MATEC Web of Conferences **106**, 02012 (2017) DOI: 10.1051/matecconf/201710602012
SPbWOSCE-2016

To the problem 6 of emplacement of triangular geometric net on the sphere

Vladimir Travush[1], *Vasilij* Antoshkin[2,*], *Irina* Erofeyeva[2]

[1] RAABS, BolshayaDmitrovka, 24,Moscow,107031, Russia
[2]Mordovian State University, Bolshevistskaya, 68, Saransk,430005, Russia

Abstract. Thesphere creates the minimal surface of enclosing structures and has unique resource saving qualities which makes it indispensable in the construction of "smart buildings». One of the methods of formation of triangular networks in the sphere was investigated. Conditions of the problem of locating a triangular network in the area were established. The evaluation criterion of solution effectiveness of the problem is the minimum number of type-sizes of dome panels, the possibility of pre-assembly and pre-stressing. The solution of the problem of the triangular network emplacement in a compatible spherical triangle on the sphere variant was provided. The problem of the emplacement of regular and irregular hexagons on the sphere, inscribed in a circles, i.e., flat figures or composed ones of spherical triangles with minimum dimensions of the ribs, has an effective solution in the form of a network, formed on the basis of minimum radii circles, i.e., circles on a sphere obtained by the touch of three adjacent circles whose centers are at the shortest distance from each other. The optimization of triangular geometric network on a sphere on the criterion of minimum sizes of elements can be solved by emplacementin the system the irregular hexagons inscribed in circles of minimal sizes, the maximum of regular hexagons.

1 Introduction

The sphere creates the minimal surface of enclosing structures and has unique resource saving qualities, which makes it indispensable in the construction of «smart buildings». The problem of the emplacement of regular and irregular hexagons on the sphere, inscribed in a circles, i.e., flat figures or composed ones of spherical triangles (See Figure 1) with minimum dimensions of the ribs, has an effective solution in the form of a network, formed on the basis of minimum radii circles, i.e., circles on a sphere obtained by the touch of three adjacent circles whose centers are at the shortest distance from each other. [1-3, 5-8].However, sometimes there is a possibility of emplacement of two evenly alternating rows of regular hexagons, starting from the equator (Figure 1).

The emplacement of the regular hexagons inscribed in circles, is being realized on the example of cutting, shown in figure 1, and its fragment, shown in Figure 2. To solve this problem (Figure 2), the spherical right triangle O_1KO_2 and spherical triangles O_1GK and

*Corresponding author: antovd@mail.ru

O_2GK with the parties-arcs, are equal, respectively, to the radii $r_1$andr_2 need to be considered. There is a rectangular spherical triangle with internal corners of 90 and 60 degrees and a leg arch, equal a. It is required to determine sizes of radii ρ_1 and ρ_2 between the O_{11}, O_{12} and O_2 centers of circles and, thus, position of all centers of circles of the first two rows of hexagons will be defined.

To determine the centers of regular hexagons of unequal radii of the first two rows, some additional requirements for the location of their circles are entered. At first, substituting hat the circles cross each other at two points and then they, respectively, might be able to fit regular hexagons. Secondly, it is not considered the influence of the first condition of the further complication of builds.

2 To the problem 6 of emplacement of triangular geometric net on the sphere

Previously the parameters in this spherical triangle of O_1KO_2 were determined. It is given that the interior angles of a triangle$A = 30^0, B = 60°$, $O_1G = r_1$, $GO_2 = r_2$, $O_1K =$ $= b = b_1 + b_2$, where r_1, and r_2- are the radii of the circles describing the hexagons of the first and second series in polar angle; b_1 and b_2- are the legs of the rectangular spherical triangles in the form of polar angles; *a* and *2c*– side hexagons in the form of polar angles; *A*, *B*, and *C* and *D*– respectively, interior angles on the sphere for spherical hexagons, inscribed in circumferences of the first series.

Using the known expressions Napier [4,14] for the sides and corners of a rectangular spherical triangles O_1GL and KGL, it's gotten

$$C = 90° - D,$$

$$\sin 30^0 \sin r_2 = \sin c, \qquad (1)$$

$$\cos C = \frac{\text{tg c}}{\text{tg}\, r_2}$$

$$\sin 2c \sin D = \sin b,$$

$$\sin 30^0 \sin r_1 = \sin b. \qquad (2)$$

From triangles O_1GK andO_2GK the magnitude of b and the desired radii r_1 andr_2 will be found

$$\cos r_1 \cos \text{b} + \sin r_1 \sin \text{b} \cos 30^0 = \cos^2 r_2 + \sin^2 r_2 \cos 60^0. \qquad (3)$$

Do the changing of system of equations using the formulas (1, 2) of spherical trigonometry [14],

$$0.5 \sin r_2 = \sin c,$$

$$\sin D = \frac{\text{tgc}}{\text{tg}r_2};$$

$$\sin 2c \frac{\text{tg c}}{\text{tg}r_2} = \sin 30^0 \sin r_1;$$

DOI: 10.1051/matecconf/201710602012

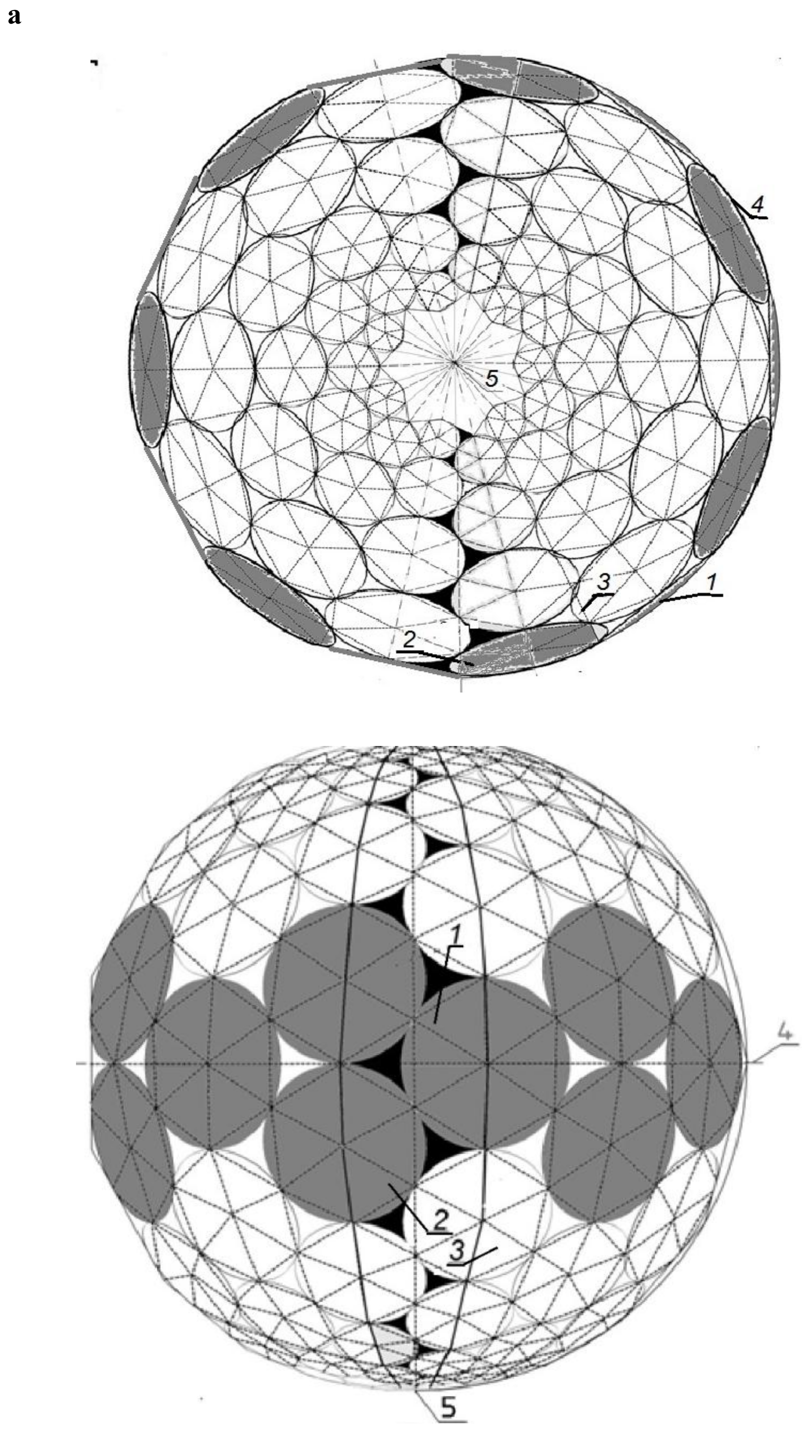

Fig.1. Geometricnetworkonasphereofhexagons, inscribed in circumferences, based on compatibles phericaltriangles(segments) b-90-90°: a – side view; b – top view; 1 – hexagons(regular hexagons gray) of the first and second number of internal 90° degree angles; 3 – too third row, respectively; 4 – the line of the equator; 5 – point of zenith.

From where it's gotten

$$2\text{tg c} \sin 2c = \sin r_1 \text{ tg } r_2 ;$$

$$4 \sin^2 \text{c} = \sin r_1 \text{ tg } r_2 ;$$

$$\text{ctg } r_2 \sin^2 r_2 = \sin r_1 ;$$

$$\sin r_2 \cos r_2 = \sin r_1 ; \tag{4}$$

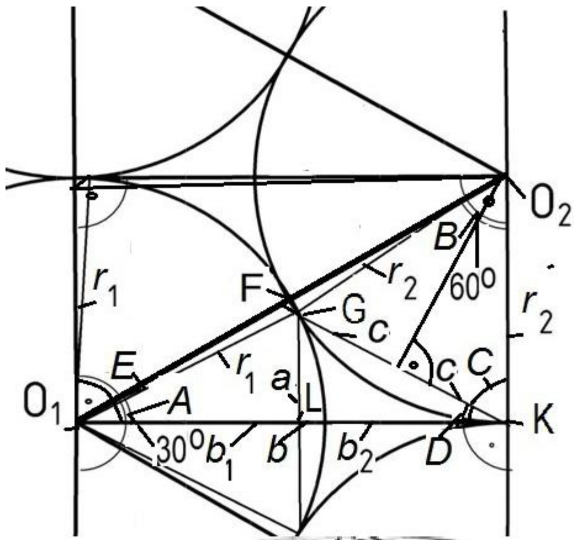

Fig.2. Determination of the position of the center O_2 of the hexagons of unequal radii in the spherical triangle b-90-90^0.

The ratio (4)will be put in the first equation (3)

$$\sqrt{1- \sin^2 r_2 \cos^2 r_2} \cos \text{b} + \sin r_2 \cos r_2 \sin \text{b} \cos 30^0 =$$
$$= \cos^2 r_2 + \cos 60^0 \sin^2 r_2 ;$$

$$\cos \text{b} \sqrt{1 - \frac{1}{1+\text{tg}^2 r_2} \frac{1}{1+\text{ctg}^2 r_2}} + \sin \text{b} \cos 30^0 \sqrt{\frac{1}{1+\text{tg}^2 r_2} \frac{1}{1+\text{ctg}^2 r_2}} = \frac{1}{1+\text{tg}^2 r_2} + \frac{0.5}{1+\text{ctg}^2 r_2} ;$$

$$\cos \text{b} \sqrt{1 - \frac{1}{1+\text{tg}^2 r_2} \frac{\text{tg}^2 r_2}{1+\text{tg}^2 r_2}} + \sqrt{\frac{1}{1+\text{tg}^2 r_2} \frac{\text{tg}^2 r_2}{1+\text{tg}^2 r_2}} \sin \text{b} \cos 30° = \frac{1}{1+\text{tg}^2 r_2} + \frac{0.5\text{tg}^2 r_2}{1+\text{tg}^2 r_2} ;$$

$$\cos \text{b} \sqrt{(1 + \text{tg}^2 r_2)(1 + \text{tg}^2 r_2) - \text{tg}^2 r_2} + \text{sinb} \cos 30^0$$

$$\text{tg } r_2 = 1 + 0.5\text{tg}^2 r_2 ;$$

$$\cos \text{b} \sqrt{(1 + \text{tg}^2 r_2)(1 + \text{tg}^2 r_2) - \text{tg}^2 r_2} =$$
$$= 1 + 0.5 \text{ tg}^2 r_2 - \text{sinb} \cos 30^0 \text{ tg } r_2 ;$$

$$\cos^2 \text{b} ((1 + \text{tg}^2 r_2)(1 + \text{tg}^2 r_2) - \text{tg}^2 r_2) =$$
$$= (1 - \text{sinb} \cos 30^0 \text{ tg } r_2 + 0.5 \text{ tg}^2 r_2)^2 ;$$

Substitute $x = \text{tg}r_2$

$$\cos^2 \text{b} \, (1 + x^2)^2 - x^2) =$$

$$= (1 - \text{sinb} \cos 30^0 \, x + 0.5 \, x^2)^2 ;$$

$$\cos^2 \text{b} \, (1 + 2x^2 + x^4 - x^2) =$$

$$= (1 - 2 \, \text{sinb} \cos 30^0 \, x + \sin^2 \text{b} \cos^2 30^0 \, x^2 +$$

$$+(1 - \text{sinb} \cos 30^0 \, x) \, x^2 + 0.25x^4);$$

$$\cos^2 \text{b} + \cos^2 \text{b} \, x^2 + \cos^2 \text{b} \, x^4 =$$

$$= 1 - 2 \, \text{sinb} \cos 30^0 \, x + \sin^2 \text{b} \cos^2 30^0 \, x^2 +$$

$$+x^2 - \text{sinb} \cos 30^0 \, x^3 + 0.25 \, x^4 ;$$

$$(\cos^2 \text{b} - 0.25)x^4 + \text{sinb} \cos 30^0 \, x^3 + (\cos^2 \text{b} - \sin^2 \text{b} \cos^2 30^0 - 1) \, x^2 + 2 \, \text{sinb} \cos 30^0 \, x + \cos^2 \text{b} - 1 = 0. \quad (5)$$

3 Results

Byaccepting $b = 20^0$, it’s gotten to

$$0.63302222156x^4 + 0.296198132726x^3 - 0.20471111227089x^2 + 0.59239626545x - 0.1169777784405 = 0. \quad (6)$$

Where from the relation

$$\text{tgr}_2 = 0.20582743305806134, \ \text{r}_2 = 11.630615.$$

From triangle O_2GK, expressing the cosine of the angle c

$$\sin c = 0.5 \sin r_2 , \ c = 5.78527776^0 .$$

All values in the expression (3) are determined, excepting the polar angle of the radius r_1 .Thus, the radius of the second circle will be determined from the Equation(4).

$$\sin r_2 \cos r_2 = \sin r_1 .$$

From where it will be received $r_1 = 11.388580819^0$.

To solve the equation (6) the free software Sclab 5.4.1 was used - The free platform for Numerical Computation (analogue Matlab) [15].

From figure 2 it is seen that the optimal triangular network on the sphere is obtained if these are right spherical triangles hexagons inscribed in circles of the smallest radius [1] and placed in the first two rows are compatible spherical triangles sphere *b*-90-90^o.

For a given position of the center of the first rows of hexagons inscribed in a circle, the angles change, and hence, the optimization are possible only for centered hexagons centered O_3, andfurther[8, 9, 16-22].

4 Conclusions

These solutions allow to realise the algorithms of approximation the triangular geometric network with the maximum number of regular hexagons and prepare options for optimization of spherecutting.

References

1. V.D. Antoshkin, V.T. Erofeev, V.I. Travush, V.I. Rimshin, V.L. Kurbatov, Modern Applied Science **3**, 46-50 (2015)
2. V. D. Antoshkin, Regional architecture and engineering **3(24)**, 112-121 (2015)
3. V.D.Antoshkin, S.S. Gudozhnikov, O.I. Perfilieva, I.V. Erofeeva, Proceedings of the Thirteenth International scientific-technical conference **12(2)**, 4-15 (2014)
4. V.D. Antoshkin., V.I. Nikonov, Fundamental research **11-8**, 1669-1673 (2014)
5. V.D. Antoshkin, A.G. Konovalov,Ogarev-Online **13(54)**, 6-9 (2015)
6. V.D.Antoshkin, G.V. Kurbakov, V.S. Bochkin, StredoevropskyVestnik pro Vedu a Vyzkum **83**, 1 (2015)
7. D.V. Alekseevskii, E.B. Vinberg, D.I. Solodovnikov, *Geometry of spaces of constant curvature. Fundamental Directions* (VINITI, 1998)
8. V.D. Antoshkin, V.G. Kurganski,StredoevropskyVestnik pro Vedu a Vyzkum **83**, 2 (2015)
9. V.T. Erofeev, O.V.Startsev, V.D. Antoshkin, S.S. Gudozhnikov, E.G. Smolkin, I.V. Boldin, A.Y. Makhankov, Basic research **9-12**, 2630-2638 (2014)
10. W. Lienhard, Elem. Math**66 (2)**, 74-82 (2011).
11. V.F. Manukhov, A.S. Turchin, *Glossary of geodetic terms: proc. Manual* (PublishingMord. Univ., Saransk, 2005)
12. B.V. Miryaev, Regional architecture and engineering **3**, 122-125 (2012)
13. A.G. Konovalov, V.D.Antoshkin, Proceedings of the Thirteenth International scientific-technical conference **14(2)**, 48-53 (2015)
14. G.A.Korn, T.M.Korn, Mathematical Handbook for Scientists and Engineers, **1151** (2000)
15. Software Scilab 5.4.1 - The free platform for Numerical Computation,
16. S. Çarbaş, M. P. Saka, Asian journal of civil engineering **10(1)**, 97—112 (2009)
17. A. Behzad, M. Hamid, A.Amran, Applied Mechanics and Materials **110-116(39)**, 956—964 (2012)
18. A. Lopez, I. Puente and M. A. Serna, Proceedings of the international symposium, **24-28(6)**, 639-644 (2002)
19. V. I. Travush, V. D. Antoshkin, , I. V. Yerofeyeva, , D. V. Antoshkin, Team spherical shell, **2564545**(2014)
20. V. I. Travush, V. D. Antoshkin, V. T. Erofeyev, S. S. Gudozhnikov, Construction and reconstruction **6(50)**, 36-48 (2013)
21. V. I. Travush, V. D. Antoshkin, V. T. Erofeev, MATEC, **86** (2016)
22. V. I. Travush, V. D. Antoshkin, V. T. Erofeev, MATEC, **86** (2016)