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Modelling of moving load effect on concrete pavements

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Abstract. Concrete roads may have different layout arrangement. The structures made of isolated prefabricated slabs are used for the construction of temporary roadways. The paper deals with numerical simulation of moving load effect on isolated slabs on elastic foundation. The computing model of a lorry and computing model of a slab are introduced. The deflections at the middle of the slab and tire forces of vehicle are modeled under various conditions. The influence of speed of vehicle motion and influence of initial conditions are evaluated. The results are presented in time domain in graphical and numerical manner.

1 Introduction

Concrete slabs are widely used in road building. Isolated plates are usually used for the temporary pavement structures. Knowledge of stress and strain is required for correct design of the slabs. Numerical methods enable to simulate moving load effect in dynamic regime with satisfactory precision. The computing models of vehicles can be created on various levels. For the purpose of this task the plane computing model is adopted. The pavement is modelled as the thin slab on elastic foundation. The assumption about deflection plate of the slab is introduced to reduce the partial differential equation of motion on ordinary differential equation. The problem is solved by numerical way. The results depend on the speed of the vehicle and on its initial conditions. The deflections in the middle of the slab and tire forces are calculated. The results are presented in numerical and graphical manner.

2 Computing model of vehicle

A plane computing model of the Tatra 815 lorry is used for the solution of this problem [1]. It is the multibody computing model with 8 degrees of freedom, Fig. 1. The model has three mass bodies with five degrees of freedom and three massless degrees of freedom. Functions $r_i(t)$, $(i = 1 - 5)$ describe the vibration of mass bodies. The model can simulate the heave and pitch effects of mass bodies. The model is excited kinematically by running along the road. Contact forces $F_j(t)$, $(j = 3, 4, 5)$ correspond to the massless degrees of freedom. The

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equations of motion for the calculation of functions $r_i(t)$ have the form of ordinary differential equations.

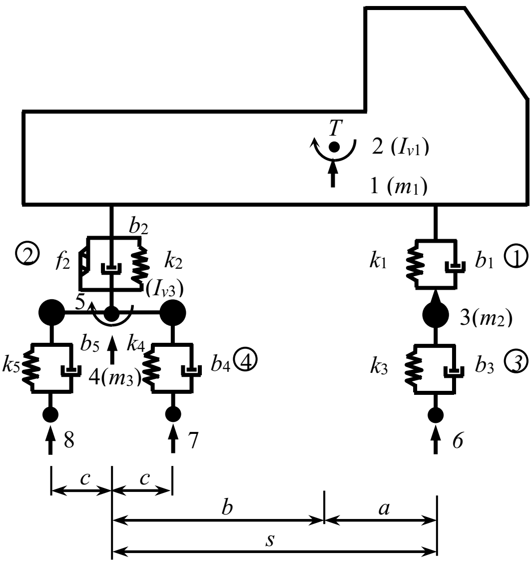

Fig. 1. Plane computing model of the Tatra 815 vehicle.

$$\ddot{r}_1(t) = -\{+k_1 \cdot d_1(t) + b_1 \cdot \dot{d}_1(t) + k_2 \cdot d_2(t) + b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dot{d}_2(t)/\dot{d}_c\} / m_1 ,$$

$$\ddot{r}_2(t) = -\{-a \cdot k_1 \cdot d_1(t) - a \cdot b_1 \cdot \dot{d}_1(t) + b \cdot k_2 \cdot d_2(t) + b \cdot b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dot{d}_2(t)/\dot{d}_c\} / I_{y1} ,$$

$$\ddot{r}_3(t) = -\{-k_1 \cdot d_1(t) - b_1 \cdot \dot{d}_1(t) + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t)\} / m_2 ,$$

$$\ddot{r}_4(t) = -\{-k_2 \cdot d_2(t) - b_2 \cdot \dot{d}_2(t) - f_2 \cdot \dot{d}_2(t)/\dot{d}_c + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t) + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t)\} / m_3 ,$$

$$\ddot{r}_5(t) = -\{-c \cdot k_4 \cdot d_4(t) - c \cdot b_4 \cdot \dot{d}_4(t) + c \cdot k_5 \cdot d_5(t) + c \cdot b_5 \cdot \dot{d}_5(t)\} / I_{y3} . \quad (1)$$

The terms for calculation of tire forces are

$$F_3(t) = -G_3 + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t) = -g \cdot \left(m_1 \cdot \frac{b}{s} + m_2 \right) + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t),$$

$$F_4(t) = -G_4 + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t) = -\frac{1}{2} \cdot g \cdot \left(m_1 \cdot \frac{a}{s} + m_3 \right) + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t),$$

$$F_5(t) = -G_5 + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t) = -\frac{1}{2} \cdot g \cdot \left(m_1 \cdot \frac{a}{s} + m_3 \right) + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t). \quad (2)$$

Values $d_i(t)$, (i = 1 – 4) represent the deformation of connected members of the model in time t. The derivations with respect to time are denoted by the dot above the symbol of dependent variable. G_j (j = 3, 4, 5) represents the gravity forces acting in the contact points.

Symbols k, b, f describe the stiffness, damping and friction properties of connecting members, m, I_y represent the mass and mass moment of inertia of mass bodies.

3 Computing model of slab

The slab computing model is created in the sense of the Kirchhoff theory of thin slabs on elastic foundation [2]. The equation of motion describing the slab vibration has the form

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+K\cdot w+\mu\frac{\partial^2 w}{\partial t^2}+2\mu\omega_b\frac{\partial w}{\partial t}=p(x,y,t)\,. \tag{3}$$

It is a partial differential equation, which will be solved using the Fourier method. The assumption about the shape of deflection plate of slab $w_0(x,y)$ due to the load is adopted and wanted function $w(x,y,t)$ describing the shape of deflection plate of the slab in time t is expressed as

$$w(x,y,t)=q(t)w_0(x,y)\,, \tag{4}$$

where

$$w_0(x,y)=\sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}\,. \tag{5}$$

Function $w_0(x,y)$ is the known function and it depends on coordinates x, y only. Function $q(t)$ is the unknown function and it dependents on time t. Function $q(t)$ has the meaning of the generalized Lagrange coordinate. It represents the proportionality coefficient between dynamic and static deflection in the time t. The meaning of other symbols is as follows: D represents the slab stiffness [Nm^2/m], K is the modulus of compressibility of elastic foundation [N/m^3], μ is the mass intensity of the slab [kg/m^2], ω_b is damped angular frequency [rad/s], l_x, l_y are the dimensions of the slab [m]. Expression $p(x,y,t)$ on the right hand side of equation (3) is the intensity of continuous dynamic load. By substituting assumption (4) into (3) we obtain

$$\ddot{q}(t)\left\{\mu\sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}\right\}+\dot{q}(t)\left\{2\mu\omega_b\sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}\right\}+$$

$$+q(t)\left\{\sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}D\left[\left(\frac{\pi}{l_x}\right)^4+2\left(\frac{\pi}{l_x}\right)^2\left(\frac{\pi}{l_y}\right)^2+\left(\frac{\pi}{l_y}\right)^4+\frac{K}{D}\right]\right\}=p(x,y,t). \tag{6}$$

In case of discrete moving load contact force $F_j(t)$ must be transformed on continuous load $p(x,y,t)$. It may be done by the advance proposed by Dirac [2]

$$p(x,y,t)=\sum_j \varepsilon_j F_j(t)\delta(x-x_j)\delta(y-y_j), \tag{7}$$

$$p(x,y,t)=\sum_j \varepsilon_j \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_{mn,j}(t)\sin\frac{m\pi x}{l_x}\sin\frac{n\pi y}{l_y}\,, \tag{8}$$

where

$$p_{mn,j}(t)=\frac{2}{l_x}\frac{2}{l_y}\int_0^{l_x}\int_0^{l_y}F_j(t)\delta(x-x_j)\delta(y-y_j)\sin\frac{m\pi x}{l_x}\sin\frac{n\pi y}{l_y}\,dxdy=F_j(t)\frac{4}{l_xl_y}\sin\frac{m\pi x_j}{l_x}\sin\frac{n\pi y_j}{l_y} \tag{9}$$

$\varepsilon_j=1$ when the force F_j acts on the slab, otherwise $\varepsilon_j=0$. Then

$$p(x,y,t)=\sum_j\varepsilon_j\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}F_j(t)\frac{4}{l_xl_y}\sin\frac{m\pi x_j}{l_x}\sin\frac{n\pi y_j}{l_y}\sin\frac{m\pi x}{l_x}\sin\frac{n\pi y}{l_y}\,. \tag{10}$$

The convergence of the series in equation (10) is possible without major detriment to accuracy, taking into account only the first member of the series. Then the expression (10) is simplified

$$p(x,y,t)=\sum_j\ \varepsilon_j\cdot F_j(t)\cdot\frac{4}{l_xl_y}\cdot\sin\frac{\pi x_j}{l_x}\cdot\sin\frac{\pi y_j}{l_y}\cdot\sin\frac{\pi x}{l_x}\cdot\sin\frac{\pi y}{l_y}\,. \tag{11}$$

Substituting $F_j(t)$ in equation (11) by equation (2) and replacing $p(x,y,t)$ in equation (6) by (11) we obtain the final shape of the slab equation of motion. It can be written in a simplified form as

$$\ddot{q}(t)=-[\ \dot{q}(t)\cdot a_{d2}+q(t)\cdot a_{d3}+r_3(t)\cdot a_{d4}+\dot{r}_3(t)\cdot a_{d5}+r_4(t)\cdot a_{d6}+\dot{r}_4(t)\cdot a_{d7}+$$
$$+r_5(t)\cdot a_{d8}+\dot{r}_5(t)\cdot a_{d9}+a_{d10}+a_{d11}+a_{d12}\ ]/a_{d1}\,. \tag{12}$$

4 Vehicle and slab parameters and numerical solution

The vehicle computing model has the following mass parameters: m_1 = 11475 kg, m_2 = 455 kg, m_3 = 1070 kg, I_{y1}=31149 kgm^2, I_{y3}=466 kgm^2. The gravity forces acting in contact points are: $G_3=(m_1\cdot b/s+m_2)g$ = 33208 N, $G_4=G_5=0.5(m_1\cdot a/s+m_3)g$ = 47161 N.

The model of the slab on layered foundation showed in Fig. 2 has following parameters: slab depth h = 0.24 m, slab proportions in longitudinal and transverse direction l_x = 6.0 m; l_y = 3.75 m, concrete modulus of elasticity E = 37500 MPa, Poisson ratio ν = 0.20. The layers of foundation 2 – 5 are included in the computing model as Winkler elastic foundation. The modulus of foundation compressibility K = 171.8 MN/m^3 was calculated by the use of computer program LAYMED [3]. Mass intensity of slab $\mu=\rho\cdot h$ = 2500·0.24 = 600 kg/m^2, damping angular frequency ω_b = 0.1 rad/s.

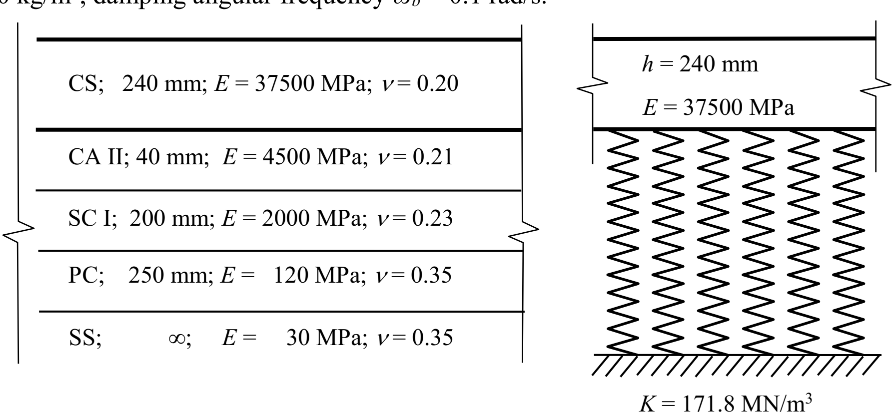

Fig. 2. Composition of the pavement with concrete slab.

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Synchronous vibration of vehicle – slab in our case is described by a system of the 2nd order ordinary differential equations. Equations (1) describe the vibration of vehicle and equation (12) describes the slab vibration. The computer program in MATLAB was created for the numerical solution of equations of motion and for displaying of the obtained results. The 4th order Runge-Kutta step-by-step integration method was used for the numerical solution [4].

5 Results of numerical solution

Vehicle always enters the slab vibrant. The results of solution are influenced by vehicle initial conditions at the moment of vehicle entry on the slab. Two variants were considered during numerical solution, Fig. 3.

Variant a – vehicle springs are aggravated, mass body m_1 amplitude is down oriented from the equilibrium position. Initial conditions are as follows: $r_1(0)$ = –0.01 m, $r_2(0)$ = +0.003 m, $r_3(0) - r_5(0)$ = 0.0 m, $\dot{r}_1(0) - \dot{r}_5(0)$= 0.0 m/s.

Variant b – vehicle springs are lightened, mass body m_1 amplitude is up oriented from the equilibrium position. Initial conditions are as follows: $r_1(0)$ = +0.01 m, $r_2(0)$ = –0.003 m, $r_3(0) - r_5(0)$ = 0.0 m, $\dot{r}_1(0) - \dot{r}_5(0)$= 0.0 m/s.

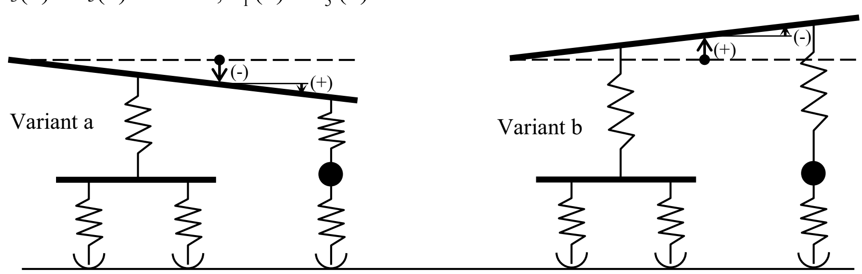

Fig. 3. Initial conditions of vehicle.

Numerical solution was realized in dependence on the speed of the vehicle motion in the interval of speeds 5 – 130 km/h with the step of 5 km/h. For every speed of the vehicle motion the time course of the middle slab vertical displacements and the time course of contact forces were calculated. Maxima of middle slab vertical displacements and dispersion of contact forces are plotted versus speed of the vehicle motion. Demonstration of the obtained results for Variant a is presented in Figs. 4, 5, 6, 7 and for Variant b in Figs. 8, 9, 10, 11.

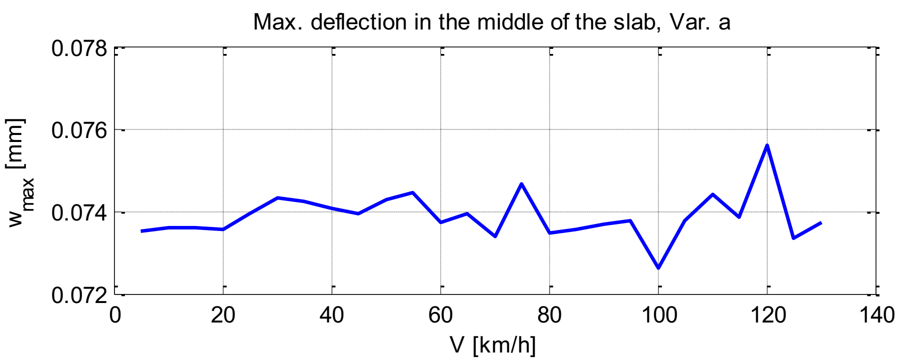

Fig. 4. Maximal deflection in the middle of the slab, Variant a.

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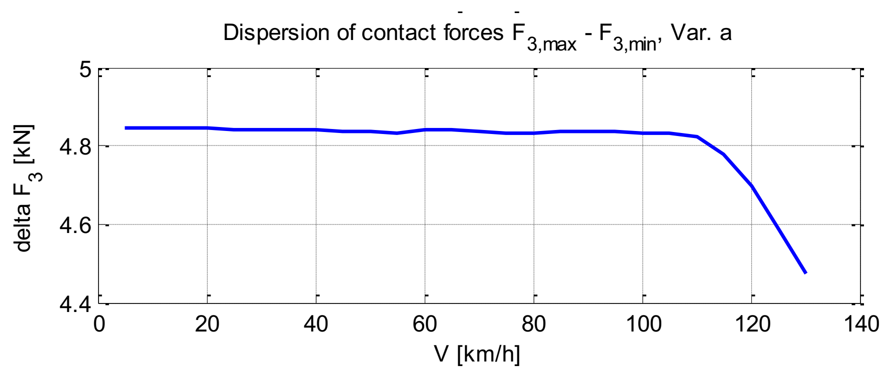

Fig. 5. Dispersion of contact force F_3, Variant a.

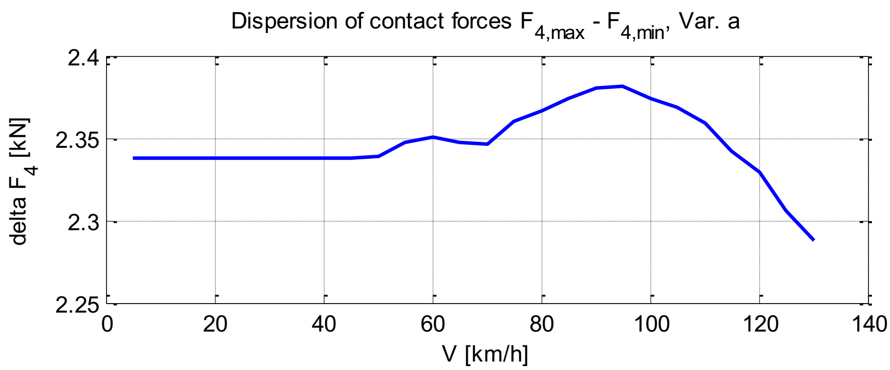

Fig. 6. Dispersion of contact force F_4, Variant a.

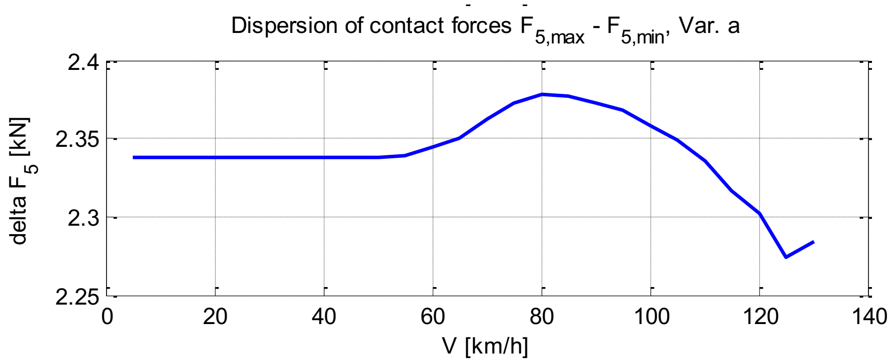

Fig. 7. Dispersion of contact force F_5, Variant a.

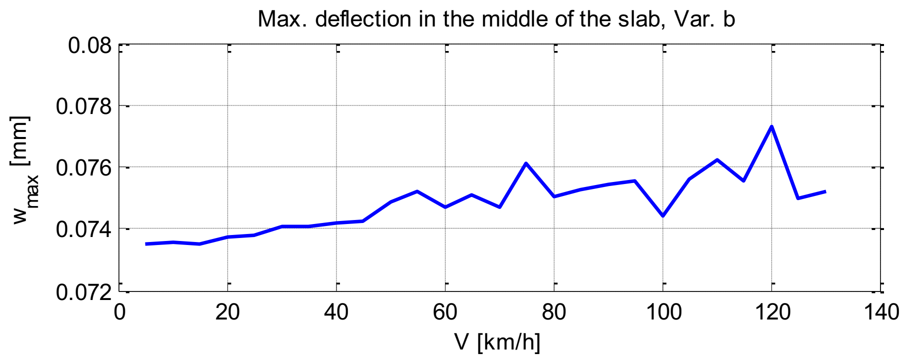

Fig. 8. Maximal deflection in the middle of the slab, Variant b.

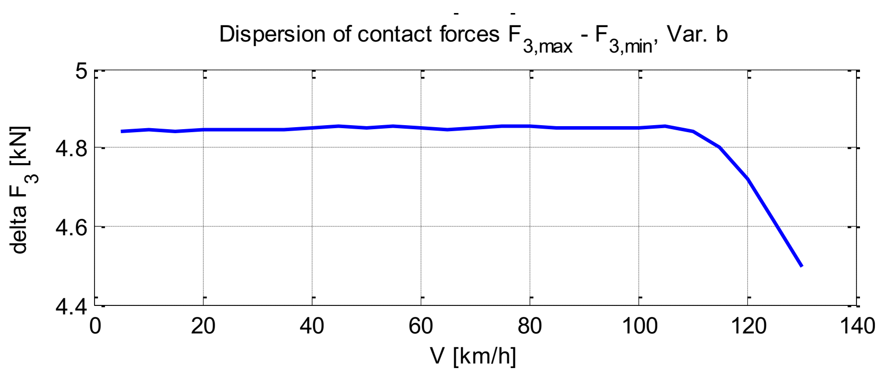

Fig. 9. Dispersion of contact force F_3, Variant b.

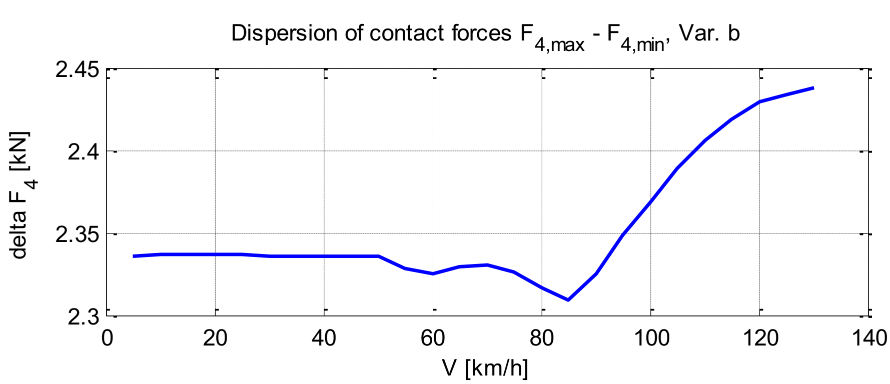

Fig. 10. Dispersion of contact force F_4, Variant b.

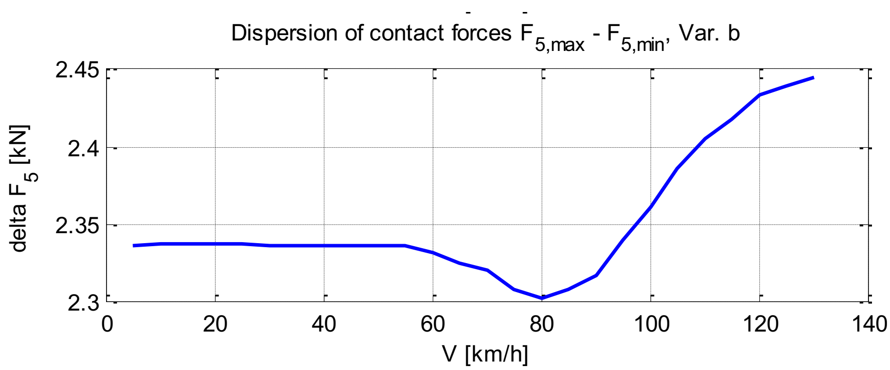

Fig. 11. Dispersion of contact force F_5, Variant b.

6 Conclusion

The movement of vehicles along isolated concrete slabs on the elastic foundation is a real engineering task and can be solved in a numerical way. The slab dynamic deflections, internal forces and the vehicle tire forces can be observed. The task was analyzed under two variants of the vehicle initial conditions. The vehicle enters the slab with aggravated springs (Variant a) and the vehicle enters the slab with lightened springs (Variant b).

The absolute maxima of the middle slab vertical dynamic deflections in the interval of speed of vehicle motion 5 – 130 km/h are practically the same for both variants (Variant a, w_{max} = 0.07562 mm, V = 120 km/h, Variant b, w_{max} = 0.07728 mm, V = 120 km/h). The absolute minima of the middle slab vertical dynamic deflections in the interval of speed of vehicle motion 5 – 130 km/h are different for individual variants (Variant a, w_{min} = 0.07261 mm, V = 100 km/h, Variant b, w_{min} = 0.07349 mm, V = 5 km/h).

The dispersion of contact forces F_3 is practically the same in both cases (Variant a, $\Delta F_{3,min}$ = 4.4727 kN, V = 130 km/h, $\Delta F_{3,max}$ = 4.8439 kN, V = 15 km/h, Variant b, $\Delta F_{3,min}$ = 4.4967 kN, V = 130 km/h, $\Delta F_{3,max}$ = 4.8548 kN, V = 55 km/h). The dispersion of contact forces F_4 and F_5 is practically the same but for individual variants different (Variant a, $\Delta F_{4,min}$ = 2.2876 kN, V = 130 km/h, $\Delta F_{4,max}$ = 2.3812 kN, V = 95 km/h, $\Delta F_{5,min}$ = 2.2739 kN, V = 125 km/h, $\Delta F_{5,max}$ = 2.3781 kN, V = 80 km/h, Variant b, $\Delta F_{4,min}$ = 2.3099 kN, V = 85 km/h, $\Delta F_{4,max}$ = 2.4384 kN, V = 130 km/h, $\Delta F_{5,min}$ = 2.3028 kN, V = 80 km/h, $\Delta F_{5,max}$ = 2.4454 kN, V = 130 km/h).

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