

# Experimental study of horizontal forces of pedestrian dynamics

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**Abstract.** At present similar detailed examination of pedestrian dynamics has been brought about chiefly by the requirements of determining the loads on footbridges. In their case – contrary to bridges – the magnitude of dynamic response is more important than that of static response, as it determines also the serviceability of the footbridge with reference to pedestrian comfort. The forces frequencies of step or strides for different walking velocities is the most important for the further analysis and analysis of the mechanisms.

## 1 Dynamic response of footbridge in the horizontal direction

The excessive lateral sway motion of the footbridge by the people walking across has great attention in literature due to some examples: especially the very flexible Millennium footbridge in London.

The human organism is more sensitive on horizontal vibration than to the vertical one. The warning example was the London Millennium Footbridge, which after pedestrian traffic showed insufficient stiffness in the horizontal direction and had to be closed. Followed suppress horizontal vibration using the dampers [2], which cost a city about 5 mil. Pounds (original price of 18.2 mil. Pounds). That was the case when the absurd proposal by architect the designer agreed to the project.

### 1.1 One pedestrian

For an approximate solution of acceleration, assuming that the footbridge is simple beam and the pedestrian is in resonance with the frequency of the footbridge, in this case Kreuzinger recommends [6]

$$a_1 = \frac{\alpha_1 \cdot 700}{M \cdot \xi \cdot k_1} \quad (1)$$

where  $\alpha_1 = 0,10$ , when  $f_{str} < 1,25$  Hz a  $f_p = 2 f_{str}$ ,

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$\alpha_1 = 0.05$ , when  $f_{str} > 1.25$  Hz and  $f_p = f_{str}$ ,

$k_1 = 0.75$

$M$  – mass of footbridge [kg],

700 – weight of pedestrians [N],

$f_p$  – step frequency,

$f_{str}$  – frequency of construction

## 1.2 The group and the continuous stream of pedestrians

Approximate solution of acceleration  $a_{hor}$  recommends Kreuzinger [6] by the expression

$$a_{hor} = a_1 \cdot N_{ef} \cdot k_{hor} \quad (2)$$

where  $N_{ef} \approx 0.2 N$ ,  $k_{hor} = 1,0$ , when  $f_{str} \leq 1.25$  Hz,

$k_{hor} = 0.5$ , when  $f_{str} = 1.75$  Hz,

$k_{hor}$  approaching 0, when  $f_{str}$  approaching 2.5 Hz.

Intermediate values  $k_{hor}$  can be interpolated linearly.

For the construction in resonance with the evenly distributed mass along its length and uniformly loaded, with the shape of the excited vibration of a sine wave, calculates the maximum acceleration in the  $j$ -th shape by the Stoyanoff [8]

$$a_{j,\max}^{hor} = \frac{2 \cdot c_R \cdot N \cdot \alpha_j^H \cdot w_p}{\pi \cdot m_{deck} \cdot l \cdot \xi_j} \quad (3)$$

where  $c_R$  is a correlation coefficient 0,2 (for vandals 1,0),

$N$  – the number of pedestrians,

$\alpha_j^H = 0.125$ ,

$w_p = 700$  N,

$m_{deck} \cdot l$  – mass of footbridge,

$\xi_j$  – damping ( $j$ -th shape of vibration).

## 1.3 Sensitivity of pedestrians

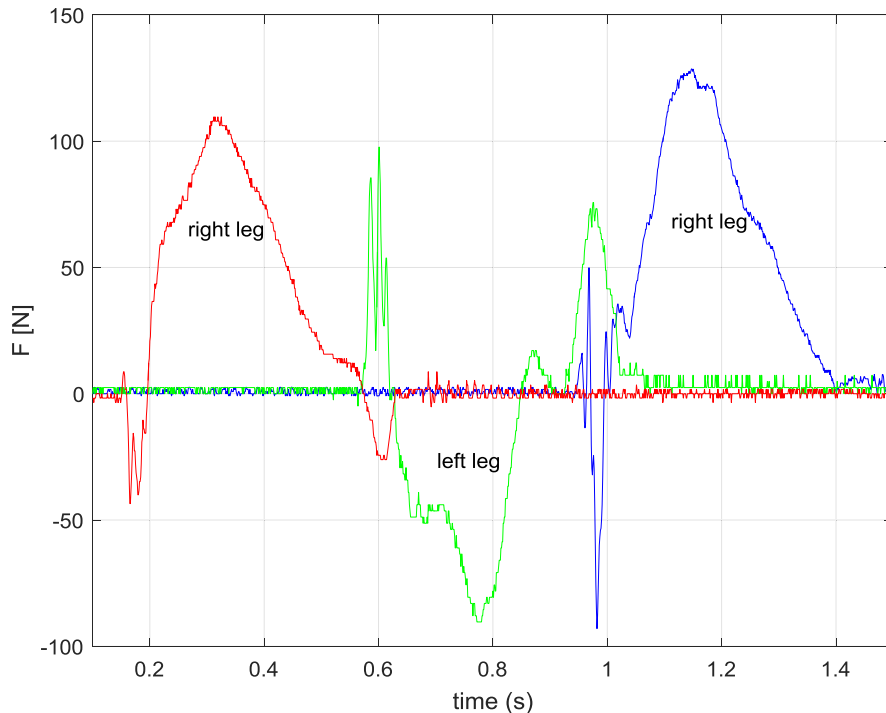
Reaction of the human body to the pad movement in the case of footbridges, instead of bridges, is important for evaluating of pedestrian comfort. In most cases is evaluated acceleration of motion [1, 2], (in the vertical and horizontal direction), as well as a velocity or displacement. From the results of many experiments [3] have been recommended limits of dynamic responses. In general confirmed, that the man is less sensitive to the vibration on the footbridge than the vibrations in residential or in other buildings. Some researchers have found that the recommended limit of pedestrian comfort depends on the time of crossing over the footbridge - with increasing duration decreases its value. Other researchers declare, on the contrary, that the pedestrian "get accustomed" to the movement and limit comfort is unchanging.

In the assessment and determination of boundaries between acceptable and unacceptable dynamic response (boundary comfort) from the viewpoint the human body, it is important to realize, that it depends on the degree of subjectivity, based only on experience<sup>†</sup>.

## 2 Lateral horizontal load

The lateral horizontal force depends on the weight of pedestrian on the speed of the walking and on the length of the stride.

Typical time histories of horizontal lateral loads (right, left, right leg) are on Fig. 1.



**Fig. 1.** Typical time histories of horizontal lateral loads.

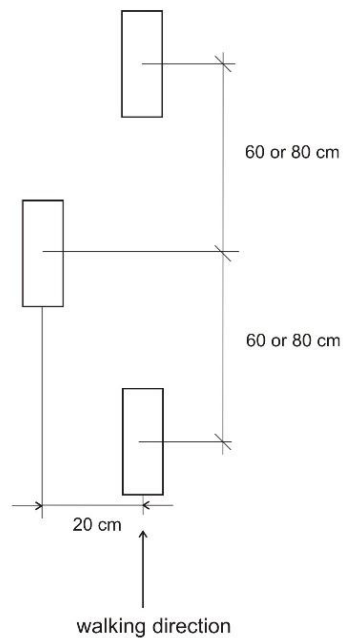
## 3 Sensors of the lateral horizontal load

The sensor is the series of three steel strips, 370 mm long, 30 mm wide, supported by boundary box. The deflection stress of steel strips was measured by strain gauges which register their horizontal deflection. On the top of steel strips is the plate from soft material, which guarantees the participation of all strips. The sensor is shown on Fig. 2. The position of sensors in plain view is drawn on Fig. 3.

<sup>†</sup> The human body is very sensitive : the second step of Meralli–Cancani–Sieberg scale (it is the acceleration 2.5 up to 5.0 mm/s<sup>2</sup>) is the start of the human feeling.

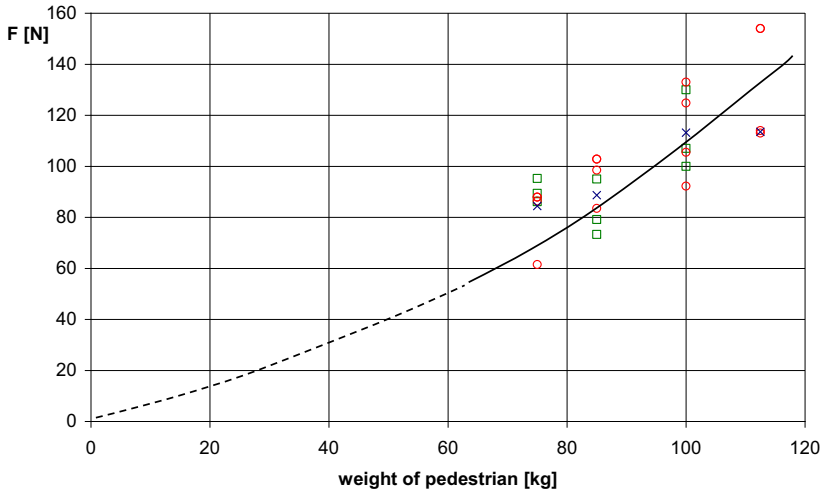


**Fig. 2.** The sensor.



**Fig. 3.** Sensors positions for lateral horizontal loads.

In Fig. 4 are results of all our experiments. The weights of pedestrians was from 700 N up to 1125 N and the walking speed from 0.45 m/s up to 1.44 m/s (from 1.6 km/h up to 5.1 km/h).



**Fig. 4.** Relation between weight of pedestrian and lateral horizontal load.  
 ○ – stride length 60cm, □ – stride length 80cm, × – the mean value.

### 4 The dynamic load in the horizontal lateral direction

Stoyanoff [8] gave the formula for the load

$$F(t) = c_R \cdot N \cdot \alpha \cdot w_P \cdot \cos \Omega t \tag{4}$$

- where  $c_R$  (correlation coefficient  $\approx 1$ )
- $N$  number of pedestrians (  $20 \div 25$  persons)
- $\alpha$  dynamic coefficient (0.125)
- $w_P$  weight of the pedestrian
- $\Omega$  dominant walking circular frequency (commonly  $f = 1$  Hz)

### 5 Power spectral density of footbridge displacement

Pedestrian(s) - induced forced vibrations have two components - transient and stationary. In case of standard values of damping ( $\xi = 0.015$ ) and stride frequency ( $f_s = 2$  Hz) the excited amplitude attains about 60% of its maximum after 10 strides, about 85 % of its maximum after 20 strides and the absolute maximum only after 60 strides. (i.e. after 30 seconds of walking). Hence it follows that the first component, i.e. transient vibrations, is in-significant for long footbridges.

After this verification let us derive the power spectral density for stationary displacement. The power spectral density of the consolidated displacement of a point  $s$  is

$$G_{v_s v_s}(f, s) = \sum_{n=1}^{\infty} |H_n(f, s)|^2 \int_0^l \int_0^l v_n(x_1) \cdot v_n(x_2) \cdot G_{p_1 p_2}(f, x_1, x_2) \cdot dx_1 \cdot dx_2 \tag{5}$$

where we have neglected the influence of correlation among natural vibration modes, where:

$$H_n(f, s) = \frac{v_n(x_s)}{M_n \left[ (f_n^2 - f^2) + 2if_b f \right] \cdot 4\pi^2} \quad (6)$$

And  $G_{p_1 p_2}$  is the cross spectral density of loads in points  $x_1$  and  $x_2$ .

The cross spectral density of loads can be expressed by means of the power spectral density of loads  $G_{pp}$  and the function  $g_{1,2}(f)$ , i.e.

$$|G_{p_1 p_2}(f)| = g_{1,2}(f) \cdot G_{pp}(f) \quad (7)$$

So far there are very few experimental verifications of the first quantity of the right-hand side of Eq. (7). According to our measurements and according to [7] it is possible to express  $g_{1,2}(f)$ , by means real coherence

$$g_{1,2}(f) = \frac{|G_{p_1 p_2}(f)|}{G_{pp}(f)} = \sqrt{\text{coh}(f)} \quad (8)$$

in the form of

$$\sqrt{\text{coh}(f)} = \exp\left(-\frac{\kappa f \cdot |x_1 - x_2|}{N}\right) \quad (9)$$

where  $k$  is so called the damping coefficient and  $N$  the number of pedestrians filling the whole deck area.

According to our measurements, see e.g. [7]<sup>‡</sup>, the value of the damping coefficient is approximately  $k \approx 60$  to  $80$

If we express the power spectral density of loads for the correlation function of one process  $x(t)$ , see [5], with mean values

we obtain

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t) \cdot x(t+\tau) dt, \quad \text{for } T \rightarrow \infty \quad (10)$$

Power spectral density is related to it by the expression

$$G(f) = 4 \int_0^{\infty} R(\tau) \cos(2\pi f \tau) d\tau \quad (11)$$

The correlation function of loads can be determined with sufficient accuracy from the formula for the amplitude of the dynamic component of the vertical force. From the formula for the power spectral density of point as we obtain the variance of the displacement

$$\sigma_v^2 = \int_0^{\infty} G(f) df \quad (12)$$

<sup>‡</sup> footbridge length  $l = 51$  m,  $N = 408$  pedestrians,  $f = 1.5$  Hz

## 6 Conclusion

The time histories of lateral forces were registered and statistical analyzed: the authors received the lateral force dependence on the walking velocity.

The important results of measurements done in the ITAM laboratory and on the footbridges of various supportive systems follow:

- 1) Mutual relations among the stride frequency, step frequency, step length, dynamic coefficient and the striding velocity depend on individual body characteristics of a pedestrian.
- 2) The dynamic coefficient for a given pedestrian can be larger than for a group of pedestrians, if they do not move in a synchronous way.
- 3) The obtained dynamic coefficients are of use for computations of load exerted by a single pedestrian, a group of pedestrians, a connected stream of pedestrians, and vandals, and for the computation of responses of footbridges with different supportive systems.

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