchanger presented in [2] is considered for further analysis. A steady state thermal performance comparison is conducted between overall counter flow, overall parallel flow, and pure cross flow. In each instance, the matrix approach is employed to study the heat exchanger performance for each individual pass. Thereafter, a parametric study is performed on the same six-pass cross flow heat exchanger subjected to counter, parallel, and pure cross flow circuiting by varying the NTU and capacity rate ratios. For each case, the effectiveness of the heat exchanger is plotted against the significant dimensionless parameters.

Numerous other papers have been reported in the literature pertaining to steady state heat exchanger performance modeling, and only the most relevant papers are reported herein. Pignotti and Shah [3] considered the effectiveness and NTU relationships for heat exchangers with complex flow arrangements. Heat exchanger terminal temperatures, surface area, and fluid flow rates were presented in dimensionless form in terms of heat exchanger effectiveness, number of transfer units, and heat capacity rate ratios. Explicit effectiveness-NTU relationships were obtained for a total of 18 complex heat exchanger flow arrangements, and the results were summarized in tabular format. Domingos [4] presented general method for calculating the total effectiveness and intermediate temperatures of assemblies of heat exchangers. The assemblies could consist of associations of any types of heat exchanger. The method utilized a transformation that related the inlet and outlet temperatures of the fluid streams, and thus permitted the derivation of closed form expressions. Shah and Pignotti [5] examined complicated heat exchanger flow arrangements and related them to simple forms for which either a solution existed, or an approximate solution could be obtained. Chen and Hsieh [6] developed a simple and systematic procedure to determine the effectiveness and exit fluid temperatures of complex assemblies of identical heat exchangers. Three complex assemblies were chosen to illustrate the procedure. The assembly with non-identical heat exchangers was also studied to examine the general applicability of the present procedure. Baclic [7] proposed a simplified formula for cross flow heat exchanger effectiveness. The formula related effectiveness NTU and heat capacity rate ratio for cross flow heat exchangers when both the fluids are unmixed.

2 Nomenclature

A – Heat transfer surface area
C – Heat capacity rate of a fluid
c – Specific heat at constant pressure
$\dot{m}$ - Mass flow rate
n – Number of passes
NTU – Overall number of transfer units
r – Capacity rate ratio
T_A - Temperature of the external (hot) fluid
T_{Ai} - Inlet temperature of the external (hot) fluid
T_{Ao} - Outlet temperature of the external (hot) fluid
T_w - Temperature of the tube-side (cold) fluid
T_{wi} - Inlet temperature of the tube-side (cold) fluid
T_{wo} - Outlet temperature of the tube-side (cold) fluid
U_o - Overall heat transfer coefficient
Greek Letters
ε - Effectiveness, matrix approach
Subscripts
A – External fluid (hot fluid)
i – running index (1 through n)
i - Inside
j – running index (1 through n)
min – Minimum
max – Maximum
o - Overall
r – Row
sum - Sum
w – Tube-side fluid (cold fluid)
Superscripts
” – Quantity expressed on per pass basis

3 Steady state performance model

A baseline heat exchanger presented in [2] is considered for further analysis. Table 1 presents the geometry of the baseline heat exchanger, as well as other fundamental operating characteristics. The baseline heat exchanger is a six-pass cross flow heat exchanger, and it is employed for a quenching process in a chemical plant in Chattanooga, TN. The baseline heat exchanger has air in the gas-side of the heat exchanger and has chilled water in the tube-side of the heat exchanger. In this paper, the thermal performance of the baseline heat exchanger is compared between overall counter and parallel flow, and pure cross flow. Only sensible heat transfer between the fluids is considered, and there is no phase change. The number of transfer units is assumed to be uniformly distributed among the heat exchanger passes. Although fouling is a common occurrence in practice, the effects of fouling are not considered in the current study.

For the overall heat exchanger, the external fluid is assumed to be the minimum capacity rate fluid, since this is a commonly encountered situation in process industries. For a parallel and counter cross flow heat exchanger, the overall capacity rate ratio is equal to the capacity rate ratio per pass. This is because each pass encounters the same full mass flow rate of the external fluid and the tube side fluid. However, for a pure cross flow heat exchanger, the tube-side fluid is assumed to be evenly split among the tube passes. Considering the baseline heat exchanger with six passes, for a pure cross flow configuration on a per pass basis the tube-side fluid encounters one-sixth of the overall mass flow rate. Thus for a pure cross flow configuration, on a per pass basis the tube-side fluid can be the minimum capacity fluid.

Heat exchanger overall performance

For the overall heat exchanger, the external fluid (air), designated through subscript “A” is the minimum capacity rate fluid and while the tube-side fluid (chilled water) designated through subscript “w” is the maximum capacity rate fluid. The capacity rate ratio for the overall heat exchanger is given by [8,9]

$$r = \frac{C_{min}}{C_{max}} = \frac{(\dot{m} \cdot c)_A}{(\dot{m} \cdot c)_w} \tag{1}$$

Likewise, the overall NTU for the heat exchanger is expressed as [8,9]

$$NTU = \frac{U_o \cdot A_o}{C_{min}} = \frac{U_o \cdot A_o}{(\dot{m} \cdot c)_A} \tag{2}$$

The effectiveness for the overall heat exchanger, assuming both fluids to be unmixed, may be calculated as [8,9]

$$\varepsilon = 1 - \exp\left[\frac{(NTU)^{0.22}}{r}\left\{\exp\left[-r(NTU)^{0.78}\right] - 1\right\}\right] \tag{3}$$

The calculated effectiveness of the heat exchanger is related to rate of heat transfer by

$$\varepsilon = \frac{q}{q_{max}} \tag{4}$$

The heat transfer is determined by means of the following energy balance expressions [8,9]

$$q = \dot{m}_A c_A (T_{Ai} - T_{Ao}) \tag{5}$$

and:

$$q = \dot{m}_w c_w (T_{wo} - T_{wi}) \tag{6}$$

The inlet temperatures of both fluids are known quantities. In that case the maximum heat transfer is given by [8,9]

$$q_{max} = C_{min}(T_{Ai} - T_{wi}) = \dot{m}_A c_A (T_{Ai} - T_{wi}) \tag{7}$$

Employing Equations 4 and 7, the rate of heat transfer between the two fluids in the heat exchanger can be determined, and from that calculation the discharge temperatures of both the fluids can readily be evaluated.

Intermediate thermal performance for parallel and counter cross flow heat exchangers using the matrix approach

The baseline heat exchanger as described in Table 1 is a six-pass cross flow heat exchanger. Figures 1 and 2 depict the flow circuiting of a six-pass parallel and counter cross flow heat exchangers.

Table 1. Baseline heat exchanger operating characteristics

Baseline heat exchanger	
Item	**Description**
Number of passes	6
Fin material	Aluminium
Fin thickness	0.00024 m
Fin density	315 fins/m
Tube material	Copper
Tube OD	0.016 m
Tube thickness	0.0008 m
Number of rows per pass	1
Number of tubes	46
Tube length	1.65 m
Longitudinal pitch	0.038 m
Transverse pitch	0.033 m
Mass flow rate of water	9396 kg/hr
Mass flow rate of air	17863 kg/hr
Inlet air temperature (T_{Ai})	296.3 K
Inlet water temperature (T_{wi})	284.1 K

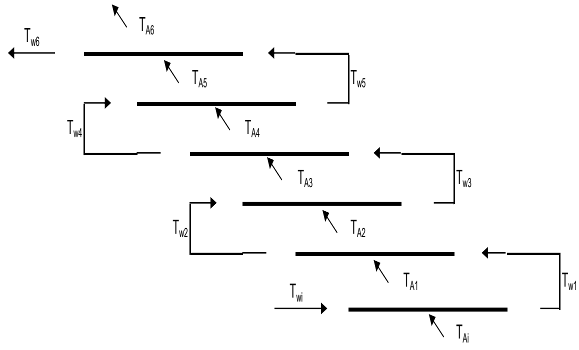

Figure 1. Flow circuiting for a six-pass parallel cross flow heat exchanger.

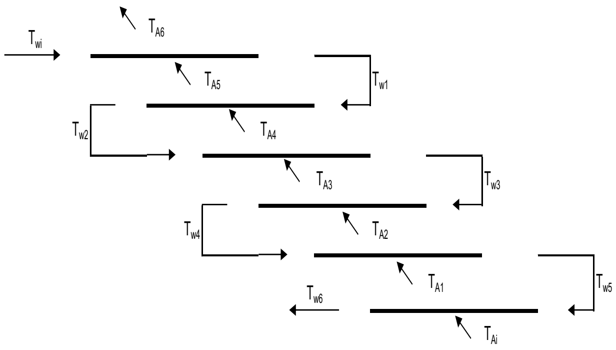

Figure 2. Flow circuiting for a six-pass counter cross flow heat exchanger.

Each pass of the cross flow heat exchanger encounters the full mass flow rate of the external fluid and tube-side fluid. Hence, the capacity rate ratio per pass may be given as

$$r'' = r = \frac{(\dot{m}\cdot c)_A}{(\dot{m}\cdot c)_w} \tag{8}$$

The surface area of the heat exchanger is assumed to be uniformly distributed among the heat exchanger passes. Therefore, the NTU per pass may be expressed as

$$NTU'' = \frac{U_o \cdot A_o/n}{C_{min}} = \frac{U_o \cdot A_o}{n(\dot{m}\cdot c)_A} \tag{9}$$

Thus by Equation 3 the heat exchanger effectiveness per pass is

$$\varepsilon'' = 1 - \exp\left[\frac{(NTU'')^{0.22}}{r''}\left\{\exp\left[-r''(NTU'')^{0.78}\right] - 1\right\}\right] \tag{10}$$

The matrix approach [1] uses the concept of an energy balance and effectiveness applied to each pass of a cross flow heat exchanger. Thus if a cross flow heat exchanger has “n” number of passes, there will be “2n” simultaneous equations to be solved. For every pass there are two unknowns, namely the discharge temperature of the external fluid and the discharge temperature of the tube-side fluid. By solving the system of simultaneous linear equations the unknown discharge temperatures exiting each pass can be readily determined. For a multi-pass parallel cross flow heat exchanger, the following generalized equations are employed to determine the intermediate and final temperatures of both the fluids in the heat exchanger. If “m” corresponds to the number of passes, “n” corresponds to twice the number of passes, and j corresponds to anywhere between “1” and “n”, then referring to [1]:

For j = 1

$$r''T_{Aj,j+1} + T_{Bj,j+1} = r''T_{Ai} + T_{wi} \tag{11}$$

For j = 2;m-1

$$r''T_{Aj-1,j} - r''T_{Aj,j+1} + T_{wj-1,j} - T_{wj,j+1} = 0 \tag{12}$$

For j = m

$$r''T_{Aj-1,j} - r''T_{Ao} + T_{wj-1,j} - T_{wo} = 0 \tag{13}$$

For j = m+1

$$T_{A12} = (1 - \varepsilon''_{r1})T_{Ai} + \varepsilon''_{r1}T_{wi} \tag{14}$$

For j = m+2;n-1

$$(1 - \varepsilon''_{rj-m})T_{Aj-(m+1),j-m} - T_{Aj-m,j-m+1} + \varepsilon''_{rj-m}T_{wj-(m+1),j-m} = 0 \tag{15}$$

For j = n

$$(1 - \varepsilon''_{rj-m})T_{Aj-(m+1),j-m} - T_{Ao} + \varepsilon''_{rj-m}T_{wj-(m+1),j-m} = 0 \tag{16}$$

Likewise, for a multi-pass counter cross flow heat exchanger, the following generalized equations can be utilized [1] to determine the intermediate and final temperatures of both the fluids in the heat exchanger:

For j = 1

$$T_{Aj,j+1} - \varepsilon''_{rj}T_{wj} = (1 - \varepsilon''_{rj})T_{Ai} \tag{17}$$

For j = 2;m-1

$$(\varepsilon''_{rj} - 1)T_{Aj-1,j} + T_{Aj,j+1} - \varepsilon''_{rj}T_{wj,j+1} = 0 \tag{18}$$

For j = m

$$(\varepsilon''_{rj} - 1)T_{Aj-1,j} + T_{Ao} = \varepsilon''_{rj}T_{wi} \tag{19}$$

For j = m+1;n-2

$$-r''T_{Aj-m,j-m+1} + r''T_{Aj-m+1,j-m+2} + T_{wj-m,j-m+1} - T_{wj-m+1,j-m+2} = 0 \qquad (20)$$

For j = n-1

$$-r''T_{Aj-m,j-m+1} + r''T_{Aj-m+1,j-m+2} + T_{wj-m,j-m+1} = T_{wi} \qquad (21)$$

For j = n

$$r''T_{A12} + T_{w12} + T_{wo} = r''T_{Ai} \qquad (22)$$

Intermediate thermal performance for pure cross flow heat exchangers using the matrix approach

Figure 3. depicts the flow circuiting of a six-pass pure cross flow heat exchanger. Each pass of the cross flow heat exchanger encounters the full mass flow rate of the external fluid and one sixth of the overall mass flow rate of the tube-side fluid.

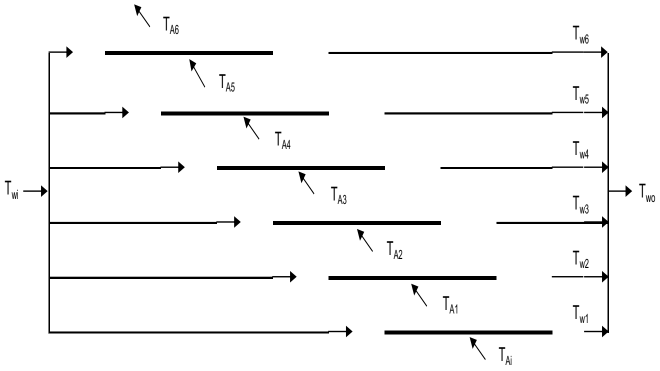

Figure 3. Flow circuiting for a six-pass pure cross flow heat exchanger.

In his analysis, it is reasonable to assume that on a per pass basis, the tube-side fluid is the minimum capacity rate fluid. Thus, the capacity rate ratio per pass is given by

$$r'' = \frac{(\dot{m}\cdot c)_w}{n(\dot{m}\cdot c)_A} = \frac{1}{n}.\frac{1}{r} \qquad (23)$$

Likewise, the NTU per pass can be expressed as

$$NTU'' = \frac{U_o \cdot A_o / n}{C_{min}} = \frac{U_o \cdot A_o / n}{(\dot{m}\cdot c)_w / n} \qquad (24)$$

Or:

$$NTU'' = \frac{U_o \cdot A_o}{(\dot{m}\cdot c)_w} = NTU \cdot r \qquad (25)$$

Assuming both fluids to be unmixed in a given pass the heat exchanger effectiveness per pass may be calculated using Equation 10.

For a multi-pass pure cross flow heat exchanger, the following generalized equations [1] are applied to determine the intermediate and final temperatures of both the fluids in the heat exchanger:

For j = 1

$$T_{Aj,j+1} + r''T_{wj} = T_{Ai} + r''T_{wi} \qquad (26)$$

For j = 2;m

$$-T_{Aj-1,j} + T_{Aj,j+1} + r''T_{wj} = r''T_{wi} \qquad (27)$$

For j = m+1

$$T_{w1} = (1-\varepsilon''_{r1})T_{wi} + \varepsilon''_{r1}T_{Ai} \qquad (28)$$

For j = m+2;n

$$-\varepsilon''_{rj-m}T_{Aj-(m+1),j-m} + T_{wj-m} = T_{wi} - \varepsilon''_{rj-m}T_{wi} \qquad (29)$$

4 Baseline heat exchanger performance

The baseline six-pass cross flow heat exchanger performance was studied for overall counter and parallel flow and pure cross flow circuiting configurations. Therein, the heat exchanger overall performance was described through the heat exchanger effectiveness. The matrix approach was employed to study the intermediate thermal performance of the heat exchanger. A Matlab code was developed to solve the system of simultaneous linear equations and to determine the intermediate and the overall thermal performance of the cross flow heat exchanger. In every instance, the external fluid was assumed to be the minimum capacity rate fluid. The external fluid and the internal fluid were considered unmixed in the analysis.

Table 2. Performance of baseline cross flow heat exchanger

Baseline cross flow heat exchanger							
6 pass Counter Flow			6 pass Parallel Flow			6 pass Pure Cross Flow	
Temp Label	Temp (K)		Temp Label	Temp (K)		Temp Label	Temp (K)
T_{Ai}	296.3		T_{Ai}	296.3		T_{Ai}	296.3
T_{A1}	293.4		T_{A1}	292.3		T_{A1}	293.5
T_{A2}	291.1		T_{A2}	290.2		T_{A2}	291.3
T_{A3}	289.3		T_{A3}	289.1		T_{A3}	289.6
T_{A4}	287.9		T_{A4}	288.5		T_{A4}	288.3
T_{A5}	286.8		T_{A5}	288.2		T_{A5}	287.4
T_{A6}	285.9		T_{A6}	288.1		T_{A6}	286.6
T_{wi}	284.1		T_{wi}	284.1		T_{wi}	284.1
T_{w1}	284.5		T_{w1}	286.0		T_{w1}	286.2
T_{w2}	285.0		T_{w2}	286.9		T_{w2}	286.8
T_{w3}	285.7		T_{w3}	287.4		T_{w3}	287.6
T_{w4}	286.5		T_{w4}	287.7		T_{w4}	288.7
T_{w5}	287.5		T_{w5}	287.8		T_{w5}	290.1
T_{w6}	288.9		T_{w6}	287.9		T_{w6}	291.9
$\varepsilon_{overall}$	**0.86**		$\varepsilon_{overall}$	**0.67**		$\varepsilon_{overall}$	**0.80**

A comparison of steady state performance was conducted for cases where the baseline heat exchanger was operated either with overall parallel or counter flow circuiting, or otherwise for pure cross flow circuiting. The specific input conditions are outlined in Table 1. A complete accounting of the equations used to characterize the heat exchanger geometry, and to evaluate such dimensionless parameters as the particular capacity rate ratio and NTU, are provided in [2], and are not included herein. The results of the present study are presented in Table 2 in terms of the overall and intermediate steady state sensible performance of the baseline heat exchanger. As expected the counter cross flow heat exchanger yielded the best heat exchanger performance, i.e., the maximum overall effectiveness. Likewise, the performance of the pure cross flow heat exchanger was intermediate between that for overall counter and parallel cross circuiting.

5 Summary

Steady state sensible performance of a cross flow heat exchanger was studied in this paper. A baseline heat exchanger presented in [2] was analyzed subject to various flow circuiting considerations, i.e., overall parallel and counter flow, as well as pure cross flow. The baseline heat exchanger‘s performance was determined for each flow circuiting, assuming operating conditions typical of design conditions.

In every instance, the heat exchanger considered in this study had six passes. However, the number of passes considered is arbitrary (subject to mandatory performance requirements), and the analysis can be readily extended to any number of passes by employing the matrix approach. The matrix approach uses fundamental analytical relationships to study the intermediate and the overall performance of a cross flow heat exchanger. The matrix approach is very simple to use, and by solving a set of generalized equations, the intermediate and the overall performance of the heat exchanger can be readily determined. Matrix analysis uses physically significant parameters such as NTU and capacity rate ratio to evaluate overall heat exchanger effectiveness, and thus provides clear information to the engineers during the initial design or selection of the cross flow heat exchanger. With the availability of intermediate temperatures, the matrix approach can be utilized to optimize size, material, weight, and initial cost of the cross flow heat exchanger. Furthermore, the matrix method can assist with thermal stress analysis of heat exchanger, since it provides detailed information pertaining to local temperature variations in the heat exchanger.

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