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Axial force and bending stiffness

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Abstract. In the solution of effects of axial force on the bending stiffness, geometrical nonlinear analysis must be used. Tensile axial force generates an increase of the bending stiffness. In case of low bending stiffness (e.g. cable) the task is simplified and it is possible to determine the tensile force needed to achieve required bending stiffness. When considering the compression force, many problems occur. This force cannot exceed the critical force. This principle is not valid, if there is an initial imperfection, which is perpendicular to the transversal load. Knowing the frequency of beam can help to identify the size of axial force. In case of compression force one gets a combination of stability and vibration.

1 Introduction

Generally, the axial force does not affect the distribution of bending moments in the beam. However, it does not mean that the axial force cannot affect the bending stiffness of a beam. Nowadays many interesting constructional elements have been used in modern structural engineering, e.g. glass plates supported by tension ropes. To analyse this type of problems, the geometric nonlinear analysis must be used [1-2].

2 Geometric nonlinear analysis of beam

In the general solution of a straight beam one needs to consider an elongation of beam, which is given by the relation between the deformed element and its original length. In a further analysis the linear elastic material will be considered.

$$\sigma = E(\varepsilon - \varepsilon_0) \tag{1}$$

where E means Young modulus and the index "0" marks an initial strain.

The analysis starts with calculation of the total potential energy and in further calculation the Ritz variation method will be used. This type of problem generally leads to the system of cubic algebraic equations, if the square is considered in the elongation. Cubic terms are mutually eliminated in the case of a straight beam

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3 Beam loaded with axial force and uniform load

In Fig. 1 there are shown two cases of beam loading, which will be analyzed.

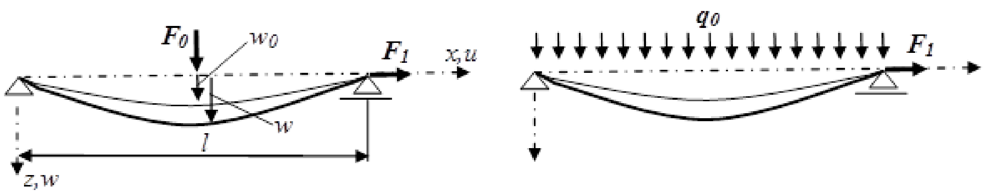

Fig. 1. Beam loaded with axial force and uniform load.

The beam with an initial imperfection will be considered. Two models of transversal loading - concentrated force in the middle and uniform load along the full length of the beam have been used.

As it was already mentioned, Ritz variation method with consideration of following general variation functions will be used.

$$w = \alpha_1 \sin\frac{\pi x}{l},\ w_0 = \alpha_0 \sin\frac{\pi x}{l},\ u = \alpha_2\, x + \alpha_3 \sin\frac{2\pi x}{l} \tag{2}$$

More detailed studies can be found in papers [3-5]. After all mathematical operations, problem leads to the equation:

$$F_E(\alpha_1 - \alpha_0) + F_1\alpha_1 = F_0\frac{2l}{\pi^2} \qquad \left\langle F_E(\alpha_1 - \alpha_0) + F_1\alpha_1 = \frac{4l^2}{\pi^3} q_0 \right\rangle \tag{3}$$

where $F_E = \dfrac{\pi^2 EI}{l^2}$ is Euler critical force, *EI* – bending stiffness, *l* – span of beam.

Equation in brackets $\langle\ \rangle$ refers to the example of system with uniform loading. In the case of a straight beam without imperfection, a check can be done considering $(\alpha_0 = 0)$ with zero axial force. Then the deflection in the middle is calculated.

$$\begin{gathered} \alpha_1 = F_0\frac{2l^3}{\pi^4 EI} = \frac{1}{48.7045}\frac{F_0 l^3}{EI}\, exact\_solution \frac{1}{48}\frac{F_0 l^3}{EI} \\ \left\langle \alpha_1 = q_0\frac{4l^4}{\pi^5 EI} = \frac{5}{382.52}\frac{q_0 l^4}{EI}\, exact\_solution \frac{5}{385}\frac{q_0 l^4}{EI} \right\rangle \end{gathered} \tag{4}$$

It is obvious that the obtained results are not identical. This is the consequence of our consideration, that deflection function has a sinus form. However, the difference in forces is only 1.5% and in uniform loads it is 0.64%.

Another simple check can be done for the case of acting just compression force and without transversal loading. Subsequently, well known result for buckling of the beam with an initial imperfection is obtained.

$$\frac{F}{F_E} = \left(1 - \frac{\alpha_0}{\alpha_1}\right) \tag{5}$$

where $F = -F_1$

4 Beam with low bending stiffness loaded with tensile axial force

As a special example can be shown a solution of beam loaded with tensile axial force while having extreme low bending stiffness. A good example for this is a steel rope. In this case the Euler critical force can be considered to be equal to zero ($F_E = 0$). Equation (3) is simplified.

$$F_1\alpha_1 = F_0 \frac{2l}{\pi^2} \qquad \left\langle F_1\alpha_1 = \frac{4l^2}{\pi^3} q_0 \right\rangle \tag{6}$$

Let us solve the example with transversal load represented by concentrated force first. The value of axial force in rope is determined:

$$F_1 = F_0 \frac{2l}{\pi^2 \alpha_1} \tag{7}$$

Stiffness in deflection is limited with a ratio $n = \frac{l}{\alpha_1}$, where values for $n = 250 – 800$ are considered. With the substitution of equation (7) the following equation is obtained.

$$F_1 = F_0 \frac{2n}{\pi^2} \tag{8}$$

In our example it can be supposed $F_0 = 1000$ N, $n = 500$. Then the force in rope is $F_1 = 101321$ N.

The most interesting fact is that the span of beam does not need to be mentioned in this equation. Taking the transverse uniform load in account, analogically the equation is obtained.

$$F_1 = q_0 \frac{4l \cdot n}{\pi^3} \tag{9}$$

In this case the values used in the example are $l = 5.0$ m, $q_0 = 200$ N/m, $n = 500$. The force in rope is $F_1 = 64503$ N.

5 Stability and vibration of beam

In the case of compressive axial force one gets the stability problems. Another interesting problem is the combination of stability and vibration of the beam [6-7]. Using linear dynamics, problem is geometrically linear and the free vibration can be obtained from the following equation:

$$\left| \mathbf{K_L} - \omega^2 \mathbf{K_M} \right|_{\det} = 0 \tag{10}$$

where $\mathbf{K_L}$ is stiffness matrix, ω – eigen circular frequency , $\mathbf{K_M}$ – mass matrix.

Usually in solution of the stability problem using the theory of linear stability is sufficient, where it is supposed that axial forces in the beam or membrane forces in the thin wall structures are not dependent on the transversal deformation. From this type of problems, one is able to determine critical forces or critical stresses and modes of buckling. Supposing the linear stability also in the problem of vibration of a given structure, equation (10) will change:

$$\left|\mathbf{K_L} - \lambda\mathbf{K_G} - \omega^2\mathbf{K_M}\right|_{\det} = 0 \tag{11}$$

where $\mathbf{K_G}$ is geometric matrix, λ – multiplier of critical load.

Incremental stiffness matrix contains also initial imperfections, level of loading and also deformations of the system. Detailed explanation of theoretical and also numerical background of the given problem was analyzed in works referred in the end [8]. At this point we will introduce only some of interesting results.

5.1 Vibration and buckling of beam

Basic example of the combination of stability and vibration is the simply supported beam loaded with the axial force at its ends. Even such a simple example shows the complexity of a given problem. In the point of the acting force, using of the moveable support is needed. With the same approach in analysis of the case of vibration, in the level of critical force the zero frequency is obtained. However, this result does not match reality. For example if the mine foreman checks timbering by knocking, the overload of supporting columns is accompanied with sound of high frequencies. In order to get the result that matching the reality, the non-moveable support must be used in the moment of vibration. Results are graphically presented in Fig. 2.

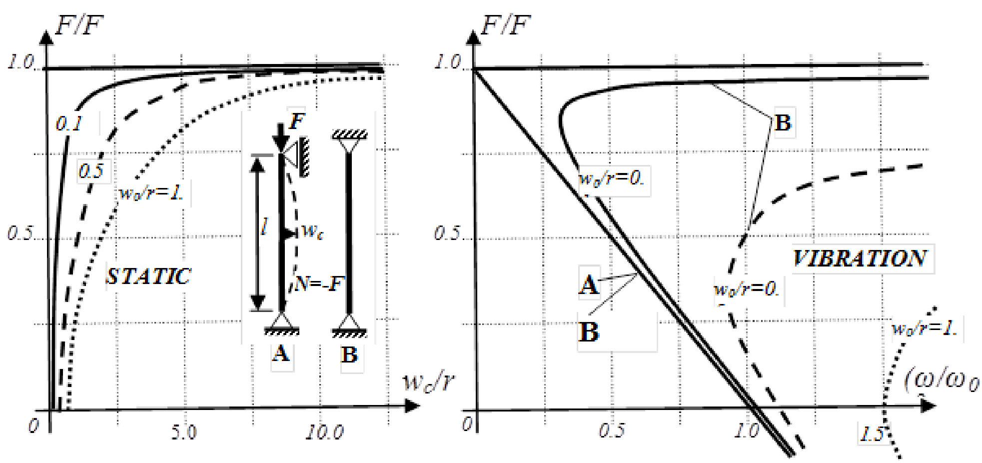

Fig. 2. Stability and vibration of a beam with initial imperfection.

5.2 Effects of the initial deformation on vibration of a beam

In the case of beam the mode of vibration is identical with the mode of buckling. That is the reason why the dependence between the load and the square of eigen circular frequency is linear. In static problems the most unfavourable mode of initial imperfection is the one identical with the mode of buckling corresponding to the lowest critical force („mode 1“). The vibration of beam is shown in Fig. 3. Although the mode of initial imperfection of the beam is close to the “second mode of buckling” it will vibrate in the first mode.

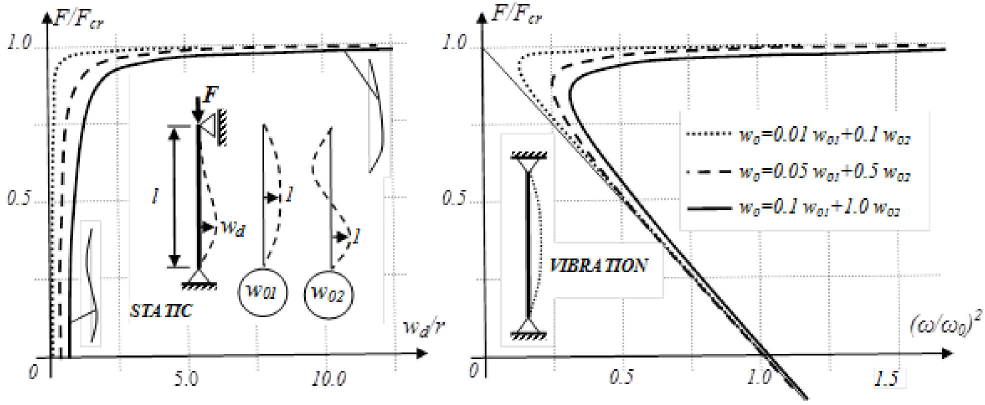

Fig. 3. Stability and vibration of a beam with initial imperfection in second mode of buckling.

5.3 Experimental measurement

As is well known precise research of the structural theory is a combination of theoretical, numerical and experimental investigations. Authors had designed special equipment for experimental laboratory measurements in order to meet this requirement. The scheme of this equipment is shown in Fig. 4. Many measurements have been made using this equipment.

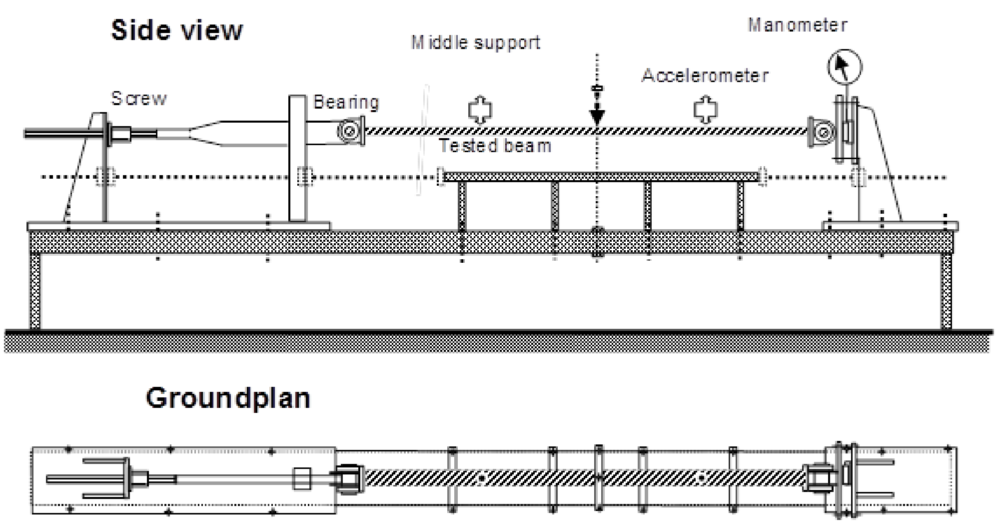

Fig. 4. Scheme of the test set-up.

6 Conclusion

The paper presents some problems of structural stability. For proper analysis of stability problems, geometric nonlinear approach is required. Just the change of compression force to tension force and the use of geometric nonlinear analysis is enough to obtain an interesting result in the determination of effects of axial force on the bending stiffness of a beam. Due to the combination of stability and vibration it is possible to determine the frequency of system considering the level of load as well as the effects of magnitude and mode of initial deformation. Comparison of theoretically and numerically obtained values of frequencies with the frequency measured on the real construction is the basis for the identification of properties or quality of structure within the scope of non-destructive methods.

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