Calculation of three-layer bent reinforced concrete elements considering fully transformed concrete deformation diagrams

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Abstract. The article deals with the method for calculating the three-layer bent reinforced concrete elements, taking into account the total deformation diagrams of different concrete layers. There are formulas and calculations in cases of presence and absence of cracks in the tension zone.

1 Introduction

Design of bent reinforced-concrete elements according to the procedures of norms providing for rectangular tension diagram in the compressed and stretched zones of concrete, respectively, for determining the strength and fracture toughness in some cases leads to substantial deviations of experimental and theoretical data [1-5]. Different ways to improve these techniques allow to bring together experimental and theoretical results, but they do not eliminate the main drawback of ignoring full of concrete deformation diagram. Noted above in some cases can lead to substantial deviations of the theoretical and experimental values [6-9]. As mentioned in a number of works of scientists, the most perspective direction for further improvement of calculation methods of reinforced concrete elements is the inclusion of formulas with complete descending branches concrete deformation diagrams in the compressed and stretched zones of the element. Using this approach, you can obtain analytical dependence describing the heavy-deformed state at all stages of the construction load. Also it provides a unified approach to the determination of the strength, toughness and crack resistance of concrete elements [1-14]. It should be noted that the vast majority of works was devoted to the development of new and innovative calculation methods of reinforced concrete bent elements of solid section. There is not enough similar work relating to the elements of the laminate section.

2 Literature review

Investigation of reinforced concrete columns with recessed longitudinal rods without transverse reinforcement is a very actually problem of modern building and constructions.

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A lot of scientists devote their scientific works to this problem. We can see the parts of this theme in the works ofD.R. Mailyan; D.R. Mailyan, V.L. Medinsky, A.G. Azizov; V.K. Osipov;V.E. Chubarov, V.K. Osipov;V.K. Osipov, D.R. Mailyan; V.K. Osipov, V.A. Zalessky; R.A. Khunagov; R.A. Khunagov; R.A. Khunagov; R.A. Khunagov; R.A. Khunagov, A.M. Blyagoz; D.R. Mailyan, V.L. Medinsky, A.G. Azizov; N.B. Aksenov, A.Y. Kubasov, A.M. Nikolaenko; N.B. Aksenov, A.Y. Kubasovat their articles : New effective design solutions of compressed concrete elements[1];More efficient use of high-strength reinforcement rod in the compressed concrete elements [2];Effective concrete for agricultural construction [3];Guidance on the computer calculation of concrete columns with mixed reinforcement [4]; Recommendations for the design of eccentrically compressed reinforced concrete three-layer structures with high reinforcement [5]; Three-layer load-bearing panel [6]; Double pre-tense reinforced concrete columns [7]; Flexible double layer evenly crimped concrete panels [8]; Impact of pre-tense uneven sections of reinforced concrete elements [9]; Calculation of two-layer pre-tense concrete panels [10]; Two-layer reinforced concrete panels with uneven compacted sections [11]; Strength concrete columns with high pre-compression fittings [12]; Features of the calculation of the girder with pre-compressed reinforcement of the upper zone s [13]; Automated calculation of bent elements combined pretension reinforcement [14].

3 Calculation of three-layer bent reinforced concrete elements considering fully transformed concrete deformation diagrams

Preparing the calculation methods, taking into account the full concrete deformation diagrams, the following design preconditions were accepted:

- tension $\sigma_b(\sigma_{bt})$and deformation $\varepsilon_b(\varepsilon_{bt})$ of each individual fiber layers of different heavy and light concrete, compressed$\overline{x}$ or stretched$\bar{y}$ zones in height varies as the transformed strain, depending on the level of pretension and strain gradient (tension) diagram "$\sigma_{b(bt)} - \varepsilon_{b(bt)}$". When these tensions may vary from zero to $K_R \gamma_{Rb\,(1,2,3)} R_{b\,(1,2,3)} \left(K_{Rt} \gamma_{Rbt\,(1,2,3)} R_{bt\,(1,2,3)}\right)$ and strain from zero to large values $K_{\varepsilon R} \gamma_{\varepsilon b}\, R_{bR}\,(K_{\varepsilon bR}\, \gamma_{\varepsilon bt}\, R_{bRt})$.

- the diagram of concrete strain is taken as the initial, recommended by SEB-FIP [1], which in this case takes the form:

$$\frac{\sigma_b}{K_R \gamma_{Rb} R_b} = \frac{K\left(\frac{\varepsilon_b}{K_{\varepsilon b} \gamma_{\varepsilon b}\, \varepsilon_{bR}}\right) - \left(\frac{\varepsilon_b}{K_{\varepsilon R} \gamma_{\varepsilon b}\, \varepsilon_R}\right)^2}{1-(K-2)[\varepsilon_b/(K_{\varepsilon R} \gamma_{\varepsilon b}\, \varepsilon_{bR})]} \quad (1)$$

This estimated diagram (1) is considered to be right for compressed and stretched fibers of different concrete layers:

- before cracking, sections remain plane during the deformation, so the hypothesis of plane sections is considered to be right; after cracking warping of sections is taken into account by the developed technique [1];

- description of the analytical chart of deformation of high strength steel and its changes caused by pretension are taken as described in [2-6];

- neutral axis diagrams of deformation and tension are the same, which is justified for a short time uploading (no time to manifest nonequilibrium deformation), and prolonged uploading is valid only for the elements in which the stiffness of the compressed and stretched zones are changed simultaneously [10].

The adoption of the prerequisites mentioned settlement allows with one voice to determine strength, toughness and crack resistance of concrete elements for any external operational impact and pretension.

The calculation starts with the selection of the initial value of an external force, certainly less destructive. Each successive value of the efforts of the new stage of the calculation is determined from the expression$N_l = N_{K-1} + \Delta N_K$ where $K = 1,2,3$ ...the forces number N.

In monotonic loading member there are two stages of the work. The first stage – is the work item without cracks in the tension zone. Due to the fact that the section is solid, extreme concrete fiber deformations are interconnected with the expressions:

$$\overline{\varepsilon_b} = \overline{\varepsilon_{bt}}\bar{x}/\bar{y} \text{or} \overline{\varepsilon_b} = \overline{\varepsilon_{bt}}\bar{x}/(h - \bar{x}) \quad (2)$$

Moment of fracture is the limit state in the first stage of the work, in which the deformation of the stretched fiber

$$\overline{\varepsilon_{bt}} = \varepsilon_{btu}\,(\varepsilon_{btu} > K_{\varepsilon R}\gamma_{\varepsilon bt}\,\varepsilon_{btR}) \quad (3)$$

and the tension

$$\overline{\sigma_{bt}} = \sigma_{btu}\,(\sigma_{btu} > K_R\gamma_{Rbt}\,\varepsilon_{bt}) \quad (4)$$

At the moment of cracking, function$M = M\left(\overline{\varepsilon_{bt}}\right)$ reaches a maximum at a value of deformation$\overline{\varepsilon_{bt}} = \varepsilon_{btu}\left(\frac{dM}{d\overline{\varepsilon_{bt}}} = 0\right)$.

The second stage of work is characterized by the presence of cracks. The deformation of the stretched fibers of the crack is assumed to be$K_\varepsilon \cdot \gamma_{\varepsilon bt} \cdot \overline{\varepsilon_{bt}}$, thus, the descending branch transformed diagram is realized "$\sigma_{bt} - \varepsilon_{bt}$". The ultimate state of the second stage of the work is the beginning of the destruction of the state in which the extreme deformations of compressed fibers reach the value $\overline{\varepsilon_b} = \varepsilon_{bu}(\varepsilon_{bu} > K_{\varepsilon R}\gamma_{\varepsilon b}\,\varepsilon_{bR})$, and the tension respectively, $\overline{\sigma_b} = \sigma_{bu}\,(\sigma_{bu} < K_R\gamma_{Rb}\,\varepsilon_b)$.

With the destruction, the function $M = M(\overline{\varepsilon_b})$reaches a maximum at the appropriate value of the deformation $\overline{\varepsilon_b} = \varepsilon_{bu}$, т.е. $dM/d\overline{\varepsilon_b} = 0$.

After the beginning of the destruction, the element continues to be deformed by declining external force. The descending branch of the diagram appears "$N - \overline{\varepsilon_b}$" или "$N - æ$".

In general, the static system of equations is written as follows:

$$b\int_{\bar{x}+\bar{y}-h_3-h_2}^{\bar{x}} \sigma_{b1}(x)dx + b\int_{\bar{x}}^{\bar{x}-h_1} \sigma_{b2}(x)dx + A_s'\sigma_s' - b\int_0^{\bar{y}-h_3} \sigma_{bt3}(y)dy - b\int_{\bar{y}-h_3}^{\bar{y}} \sigma_{bt2}(y)dy + A_s\sigma_s = 0 \quad (5)$$

$$M = b\int_{\bar{x}+\bar{y}-h_3-h_2}^{\bar{x}} \sigma_{b1}(x)(h - h_3 - h_2 + x)dx + b\int_{\bar{x}}^{\bar{x}-h_1} \sigma_{b2}(x)(h - \bar{x} + x)dx + b\int_0^{\bar{y}-h_3} \sigma_{bt3}(y)\cdot(\bar{y} - y)\,dy + b\int_{\bar{y}-h_3}^{\bar{y}} \sigma_{bt2}(y)(\bar{y} - h_3 + y)dy + A_s\sigma_s\alpha - A_s'\sigma_s'(h - \alpha') = 0 \quad (6)$$

In the first stage we accept $К_R = 1$; $К_{\varepsilon b} = 1$; $\gamma_{Rb} = 1$; $\gamma_{Rb} = 1$, that is, transformation diagram is not performed.

Due to the fact that the number of unknowns exceeds the number of equations, it was necessary to describe the relationship of unknowns using the following equations:

$$h_1 + h_2 + h_3 = h;$$

$$x + y = h \tag{7}$$

Based on the hypothesis of plane sections we can write the equation of deformation relationship of steel with extreme deformations of concrete fibers:

$$\varepsilon_s = \frac{\overline{\varepsilon_b}(\bar{y}-\alpha)}{\bar{x}} = \frac{\overline{\varepsilon_b}(h-\bar{x}-\alpha)}{\bar{x}} \tag{8}$$

$$\varepsilon_s^` = \frac{\overline{\varepsilon_{bt}}(\bar{x}-a^`)}{\bar{y}} = \frac{\overline{\varepsilon_b}(\bar{x}-\alpha)}{\bar{x}} \tag{9}$$

Solving the system of equations (5) ... (9) assuming an elastic reinforcement work, wherein:

$$\sigma_s = \varepsilon_s E_s + \sigma_{sp}\ \sigma^`_s = \varepsilon^`_s E_s - \sigma^`_{sp} \tag{10}$$

If the condition $\sigma_s \ll \sigma^`_{el}$ (where $\sigma_s \ll \sigma^`_{el}$ -the new value of the elastic limit) is not satisfied, the calculation is repeated.
In this case, the equations (5) ... (9) are added equations relating tension and deformation of high-strength steel [8-12]. In the presence of pretension, the deformations are determined first ε_{sp} , due to the efforts of pretension, and then, using the same method of stress σ_s. As a result, we obtain the solution of systems of equations $\bar{x}$, $\bar{y}$, $\overline{\varepsilon_b}$, $\overline{\varepsilon_{bt}}$,$\sigma_s$,$\sigma^`_s$.

At each stage of loading, the force is determined, perceived stretched concrete area concerning very short fibers:

$$M_{bt} = P_{bt}(y_{bt} + \bar{x})$$

$$P_{bt} = b\int_0^{\bar{x}+\bar{y}-h_1-h_2} \sigma_{bt1}(y)dy + \int_{\bar{x}+\bar{y}-h_1-h_2}^{\bar{y}} \sigma_{bt2}(y)dy \tag{11}$$

$$y_{bt} = \frac{\int_0^{\bar{x}+\bar{y}-h_1-h_2} \sigma_{bt1}(y)y_{dy} + \int_{\bar{x}+\bar{y}-h_1-h_2}^{\bar{y}} \sigma_{bt2}(y)y_{dx}}{\int_0^{\bar{x}+\bar{y}-h_1-h_2} \sigma_{bt1}(y)dy + \int_{\bar{x}+\bar{y}-h_1-h_2}^{\bar{y}} \sigma_{bt2}(y)dy}$$

When the function «M_{bt} - $\overline{\varepsilon_{bt}}$» peaks

$$\left(\frac{dM_{bt}}{d\varepsilon_{bt}} = 0\right) \tag{12}$$

The next step is performed the transformation of deformation diagrams of concrete layers, depending on the strain gradient (tension) and the effect of pretension. For this purpose, the tension diagram in concrete is used because of the pretension force, determined previously. Depending on the level and mark of the initial (pretension) and re-sign (from the external load) tension in the concrete, each concrete fiber ratios γ_{Rb} and $\gamma_{\varepsilon b}$ are determined in accordance with the established methodology [1-3].

Again, we solve the system of equations (5) ... (9), together with equations describing the connection "$\sigma_s - \varepsilon_s$" subject to transformable diagram "$\sigma_b - \varepsilon_b$".

Upon receipt of the new tension diagram in concrete, diagrams re-transformation "$\sigma_b - \varepsilon_b$" of all concrete layers is made and the original system of equations is solved again, the moments M_{bt} are defined and cracks formation is checked.

The calculation is repeated to achieve the desired convergence and then a new value is given by adding the increment efforts ΔN_K.

At each stage of the calculation, condition of the compatibility of the system is checked. If equations are incompatible (have no solutions), it means that the limit value $N_K = N_u$ is reached without the formation of cracks in the element.

The calculation of elements with cracks in the tension zone begins with the definition of an external force N_K. The first value of power takes more effort in cracking. Then the height of the crack is determined, where the height of the stretched zone and the deformation of the extreme fiber tensile zone $\overline{\varepsilon_{bt}}$ are taken as a result of the calculation of the normal section assuming no cracking when subjected to forces N_K.

The obtained value h_{crc} is put in statics equations. Then the condition is being checked,

$$h_{crc} < h_3 \tag{13}$$

If the condition is satisfied, the equation takes the form of static:

$$\int_{\bar{x}+\bar{y}-h_3-h_2}^{\bar{x}} \sigma_{b1}(x)dx + b\int_{\bar{x}}^{\bar{x}-h_1} \sigma_{b2}(x)dx + A_s^{'}\sigma_s^{'} - b\int_{\bar{x}+\bar{y}-h_1-h_2}^{\bar{y}-h_{crc}} \sigma_{bt3}(y)dy - b\int_0^{\bar{x}+\bar{y}-h_1-h_2} \sigma_{bt2}(y)dy - A_s\sigma_s = 0 \tag{14}$$

$$M = b\int_{\bar{x}+\bar{y}-h_3-h_2}^{\bar{x}} \sigma_{b1}(x)(h-h_3-h_2+x)dx + b\int_{\bar{x}}^{\bar{x}-h_1} \sigma_{b2}(x)(h-\bar{x}+x)dx + b\int_{\bar{x}+\bar{y}-h_1-h_2}^{\bar{y}-h_{crc}} \sigma_{bt3}(y)\,(\bar{y}-y)dy + b\int_0^{\bar{x}+\bar{y}-h_1-h_2} \sigma_{bt2}(y)(\bar{y}-h_3+y)dy + A_s\sigma_s\alpha - A_s^{'}\sigma_s^{'}(h-\alpha^{'}) = 0 \tag{15}$$

If the static condition is not met, then:

$$b\int_{\bar{x}+\bar{y}-h_3-h_2}^{\bar{x}} \sigma_{b1}(x)dx + b\int_{\bar{x}}^{\bar{x}-h_1} \sigma_{b2}(x)dx + A_s^{'}\sigma_s^{'} - b\int_0^{\bar{y}-h_{crc}} \sigma_{bt2}(y)dy - A_s\sigma_s = 0 \tag{16}$$

$$M = b\int_{\bar{x}+\bar{y}-h_3-h_2}^{\bar{x}} \sigma_{b1}(x)(h-h_3-h_2+x)dx + b\int_{\bar{x}}^{\bar{x}-h_1} \sigma_{b2}(x)(h-\bar{x}+x)dx + b\int_0^{\bar{y}-h_{crc}} \sigma_{bt2}(y)\,(\bar{y}-y)dy + A_s\sigma_s\alpha - A_s^{'}\sigma_s^{'}(h-\alpha^{'}) = 0 \tag{17}$$

As a result, system solutions with the original diagrams "$\sigma_{b(bt)} - \varepsilon_{b(bt)}$" ($K_R = 1$, $K_{\varepsilon b} = 1$, $\gamma_{Rb} = 1$, $\gamma_{\varepsilon b} = 1$) will get new values $\overline{x}$,$\overline{\varepsilon_{bt}}$, ε_b. The iterative process is terminated in case of convergence condition $\left(h_{crc,i} - h_{crc,(i-1)}\right)/h_{crc,i} \ll \alpha$,
where α - given criterion calculation accuracy.

Calculating each iteration, the condition of compatibility of the system should be checked. If the system is inconsistent, then the solution of the system is not available, the element is considered to be destroyed.

After that, the transformation of concrete deformation diagram is performed, depending on the deformation gradient (tension) and the effect of pretension. Again, we solve the system of equations (13) ... (14) or (15) ... (16) with the constraint equation "$\sigma_s - \varepsilon_s$", we get new values $\overline{x}$, $\overline{y}$,$\overline{\varepsilon_b}$,$\overline{\varepsilon_{bt}}$.

After reaching the desired convergence, the iterative process is finished, the final tension diagram in concrete and the height of cracks h_{crc} are defined.

Further, the strength N_K is assigned a new value and the calculation is repeated, as long as the system of static equations would be incompatible, that is, it will have no solution.

4 Conclusions

Thus, the proposed method of calculation helps to determine the tension-deformation state of the normal sections of reinforced concrete elements at any stage of the work.

Equations of equilibrium of internal and external forces in determining the effort of cracking and breaking force have an identical view, which provides a common approach to the assessment of the strength and fracture structures. This approach also makes it possible to determine the arching columns and deflections at all stages of the work.

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