Analytical Investigation on the Damping Performance of Atorque Converter in an Automotive Driveline Model

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Abstract. Torque converters are avital part of automatic transmission vehicles. In this paper the influence of a torque converter on the lowest mode of driveline vibrations (shuffle mode) is investigated. The automotive driveline is modelled by lumped parameter models, in which a steady-state torque converter model is inserted between the engine and the gearbox. A detailed mathematical model of a two-degree of freedom and a three-degree of freedom dynamic systemis given. The resulting nonlinear differential equations are linearized to obtain system transfer functions and to study damping characteristics of the torque converter. Additionally, a dynamic system model with nonlinear equations is developed in *MATLAB/Simulink* to verify the linearized system. The simulations show that the high frequency vibrations from the engine side are damped due to the torque converter and the linearized model predicts the system response accurately.

1 Introduction

Torque converters play an important role in the driving comfort, fuel economy and performance of automatic transmission vehicles. Therefore, they have been investigated analytically, numerically and experimentally by many researchers in the last decades.

Hrovat et al. [1] derived four first order nonlinear differential equations by using moment of momentum equations for pump, turbine and stator inertias, including also the energy balance equation for the fluid inertia along with the power losses. In the study the forward flow mode and the reverse flow mode equations were derived. An analysis with the bond graph theory was conducted and verified by experiments. The given equations are suitable for modelling the transient characteristics of the torque converter and investigating the parameter sensitivity. Barglazan [2] presented a theoretical model by considering the torque converter as a machine with lumped parameters. In the study, harmonic sinusoidal perturbations on the parameters of torque converter were applied and consequently a linearized dynamic model was obtained. Inthe study by Walker [3] dual clutch transmission powertrain models with 4DOF (degree of freedom) and 15 DOF were compared to study the influence of engine torque harmonics on the transient response. Free vibration analysis and shift transient simulations were conducted. By using a lower order model, only the lowest natural frequencies have been seen in the powertrain response. Rosbi et al. [4] studied the effect of a dynamic absorber to reduce shudder in automatic transmission powertrain. A linear multi-degree offreedom system has been used to conduct a vibration

analysis and the results were compared with the experiments. The shudder occurred in fifth gear of a six-speed transmission in the experiments; therefore, a theoretical model of the fifth gear for the powertrain with 13 DOF was built. By using an optimized dynamic absorber, the shudder was entirely suppressed.

The numerical modelling approach was utilized in the following works. Asl et al. [5] presented the damping characteristics of a dynamic torque converter for the torque multiplication (forward flow), coupling and reverse flow modes by *MapleSim*. The results showed that the high frequency disturbances are effectively damped from the engine/pump side to the transmission/turbine side in all three operation modes. In the work of Asl et al. [6] a math-based torque converter in the automatic driveline with a mean value engine model was modelled. By using the input of the throttle angle, the fuel consumption and mechanical power delivered to the pump of the torque converter was obtained. The gearbox is modelled by simple input-output torque with variable gear ratios. The vehicle's longitudinal dynamics were modelled considering the vehicle mass, final drive ratio, aerodynamic drag and rolling resistance. The study investigated the effect of a lock-up clutch on powertrain efficiency and ride comfort. Moreover, the engine braking and coasting phenomena were simulated.

Some researchers did experimental work on this subject such as:Castiglione et al. [7] investigated the transient torque converter behaviour and compared with the steady-state response measured in experiments. The main purpose of the study was the development of an empirical model which can describe the transient

behaviour. The simulations were based on the nonlinear equations. Six different torque converters were tested with a test bench and reverse engineering was performed for the nonlinear equation parameters like blade angles and design path radiuses. The experiments showed similar transient performance characteristics with the simulations. A linearized frequency response study was also conducted. The transient fluid moment inside the torque converter has only been seen when the speed ratio near or above coupling point and torsional frequency was between 1 to 10 Hz.

2 Objectives

The present work uses analytical and numerical methods to study the damping characteristics of a torque converter in a linear multi-degree of freedom driveline model. In the analysis, a reduced driveline model with a steady-state torque converter model is considered to evaluate the system response. The torque converter is modelled by characteristic performance curves. Only the low frequency domain, which affects the shift comfort, has been investigated. This is achieved by linearizing the nonlinear differential equations around an equilibrium point. The performance of the linearized model is compared with dynamic simulations of the reduced nonlinear driveline model.

3 Driveline model

A simple driveline model consisting of a steady state engine model, a torque converter, a two-gear transmission and an output shaft is shown in Figure 1.

For the further analytical investigations; we will take advantage of a simpler, i.e. reduced order model with two- and three-degree of freedom (DOF). These models are explained in detail in sections 3.1 and 3.2.

Figure 1. Driveline model.

3.1 Two-degree of freedom model

A reduced model of the driveline and the torque converter with two-degree of freedom is shown in Figure 2. The moment of inertias of the engine and of the pump side torque converter (primary side) are rigidly connected. The turbine side torque converter (secondary side) is connected to the mass of the vehicle, the wheels, the output shaft and the gearbox mass. These elements are combined and reduced to the shaft of the torque converter.

The 2DOF model is the most simple driveline model. Since no flexibility is taken into account (unlike the 3 DOF described in section 3.3), the fundamental torsional

mode of the driveline does not occur and driveline oscillations cannot be simulated with this model.

Figure 2. Two-degree of freedom model.

In general, this system can be described in the form of the differential equation system given in (1): M is the inertia matrix, D is the damping matrix, F is the external load matrix and φ is the angular displacement vector. In the equations, a single underscore denotes a vector and a double underscore denotes a matrix.

$$\underline{\underline{M}}\ddot{\underline{\varphi}} + \underline{\underline{D}}\dot{\underline{\varphi}} - \underline{F} = 0 \quad (1)$$

As given in Figure 2, the engine and the torque converter pump are rigidly connected and the angular displacement of the engine (φ_e) and the torque converter pump (φ_p) are equal. Similarly, the angular displacement of the reduced mass (φ_1) is equal to the angular displacement of turbine (φ_t). The governing equations defining the system in Figure 2 are presented in (2): I_e is the engine inertia, I_p is the torque converter pump inertia, I_t is the torque converter turbine inertia, I_1 and I_2 are the reduced driveline inertias, T_e is the engine torque, T_p is the torque converter pump torque, T_t is the torque converter turbine torque and d_{load} is the load damping constant.

$$\begin{aligned} (I_e + I_p)\ddot{\varphi}_e - T_e + T_p &= 0 \\ (I_t + I_1 + I_2)\ddot{\varphi}_1 + d_{load}\dot{\varphi}_1 - T_t &= 0 \\ \varphi_e &= \varphi_p \\ \varphi_1 &= \varphi_t \end{aligned} \quad (2)$$

3.2 Three-degree of freedom model

The three-degree of freedom model is shown in Figure 3. The driveline has flexibility, i.e. k_{12} . It is an extension of the 2 DOF model by separating the secondary side of the torque converter and the reduced vehicle mass by a spring. Therefore, this model is able to simulate driveline vibrations.

Figure 3. Three-degree of freedom model.

The system matrix is given in (3): In addition to the inertia, M , the damping, D , and the external load, F , matrices, a stiffness matrix, K , is added to the system.

$$\underline{M}\ddot{\underline{\varphi}} + \underline{D}\dot{\underline{\varphi}} + \underline{K}\underline{\varphi} - \underline{F} = 0 \quad (3)$$

The governing equations defining the system in Figure 3 are presented in (4): φ_2 is the angular displacement of the reduced vehicle mass, k_{12} is the reduced stiffness constant and d_{12} is the reduced damping constant between the secondary side of the torque converter and the reduced vehicle mass.

$$\begin{aligned} (I_e + I_p)\ddot{\varphi}_e - T_e + T_p &= 0 \\ I_1\ddot{\varphi}_1 + d_{12}(\dot{\varphi}_1 - \dot{\varphi}_2) + k_{12}(\varphi_1 - \varphi_2) - T_t &= 0 \\ I_2\ddot{\varphi}_2 + d_{12}(\dot{\varphi}_2 - \dot{\varphi}_1) + k_{12}(\varphi_2 - \varphi_1) + d_{load}\dot{\varphi}_2 &= 0 \quad (4) \\ \varphi_e &= \varphi_p \\ \varphi_1 &= \varphi_t \end{aligned}$$

3.3 Torque converter model

The easiest mathematical model of a torque converter is via characteristic maps, i.e. the torque ratio and the capacity factor curves. The characteristic curves for the torque ratio, $r(w) = T_t/T_p$, and the capacity factor, $\kappa(w) = \dot{\varphi}_p/\sqrt{T_p}$, are functions of the speed ratio, $w = \dot{\varphi}_t/\dot{\varphi}_p$, which is the ratio of the turbine speed to the pump speed of the torque converter. A typical characteristic map of an automotive torque converter is shown in Figure 4.

Figure 4. Torque converter model inputs and outputs.

The inputs of the torque converter model consist of the angular speeds of the pump, $\dot{\varphi}_p = \omega_p$ and the turbine, $\dot{\varphi}_t = \omega_t$.

From the characteristic curves, the torque acting on the engine T_p (pump torque) and the torque acting on driveline T_t (turbine torque) can be calculated as follows:

$$\begin{aligned} T_p &= \dot{\varphi}_p^2 / \kappa^2(w) \\ T_t &= T_p r(w) \end{aligned} \quad (5)$$

As previously mentioned, this model only describes the behaviour of a torque converter in the low frequency domain. More refined models, which also take into account physical and geometrical parameters, can be found in [1] and [5-6].

4 Linearization

It can be seen that the torque converter equations are non-linear, and hence the damping performance of the 2 DOF and the 3 DOF model depend on the operation point.

Therefore, the linearized equations are valid only at the chosen equilibrium point.

The linearization is conducted by setting up the equations for the reduced model of the driveline, which are then used to find equilibrium points for a given set of torque loads. By linearizing the differential equations and inserting the equilibrium point, a set of linear differential equations can be found. In the following, the linearization equations for the 3 DOF model is presented. They can be derived for the 2 DOF model analogous to the 3 DOF model.

First, the differential equations are transformed into the structure in (6): The functions f_1 , f_2 and f_3 depend on the variables given in brackets.

$$\begin{aligned} f_1(\ddot{\varphi}_e, \dot{\varphi}_e, \dot{\varphi}_1, T_e, T_p) &\approx 0 \\ f_2(\ddot{\varphi}_1, \dot{\varphi}_1, \varphi_1, \dot{\varphi}_2, \varphi_2, T_t) &\approx 0 \\ f_3(\dot{\varphi}_1, \varphi_1, \ddot{\varphi}_2, \dot{\varphi}_2, \varphi_2) &\approx 0 \end{aligned} \quad (6)$$

Assuming continuous functions for f_1 , f_2 and f_3 , the Taylor series representations at the equilibrium state, $(\dot{\varphi}_{e0}, \dot{\varphi}_{10})$, can be obtained as in (7):

$$\begin{aligned} f_1 &\approx f_1(\varphi_{10}, \varphi_{e0}) + \frac{\partial f_1}{\partial \ddot{\varphi}_e} \Delta \ddot{\varphi}_e + \frac{\partial f_1}{\partial \dot{\varphi}_e} \Delta \dot{\varphi}_e + \dots \\ &\dots + \frac{\partial f_1}{\partial \dot{\varphi}_1} \Delta \dot{\varphi}_1 + \frac{\partial f_1}{\partial T_e} \Delta T_e + \frac{\partial f_1}{\partial T_p} \Delta T_p \approx 0 \\ f_2 &\approx f_2(\varphi_{10}, \varphi_{e0}) + \frac{\partial f_2}{\partial \ddot{\varphi}_1} \Delta \ddot{\varphi}_1 + \frac{\partial f_2}{\partial \dot{\varphi}_1} \Delta \dot{\varphi}_1 + \dots \\ &\dots + \frac{\partial f_2}{\partial \varphi_1} \Delta \varphi_1 + \frac{\partial f_2}{\partial \dot{\varphi}_2} \Delta \dot{\varphi}_2 + \frac{\partial f_2}{\partial \varphi_2} \Delta \varphi_2 + \frac{\partial f_2}{\partial T_t} \Delta T_t \approx 0 \\ f_3 &\approx f_3(\varphi_{10}, \varphi_{e0}) + \frac{\partial f_3}{\partial \dot{\varphi}_1} \Delta \dot{\varphi}_1 + \frac{\partial f_3}{\partial \varphi_1} \Delta \varphi_1 + \dots \\ &\dots + \frac{\partial f_3}{\partial \ddot{\varphi}_2} \Delta \ddot{\varphi}_2 + \frac{\partial f_3}{\partial \dot{\varphi}_2} \Delta \dot{\varphi}_2 + \frac{\partial f_3}{\partial \varphi_2} \Delta \varphi_2 \approx 0 \end{aligned} \quad (7)$$

It should be noted that the deviation variables of the turbine torque, ΔT_t , and the pump torque, ΔT_p , are also a function of the angular speeds, $\dot{\varphi}_e$ and $\dot{\varphi}_1$. By using the chain rule in partial differentiation, these terms should be replaced in (7) by the expressions given in (8):

$$\begin{aligned} \Delta T_t &= \frac{\partial T_t}{\partial \dot{\varphi}_e} \Delta \dot{\varphi}_e + \frac{\partial T_t}{\partial \dot{\varphi}_1} \Delta \dot{\varphi}_1 \\ \Delta T_p &= \frac{\partial T_p}{\partial \dot{\varphi}_e} \Delta \dot{\varphi}_e + \frac{\partial T_p}{\partial \dot{\varphi}_1} \Delta \dot{\varphi}_1 \end{aligned} \quad (8)$$

4.1 Linearization – 2 DOF model

For the 2 DOF model, the functions f_1 and f_2 are shown in (9):

$$\begin{aligned} f_1 &= (I_e + I_p)\ddot{\varphi}_e - T_e + T_p = 0 \\ f_2 &= (I_t + I_1 + I_2)\ddot{\varphi}_1 + d_{load}\dot{\varphi}_1 - T_t = 0 \end{aligned} \quad (9)$$

By applying (7) the linearized differential equations for the 2 DOF model at the equilibrium point are determined by (10):

$$\begin{bmatrix} I_p + I_e & 0 \\ 0 & I_t + I_1 + I_2 \end{bmatrix} \begin{bmatrix} \Delta \ddot{\phi}_e \\ \Delta \ddot{\phi}_1 \end{bmatrix} + \dots \begin{bmatrix} \frac{\partial T_p}{\partial \dot{\phi}_e} & \frac{\partial T_p}{\partial \dot{\phi}_1} \\ -\frac{\partial T_t}{\partial \dot{\phi}_e} & -\frac{\partial T_t}{\partial \dot{\phi}_1} + d_{load} \end{bmatrix} \begin{bmatrix} \Delta \dot{\phi}_e \\ \Delta \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} \Delta T_e \\ 0 \end{bmatrix} \quad (10)$$

4.2 Linearization – 3 DOF model

For the 3 DOF model, the functions, f_1 , f_2 and f_3 are presented in (11):

$$\begin{aligned} f_1 &= (I_e + I_p) \ddot{\phi}_p + T_p - T_e \\ f_2 &= (I_t + I_1) \ddot{\phi}_t + d_{12} (\dot{\phi}_t - \dot{\phi}_2) + k_{12} (\phi_t - \phi_2) - T_t \\ f_3 &= I_2 \ddot{\phi}_2 + d_{12} (\dot{\phi}_2 - \dot{\phi}_t) + k_{12} (\phi_2 - \phi_t) + d_{load} \dot{\phi}_2 \end{aligned} \quad (11)$$

Consequently, one obtains the linear differential equations by applying (7) as stated in (12):

$$\begin{bmatrix} I_p + I_e & 0 & 0 \\ 0 & I_1 + I_t & 0 \\ 0 & 0 & I_2 \end{bmatrix} \begin{bmatrix} \Delta \ddot{\phi}_e \\ \Delta \ddot{\phi}_1 \\ \Delta \ddot{\phi}_2 \end{bmatrix} + \dots \begin{bmatrix} \frac{\partial T_p}{\partial \dot{\phi}_e} & \frac{\partial T_p}{\partial \dot{\phi}_1} & 0 \\ -\frac{\partial T_t}{\partial \dot{\phi}_e} & -\frac{\partial T_t}{\partial \dot{\phi}_1} + d_{12} & -d_{12} \\ 0 & -d_{12} & d_{12} + d_{load} \end{bmatrix} \begin{bmatrix} \Delta \dot{\phi}_e \\ \Delta \dot{\phi}_1 \\ \Delta \dot{\phi}_2 \end{bmatrix} + \dots \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_{12} & -k_{12} \\ 0 & -k_{12} & k_{12} \end{bmatrix} \begin{bmatrix} \Delta \phi_e \\ \Delta \phi_1 \\ \Delta \phi_2 \end{bmatrix} = \begin{bmatrix} \Delta T_e \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

4.3 The state space representation

The inertia and damping matrices for the 2DOF system are given in (13): For the sake of simplicity, the Δ -symbol will be dropped in the equations from now on.

$$\begin{aligned} \underline{M} \cdot \ddot{\underline{\phi}} + \underline{D} \cdot \dot{\underline{\phi}} &= \underline{F} \\ \underline{M} &= \begin{bmatrix} I_p + I_e & 0 \\ 0 & I_t + I_1 + I_2 \end{bmatrix} \\ \underline{D} &= \begin{bmatrix} T_{p,\dot{\phi}_e} & T_{p,\dot{\phi}_1} \\ -T_{t,\dot{\phi}_e} & -T_{t,\dot{\phi}_1} + d_{load} \end{bmatrix} \\ \underline{F} &= \begin{bmatrix} T_e \\ 0 \end{bmatrix} \end{aligned} \quad (13)$$

The inertia and damping matrices are given in (14) for the 3 DOF system:

$$\begin{aligned} \underline{M} \cdot \ddot{\underline{\phi}} + \underline{D} \cdot \dot{\underline{\phi}} + \underline{K} \cdot \underline{\phi} &= \underline{F} \\ \underline{M} &= \begin{bmatrix} I_p + I_e & 0 & 0 \\ 0 & I_1 + I_t & 0 \\ 0 & 0 & I_2 \end{bmatrix} \\ \underline{D} &= \begin{bmatrix} T_{p,\dot{\phi}_e} & T_{p,\dot{\phi}_1} & 0 \\ -T_{t,\dot{\phi}_e} & -T_{t,\dot{\phi}_1} + d_{12} & -d_{12} \\ 0 & -d_{12} & d_{12} + d_{load} \end{bmatrix} \\ \underline{K} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_{12} & -k_{12} \\ 0 & -k_{12} & k_{12} \end{bmatrix}, \underline{F} = \begin{bmatrix} T_e \\ 0 \\ 0 \end{bmatrix} \end{aligned} \quad (14)$$

The torsional mode frequency is assumed to be the square root of the ratio of the output shaft stiffness and the inertia, $\sqrt{k_{out}/I_{out}}$, which is assumed to be at 5 Hz.

The resulting system of equations containing mass, damping and stiffness matrices are then transferred to a standard state space representation for continuous linear time invariant systems, which is shown in (15). The vector of state variables is indicated by x , u is the input vector, y is the output vector, A is the system matrix, B is the input matrix and C is the output matrix. From the state space representation, the matrices A , B and C can be found and the transfer functions can be calculated for each of the input and output parameters.

$$\begin{aligned} \dot{x} &= \underline{A}x + \underline{B}u \\ y &= \underline{C}x \end{aligned} \quad (15)$$

The sizes of the matrices depend on the number of the degrees of freedom.

The input vector u and the state vector x for the 2 DOF system are given in (16).

$$\begin{aligned} x &= \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \phi_p \\ \phi_t \\ \dot{\phi}_p \\ \dot{\phi}_t \end{bmatrix}, u = \begin{bmatrix} T_e \\ 0 \end{bmatrix} \\ \underline{A} &= -\underline{M}^{-1}\underline{D}, \underline{B} = \underline{M}^{-1}, \underline{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (16)$$

The input and output vectors for the 3DOF system are presented in (17): The identity matrix is represented by I .

$$\begin{aligned} x &= \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \phi_p \\ \phi_t \\ \phi_2 \\ \dot{\phi}_p \\ \dot{\phi}_t \\ \dot{\phi}_2 \end{bmatrix}, u = \begin{bmatrix} T_e \\ 0 \\ 0 \end{bmatrix} \\ \underline{A} &= \begin{bmatrix} 0 & I \\ -\underline{M}^{-1}\underline{K} & -\underline{M}^{-1}\underline{D} \end{bmatrix}, \underline{B} = \begin{bmatrix} 0 \\ \underline{M}^{-1} \end{bmatrix} \\ \underline{C} &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (17)$$

The transfer function can be calculated in the form of the state space representation by the Laplace transformation as in (18). A detailed explanation on the derivation of (18) can be found in [8].

$$G(s) = \frac{Y(s)}{U(s)} = \underline{\underline{C}}(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{\underline{B}} \quad (18)$$

5 Simulations

In this study, the system responses due to variations of the engine torque are investigated. In particular, the response of the angular speeds ω_p and ω_t due to an increase of the engine torque by 1Nm (step function) or due to a Dirac impulse are compared. The simulations are performed with *MATLAB/Simulink*.

In the following Figures 5-10, the results denoted by “exact” are obtained from the dynamic *Simulink* model and the results denoted by “linearized” are obtained from the solution of linearized equations (see Eqs. (10), (12), (13), (14), (16), (17) and (18)).

In order to find the equilibrium point, a time-constant engine torque (or pump torque) of about $T_{e0} = T_{p0} = 30 \text{ Nm}$ and a constant load (or turbine torque) of $T_{t0} = 55.85 \text{ Nm}$ are assumed. This yields the equilibrium points for the speeds $\omega_{p0} = 104.3 \text{ rad/s}$ and $\omega_{t0} = 55.85 \text{ rad/s}$.

Model parameters used in the analytical calculations and the simulations are given in Table 1.

Table 1. Model parameters.

I_e , engine inertia	0.5 kgm ²
I_p , pump inertia	0.092 kgm ²
I_t , turbine inertia	0.026 kgm ²
r_{whl} , wheel radius	0.35 m
m_{veh} , vehicle mass	2000 kg
$I_{veh} = m_{veh} r_{whl}^2$, vehicle inertia	245 kgm ²
f_{shuffl} , torsional mode frequency	5 Hz
I_{out} , output shaft inertia	24.5 kgm ²
k_{out} , stiffness of output shaft	24181 Nm/rad
d_{out} , damping of output shaft	6.28 Nms/rad
d_{load} , reduced load damping constant	1 Nms/rad
g_1 , gear ratio of 1st gear	10
$I_1 = I_{out} / g_1^2$, reduced mass inertia	0.245 kgm ²
$I_2 = I_{veh} / g_1^2$, reduced mass inertia	2.45 kgm ²
$k_{12} = k_{out} / g_1^2$, reduced stiffness constant	241.8 Nm/rad
$d_{12} = d_{out} / g_1^2$, reduced damping constant	0.0628 Nms/rad

5.1 Unit-step response

In order to find the unit step response of the system, the input given in (19) is applied to the system. The function $H(t)$ denotes the Heaviside step function, which is defined as in (20):

$$T_e(t) = T_{e0} + H(t) \quad (19)$$

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \quad (20)$$

The step responses of the angular velocities of the pump, the turbine and the additional mass for the 2DOF and the 3DOF models are shown in Figure 5 and Figure 6. In both figures zoomed views up to the first second of the simulation time are given under the full time range plots.

Figure 5. Step time response for the 2DOF model.

Figure 6. Step time response for the 3DOF model.

The linearized model and the exact model yield the same results in both 2 DOF and 3 DOF systems.

The steady state responses of the 2 DOF and 3 DOF models are also the same. The pump speed has a steady state value of 2 rad/s and the turbine speed has a steady state value of 1.4 rad/s after 10 seconds. However, it can be seen that the turbine side has speed oscillations

because of the additional mass about 5 Hz in the 3 DOF model. Since the 2 DOF model has no flexibility, this effect cannot be observed in the 2 DOF model.

The vibrations on the turbine side of the 3 DOF model are eliminated after 2 to 3 time periods by the torque converter.

5.2 Impulse response

The impulse input is given in (21): The function $\delta(t)$ denotes the Dirac delta function.

$$T_e(t) = T_{e0} + \delta(t) \quad (21)$$

Again, the dynamic simulation from the exact and the linearized models have the same impulse response (see Figure 7 for the 2 DOF and Figure 8 for the 3 DOF). Furthermore, the 2 DOF and the 3 DOF models display similar steady state behaviour.

The oscillations of the angular speed of the turbine can be observed only in the 3 DOF model as in the previous section 5.1 (see zoomed plots in Figure 8). The angular speed of the additional mass (ω_2) vanishes in the 2 DOF model.

Figure 7. Impulse time response for the 2DOF model.

Figure 8. Impulse time response for the 3DOF model.

5.3 Frequency response

In this section the transient results due to time-harmonic excitation are presented. The harmonic engine torque input given in (22) is applied to the system.

$$T_e(t) = T_{e0} + \sin(2\pi ft) \quad (22)$$

In order to compare the results for several frequencies, the frequency is varied in the simulations every 10 seconds, e.g. between 20 and 30 s, the excitation frequency is 10Hz (see Table 2).

Table 2. Frequency input time intervals.

Time range	Frequency
0 s < t < 10 s	f = 50 Hz
10 s < t < 20 s	f = 25 Hz
20 s < t < 30 s	f = 10 Hz
30 s < t < 40 s	f = 1 Hz

Figure 9 and Figure 10 show the results for the 2 DOF and 3 DOF models. The results for the pump (left) and the turbine speeds (right) — are shown for the linearized and the exact model. The zoomed plots for one second time range at 10 Hz and at 1 Hz are also presented. The transfer functions of the linearized model, $G = \Delta\omega_p / \Delta T_e$ and $G = \Delta\omega_t / \Delta T_e$, are shown at the bottom.

Figure 9. Linearized pump and turbine speed outputs for 2DOF model.

In both 2 DOF and 3 DOF models, the steady state values of the angular speeds are for the pump 104.3 rad/s and for the turbine 55.85 rad/s respectively. They match with the equilibrium point which is chosen for the simulations.

The results for the turbine and the pump speeds indicate that the higher frequency vibrations are damped by the torque converter, which can be seen also in the zoomed views in Figure 9 and Figure 10. Moreover, the

amplitude of speed oscillations of the pump speed for the 3 DOF model is slightly higher than the ones for the 2 DOF model.

Figure 10. Linearized pump and turbine speed outputs for 3DOF model.

There is a significant difference between the transfer function plots of the 2 DOF and 3 DOF models at the natural frequency of the driveline.

The torsional mode resonance can be seen from the transfer function plot between the turbine speed and the engine torque for the 3DOF model in Figure 10. However, the transfer function looks different for the 2DOF model in Figure 9 which demonstrates only the fundamental mode. Since, the 2 DOF model has no flexibility, the effects of the natural frequency cannot be observed. Therefore, the 2 DOF model is inconvenient to consider excitations in the system around the natural frequency.

The exact and linearized solutions match perfectly for both the 2 DOF and the 3 DOF models.

6 Conclusions

The aim of the work was to analyse the damping characteristics of a torque converter in the low frequency domain by modelling a simple automotive driveline model. Since even the simplest torque converter model is described by a nonlinear relation, the linearization of the nonlinear differential equations is unavoidable in order to get a better understanding of which frequencies are mainly attenuated by a torque converter.

The driveline models with 2 DOF and 3 DOF are investigated. These systems are simulated by two methods; the linearization and the nonlinear dynamical modelling in *Simulink*. The system transfer functions are evaluated to observe the damping effect of torque converter on the small variations in the input of the engine torque.

Torsional mode resonance was observed in the transfer function plot between the turbine speed and the engine torque of the 3DOF model. However, the 2 DOF model was not sufficient to demonstrate the effects of the natural frequency.

In the frequency response analysis, it has been seen that the torque converter damps higher frequency vibrations in the low frequency domain effectively.

The linearized model used in the present study provided an accurate approximation of the system response around the chosen equilibrium point. It can be concluded that, by using the linearized system equations along with the reduced degrees of freedom models, the computational effort can be decreased significantly.

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