

Stability analysis of an open shallow cylindrical shell with imperfection under external pressure

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Abstract. Elastic shallow generalized cylindrical shells of an open cross-section subjected to the various forms of external pressure are analysed in the paper numerically using the finite element method. Load - displacement paths are calculated for the perfect and imperfect geometry, respectively. Special attention is paid to the influence of initial geometric imperfection on the limit load level of fundamental equilibrium path of nonlinear analysis. ANSYS system was used for analysis, arc-length method was chosen for obtaining fundamental load - displacement path of solution.

1 Introduction

Cylindrical shells with open cross-section are structural elements widely used in many branches of engineering. Such shells occur as parts of aircraft and marine structures in mechanical engineering, create covers of large span structures in civil engineering. Their middle surface is generally created by sliding generator along any curve (directrix). Presented paper considers only right cylinders and the circle, parabola, vertical and horizontal ellipses as a directrix. The effort was to assume comparable value of the average curvature of the considered curves.

These shells subjected to the external pressure are liable to the buckling due to dominant compression membrane forces within the shell. It is the reason, why the stability problem has been analysed since the beginning of the twenty century. It was then when the first very slender structures of barrel shells appeared. Among the basic books dealing with this problematics are included works by Bushnell [1] and by Yamaki [2]. The shell stability handbook [3] treats various important shell buckling problems of practical interest.

Solving stability of the thin shell, it is often insufficient to determine the elastic critical load from eigenvalue buckling analysis, i.e. the load, when perfect shell starts buckling. Nonlinear analysis is necessary, resulting in a full load-displacement response. It is also necessary to include initial imperfections of real shell into the solution and determine limit load level more accurately. Significant works dealing with imperfections in real structures are [4-5]. Experience shows that small deviations from the prescribed geometry of a perfect thin-walled cylinder may cause a strong reduction of the buckling load and of the load-carrying capacity of shell. When the perfect geometry is broken ability to use membrane action is heavily reduced.

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2 FEM in nonlinear theory

The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the imperfect shell. Applying the finite element method to the problem of geometrical nonlinearity, problem leads to a set of nonlinear algebraic equations. This system can be written as:

$$\mathbf{K}_{inc} \Delta \mathbf{a} + \mathbf{F}_{int} - \mathbf{F}_{ext} - \Delta \mathbf{F}_{ext} = \mathbf{0}, \quad (1)$$

where \mathbf{K}_{inc} is the incremental stiffness matrix of shell, \mathbf{F}_{int} is the internal force vector of shell, \mathbf{F}_{ext} is the external load of shell and $\Delta \mathbf{F}_{ext}$ is the increment of the external load of shell.

The solution of presented system cannot be achieved directly, and is based on a step by step incremental process that causes a deviation from the equilibrium nonlinear path. To correct this deviation, an iterative technique such as the Newton-Raphson method is used. Murray and Wilson [6] first presented idea of combining incremental (Euler) and iterative (Newton-Raphson) methods for solving nonlinear problems. Early works involving critical points and snap-through effect were written by Sharifi and Popov [7], and Sabir and Lock [8]. Using arc-length method to pass limit points on load-displacement paths introduced Riks in [9]. Getting through this problem using displacement control procedure presented Batoz and Dhatt [10]. Detection of critical points using arc-length method was introduced by Wriggers and Simo [11]. Works of Bathe [12] dominate in application of FEM to geometric nonlinear problems. Crisfield [13] gave the most important modification of the arc-length method suggesting the fixation of incremental length during load increment (cylindrical ALM). Fafard and Massicotte [14] and Teng and Luo [15] submitted further significant modifications of the method.

3 Numerical analysis, conclusions

Illustrative example of isotropic shallow shell loaded by the external pressure (Fig. 1) is presented. Eigenvalue buckling analysis is calculated first. These results offer an image about location of critical points of nonlinear solution, help with settings in the management of nonlinear calculation process. Results of fully nonlinear analysis follow (ideal shell and subsequently structure with initial imperfection).

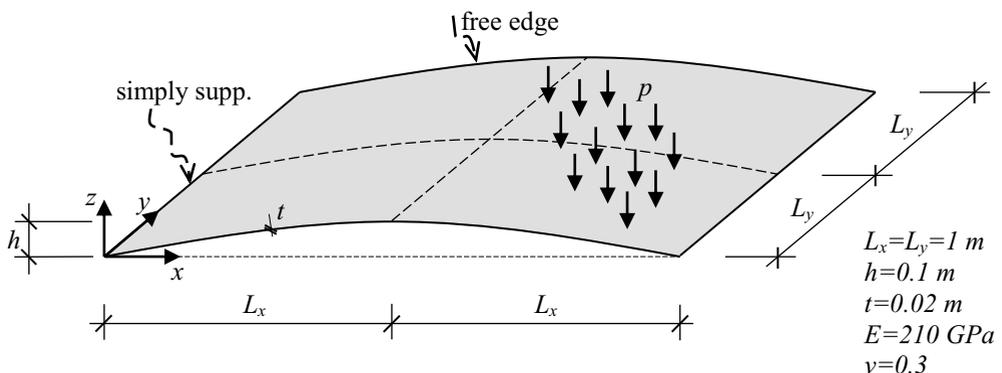


Fig. 1. Shallow cylindrical shell. Geometry and material properties.

Presented results were obtained by division into 40×40 elements. Boundary conditions are considered as simply supported along boundary generators (UX, UY and UZ applied on lines), and free boundary directrices. Uniform external pressure (in normal direction) was assumed first, later uniform pressure in vertical direction was applied. System ANSYS [16] was used for analysis, FEM model was created using element type SHELL181 (4 nodes, 6 DOF at each node). The arc-length method was chosen for analysis, the reference arc-length radius is calculated from the load increment. Only fundamental path of nonlinear solution has been presented.

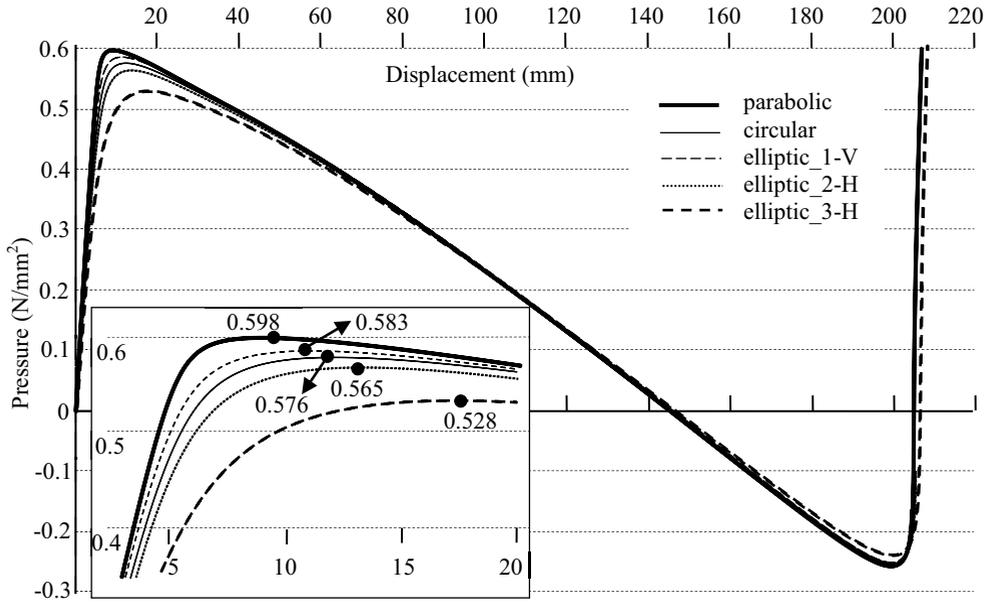


Fig. 2. Fundamental paths of perfect geometry for different shapes of directrix.

In Fig. 2, the fundamental path of the analysis of an ideal cylindrical shell having parameters from Figure 1 can be seen. Applied load acts in the normal direction. Parabolic, circular and elliptic (vertical and horizontal ellipse) shapes of arch were considered. Due to considerable shallowness of the shell ($h/2L_x=1/20$) shapes of the middle surface are very similar. However, in Fig. 2 (bottom left detail) one can see non-negligible differences in values of load in limit points of particular paths. Obtained results are analyzed and compared also in Table 1. Load in limit point of the circular arch was chosen as the comparative value. The results for the load acting in the vertical direction have been presented in the table, too. In this case, also the component of the load acting in tangential direction affects the shell; value of load acting in normal direction has been changing along the arch. Due to this, the value of load in limit point is lower (since the shell is very shallow, differences are small).

Let us now analyze the shell with geometrical imperfection. The initial displacements were assumed as the out of plane displacements only [17] as a product of selected buckling mode (dimensionless) and appropriately selected constant α_0 .

Table 1. Comparison of the loads in limit points for different shapes of directrix.

directrix	External pressure [N/mm ²] (normal direction)			Uniform pressure [N/mm ²] (vertical direction)		
	load [N/mm ²]	Δ [%]	displ. [mm]	load [N/mm ²]	Δ [%]	displ. [mm]
parabola	0.598	+3.82	9.16	0.585	+2.99	10.1
ellipse_1 (vertical)	0.583	+1.22	11.40			
circle	0.576	100	12.49	0.568	100	15.0
ellipse_2 (horizontal)	0.565	-1.91	13.29	0.559	-1.58	15.7
ellipse_3 (horizontal)	0.528	-8.33	17.66	0.525	-7.57	17.5

Solution in Fig. 3 presents, in addition to load - displacement paths of an ideal shell (symmetrical and non-symmetrical path), also paths which represent solution of the shell with geometric imperfection coincided with first buckling mode and multiplied by parameter $\alpha_0 = 0.5$ mm and 1.5 mm, respectively. The decrease of load in the limit point of load - displacement path of an imperfect shell can be seen in the detail in the top right, corresponding to the increase of the magnitude of an initial imperfection.

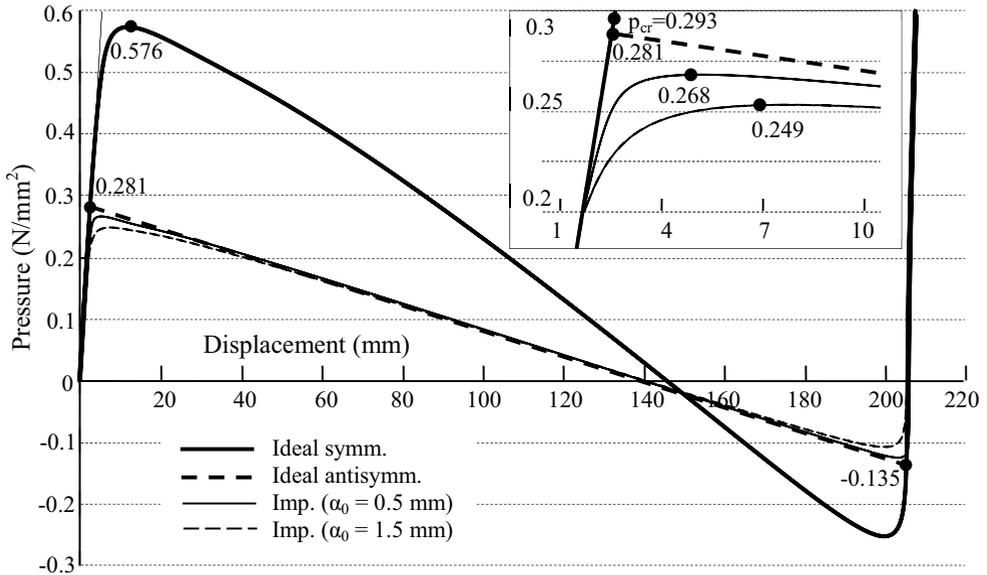


Fig. 3. Shallow circular cylindrical shell. Equilibrium paths for perfect shell and shell with geometric imperfection.

Obtained results are compared also in Table 2. Here the results from nonlinear analysis (NA) - ideal shell and shell with initial imperfections (for different values of α_0) are

compared with results from eigenvalue buckling analysis (EBA) – critical load corresponded to first eigenmode.

Table 2. Circular cylindrical shell, external pressure – results comparison.

analysis	load [N/mm ²]	Δ [%]
EBA p_{cr}	0.293	100
NA perfect shell	0.281	-4.10
NA $\alpha_0 = 0.5$ mm	0.268	-8.53
NA $\alpha_0 = 1.5$ mm	0.249	-15.02

In the first part of the paper the effect of the cross-section of a shallow shell upon its load-carrying capacity has been investigated. It has been proven that even in the case of a very shallow shell it is important how the curvature changes along the arch. The reduction of curvature from the top towards the supports is preferable.

In the second part of the paper, the effects of the initial geometric imperfection upon the load-carrying capacity of a shell has been analysed. The reduction of the load-carrying capacity in dependence on the magnitude of assumed geometric imperfection has been presented.

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