Mathematical conformity problems in the dynamic modeling of seismic stability structures

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Abstract. The article discusses the problems of field studies and dynamic stability of construction objects at design stage. It highlights the analysis results of various mathematical model approaches for seismic stability of construction structures. The mathematical aspects of their dynamic properties are represented as amplitude-frequency responses and transfer functions. The results of the study as well as the comparative analysis of the mathematical conformity of modeling dynamic properties of a construction object by the Fourier method and the Laplace method are represented. The article considers the uncertainty in the estimates of the dynamic coefficient of structures when using amplitude-frequency response of the object. Furthermore, it is identified that there is a dependence between this coefficient and the instantaneous spectrum of seismic impact when using the Fourier method. The article gives the results of the studies on causes of virtual dependence of the amplitude-frequency response and on the dynamic coefficient from the Fourier transform basis function. The Fourier transform basis function is proved to fall a long way short of the field data and represents itself some abstract description of seismic loads that do not exist in the real nature. The diagrams are given in the article, that proves the comparative estimates of the mathematical conformity and metrological validity of the construction's dynamic models when using the Fourier method and the Laplace method.od.

1 The conformity problem in dynamic simulation

In the last decade the world has witnessed a global increase in seismic activity [1-3]. Reputable scientists from leading international geophysical organizations unanimously predict an increase of seismic threat to the densely populated areas of the planet [4-7].

However, the objective for field researchers and designers (theoretical) to ensure the dynamic stability of construction projects remains one of the most challenging tasks of structural mechanics.

Nowadays digital technologies are widely used for computer simulation purposes, including software systems based on the finite element method (FEM). The questionable validity of this modeling method has already been subjected to strict metrological analysis and mathematical assessment earlier in the authors articles [8-10].

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Along with this method, one of the main tools of modeling design and seismic stability field testing of structures is their dynamic properties represented in the form of amplitude-frequency response (FR). The theoretical or experimental method of obtaining the frequency response is nothing more than the Fourier transform of dynamic model of the construction object. The frequency response generally is the original characteristic for further dynamic coefficient of the construction calculation.

However, in recent years a number of studies of the digital technologies metrological reliability (including the finite element method) confirmed the significant difficulties in the computer simulation of buildings and structures dynamic characteristics.

In addition, there has been a number of cases of the mathematical conformity violation in the algorithms, that has led to unacceptable uncertainties in the simulation results. A comparative analysis of the amplitude frequency characteristic (FR) of the construction object, obtained by the Fourier transform, and transfer functions obtained by Laplace transformation, is to be carried out in order to concretize the consideration of this problem. A comparative analysis of FR and the Laplace representation of a dynamic object model is workable in the main resonance vicinity.

In order to remove uncertainties in the mathematical term of conformity we compare two chains of successive operations with the use of Fourier and Laplace and some function $\boldsymbol{a}(\boldsymbol{t})$. In technical interpretation it would look like the following:

$$\boldsymbol{a}(\boldsymbol{t}) \Rightarrow \mathbf{F}\left[\boldsymbol{a}(\boldsymbol{t})\right] = \boldsymbol{A}(\mathrm{j}\omega) \Rightarrow \mathbf{F}^{-1}\left[\boldsymbol{A}(\mathrm{j}\omega)\right] \neq \boldsymbol{a}(\boldsymbol{t}) \text{ or } \mathbf{F}^{-1}\left[\boldsymbol{A}(\mathrm{j}\omega)\right] \approx \boldsymbol{a}(\boldsymbol{t})$$

$$\boldsymbol{a}(\boldsymbol{t}) \Rightarrow \mathbf{L}\left[\boldsymbol{a}(\boldsymbol{t})\right] = \boldsymbol{A}(\mathrm{s}) \Rightarrow \mathbf{L}^{-1}\left[\boldsymbol{A}(\mathrm{s})\right] \equiv \boldsymbol{a}(\boldsymbol{t}), \text{ where } \mathrm{s} = \mathrm{j}\omega + \mathrm{c},\ \omega = 2\boldsymbol{\pi}\boldsymbol{f}.$$

In other words, the Fourier transform does not give one-to-one correspondence between the results of direct and inverse Fourier transform in the General case.

On the contrary the Laplace transform provides such compliance.

2 Dynamic characteristics simulation by the Fourier method

In the principal resonance vicinity the object dynamic properties can be represented in the form of a mechanical oscillatory system with one resonance, for example, in the spring pendulum form, which has mass *M*, damping coefficient ξ and spring stiffness *k*, which obeys the elastic deformation equation under static loads

$$\boldsymbol{F}_{\text{load}} = \boldsymbol{kx}\,. \tag{1}$$

The free vibrations equation of the system will be as follows:

$$\ddot{x} + 2\xi\dot{x} + \frac{k}{M}x = 0 \tag{2}$$

where $\frac{k}{M} = \omega_{\text{rez}}^2$, $\omega_{\text{rez}} = 2\pi f_{\text{rez}}$, $f_{\text{rez}} = \frac{\omega_{\text{rez}}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$.

Amplitude-phase frequency characteristic (APFC) of an oscillatory mechanical system with one resonance can be represented by a complex function:

$$K(jf) = \operatorname{Re} K(f) + \operatorname{Im} K(f) = P(f) + jQ(f)\,, \quad (3)$$

where

$$P(f) = \frac{k\left(1 - \dfrac{f^2}{f_{\text{rez}}^2}\right)}{\left(1 - \dfrac{f^2}{f_{\text{rez}}^2}\right)^2 + \left(2\xi \dfrac{f}{f_{\text{rez}}^2}\right)^2}\,, \quad (4)$$

$$Q(f) = -\frac{2k\xi \dfrac{f}{f_{\text{rez}}}}{\left(1 - \dfrac{f^2}{f_{\text{rez}}^2}\right)^2 + \left(2\xi \dfrac{f}{f_{\text{rez}}}\right)^2}\,. \quad (5)$$

If $2\xi \geq 1 = 2\xi_{\text{crit}}$ then oscillatory mechanical system (oscillating link) degenerates into two aperiodic links. Then, under a sharp blow on the system, there will not be fluctuations in the system and the mechanical oscillating system will gradually return to its original state.

Using the expression (3), (4) and (5), the amplitude frequency characteristic (FR) of an oscillatory mechanical system can be represented as:

$$|K(jf)| = \frac{k}{\sqrt{\left(1 - \dfrac{f^2}{f_{рез}^2}\right)^2 + \left(2\xi \dfrac{f}{f_{рез}}\right)^2}}\,. \quad (6)$$

Phase frequency characteristic (PFR) of oscillatory link

$$\varphi(f) = arctg \frac{\operatorname{Im} K(f)}{\operatorname{Re} K(f)} = arctg \frac{Q(f)}{P(f)},$$

$$\varphi(f) = \begin{cases} -arctg \dfrac{2\xi \dfrac{f}{f_{\text{rez}}}}{\left(1 - \dfrac{f^2}{f_{\text{rez}}^2}\right)} \ \text{при} \ f \leq f_{\text{rez}} \\ -\pi - arctg \dfrac{2\xi \dfrac{f}{f_{\text{rez}}}}{\left(1 - \dfrac{f^2}{f_{\text{rez}}^2}\right)} \ при \ f > f_{\text{rez}} \end{cases}. \quad (7)$$

The magnitude of the reverse attenuation factor $\tau = \dfrac{1}{\xi}$ is called the attenuation constant, which is equal to the decay time of free resonant oscillations, during which amplitude of oscillations decreases in e times. In practice, the normalized (over the resonance period, T_{rez}) damping factor often used instead of the attenuation coefficient ξ, which is called the damping ratio and equals

$$\alpha = \frac{\xi}{T_{\text{rez}}}.$$

In the buildings and structures dynamic characteristics passports it is required to specify the experimentally derived value of the attenuation logarithmic decrement, which is equal to

$$d = \xi T_{\text{rez}} = \frac{T_{\text{rez}}}{\tau}. \quad (8)$$

Reciprocal of damping logarithmic decrement

$$\frac{1}{d} = \frac{\tau}{T_{\text{rez}}},$$

equals to the number of the mechanical oscillatory system oscillation cycles for a time equal to constant attenuation.

A normalized coordinate system is usually used to plot FR. For example, in the modulus |K| dependences form of the normalized frequency

$$\bar{f} = \frac{f}{f_{\text{rez}}}. \quad (9)$$

Plotting FR in the normalized coordinate system allows to conveniently perform a graphical calculation of the oscillatory systems mechanical damping coefficients by comparing with the nomogram of the estimated FR family with different damping factors (see Fig.1).

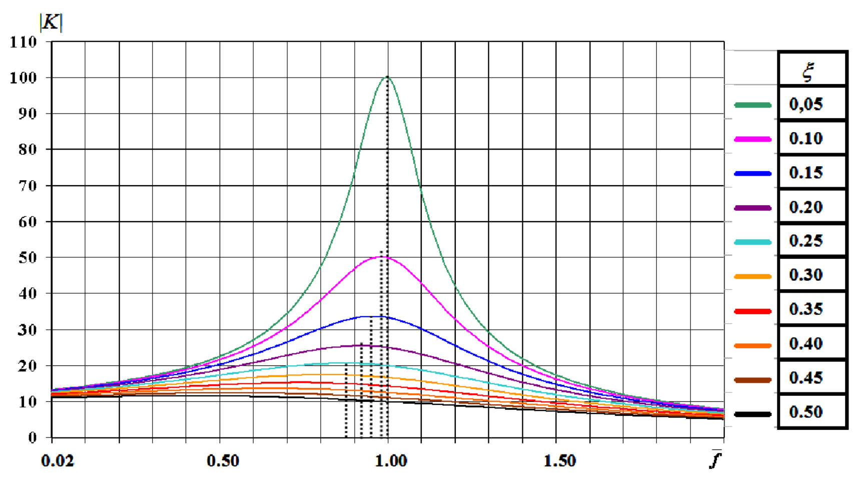

Fig.1. Nomograph family of the mechanical vibrating system frequency characteristics in the vicinity of the main resonance for 10 values of damping coefficient

For this experimental FR and PFR are built in the same scale of frequencies as standard nomograms. The nomogram Fig.1 shows a FR family in the main resonance vicinity of the mechanical oscillation link (k=10, $f_{\text{rez}} = 50$) and with different values of damping factors and with a normalized frequency scale.

A FR family nomogram in Fig.1 is a classic form of the vibrational link resonance which frequency decreases with increasing damping ratio, as noted in the figure with the vertical dashed lines.

3 Modeling of the object transfer function with using of the Laplace transform

Let's consider the dynamic model of the mechanical oscillating system, which is built in the vicinity of the main resonance with using of the Laplace transform.

The dynamic model of this system represents the transfer function of the vibrational link of the second order

$$W(s) = \frac{k}{T_{rez}^2 s^2 + 2\xi T_{rez} s + 1}, \tag{10}$$

where $s = jf + c$, $T_{rez} = 1/f_{rez}$.

The modules family of the vibrational link transfer functions has the same FR. Only an integrated variable $s = jf + c$ is used instead of an imaginary variable jf :

$$|W(s)| = \sqrt{P(s)^2 + Q(s)^2}\,. \tag{11}$$

The physical meaning of constant c is illustrated in Fig.2, which shows that the constant c is the decay time constant reciprocal value τ of the Laplace transform basic element in the form of a damped cosine wave.

The curves family in Fig.2 can also be interpreted as different types of shock loads. These loads differ from each other by the speed drop (attenuation) in the amplitude of the shock, which is characterized as the time constant of the trailing edge shock loads.

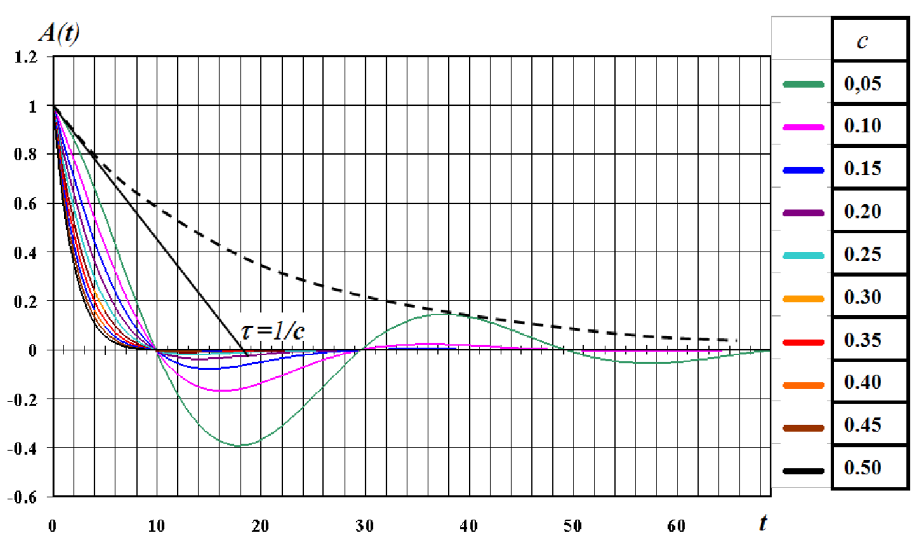

Fig.2. The basis element family of the Laplace transform with different values of the constant c

As an example let us build (see Fig.3) the transfer functions modules family in the resonance frequency vicinity for different values of constant *c* and for *k*=10 (as in the previous case). The transfer functions modules family for different values of the constant *c,* which is shown in Fig.3, has a high degree of similarity to the frequency response nomograms in Fig.1.

Despite the apparent similarity the FR graphs in Fig.1 are fundamentally different from ones of the transfer function in Fig.3 with the fact that in the first case the only parameter of the graph is the attenuation factor and the second two parameters: the attenuation coefficient and the constant *c* or the type of impact (Fig.2). In this case the difference between the impact types is at various speeds or time-constant of the trailing edge shock decay τ .

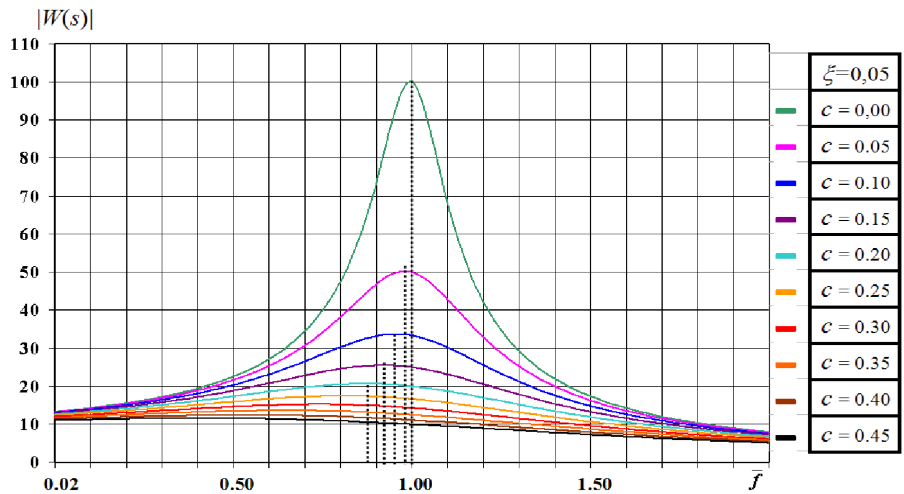

Fig.3. Nomogram of the transfer functions modules family for different values of the constant *c*

4 Conclusion

Considering the problem of the mathematical conformity of dynamic modelling, it is arguable that the FR models do not reflect the dynamic properties of structures correctly. It should also be noted that a significant difference of amplitude-phase frequency characteristics (6) from the object transfer function (10) is the mathematical conformity lack of first. This means that using dynamic Fourier model of the object (3) and module (6) is impossible to obtain a one-to-one correspondence between the amplitude-time response and phase frequency characteristics (7).

Based on the above, we can draw the following general conclusions.

While solving the project evaluation problem of the impact resistance and dynamic characteristics of the object construction security, a problem of correct use of project modeling methods with Fourier transform application arises.

To date, the Laplace transform is the only known method with mathematical conformity of the object original dynamic response in the amplitude-time domain and displaying it in the spectral domain. However, despite all the requirements, including authors, comments to GOST 31937-2011 "Buildings and structures. Rules of examination and monitoring of technical condition" and its requirements for verification of the frequency and the logarithmic damping decrement are valid and correct, as they allow to objectively assess the maximum gain of the oscillation amplitude at the resonant frequencies, and, consequently, to estimate the dynamic factor of the building structure and the system of type "object-based" and their resistance to dynamic loads.

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