Modelling of saturated soil slopes equilibrium with an account of the liquid phase bearing capacity

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Abstract. The paper presents an original method of solving the problem of uniformly distributed load action on a two-phase elastic half-plane with the use of a kinematic model. The kinematic model (Maltsev L.E.) of two-phase medium is based on two new hypotheses according to which the stress and strain state of the two-phase body is described by a system of linear elliptic equations. These equations differ from the Lame equations of elasticity theory with two terms in each equation. The terms describe the bearing capacity of the liquid phase or a decrease in stress in the solid phase. The finite element method has been chosen as a solution method.

1 Introduction

Research into the stress and strain state (SSS) of a loaded two-phase body with an account of bearing capacity of the two phases is of practical and theoretical interest. In the north of the Tyumen region, the industrial development of oil and gas fields is going on, oil and gas complex facilities, residential villages, roads on water-saturated clay and peaty soils are being constructed. One of the objectives when designing is the assessment of the strained state of water-saturated soil bases in a stable condition independent of time.

The main carrier phase is a solid one. The load-bearing capacity of the liquid phase is understood as the perception of the external load part with condition being stabilized in time. Long-term field studies of the stress and strain state and the consolidation of flooded bases of the facilities for protecting St. Petersburg from floods [1] showed that the residual pressure occurs in the pore fluid starting with one meter from the daylight surface. It grows to a certain depth and can reach 70% of the total stress induced by the load uniformly distributed on the daylight surface.

The mathematical description (the kinematic model) of residual pore pressure in the liquid phase was first made by prof. Maltsev L.E. (1992) for the one-dimensional case [2]. The generalization for a three-dimensional case and further development of the kinematic model [3-6] led us to the following: a mathematical apparatus used in elasticity theory with modifications appropriate for the two-phase medium can be applied for calculating the elastic two-phase half-plane, half-space, etc.

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2 Problem formulation

In the structural mechanics, there arise occasions when the soil mass of different geometric relief such as slopes surrounding built-up area has insufficient strength. Its movements are being observed and there is a danger of a catastrophic collapse of soil slopes under the influence of an external load. Therefore, when constructing such structures it is important to determine the minimum distance d (Figure 1), at which you have to build a facility, a structure in order to avoid collapse.

Let uniformly distributed load of constant intensity q affect a two-phase elastic foundation at a distance b (Figure 1). The foundation is defined as a rectangular trapezoid with an inclination angle θ .

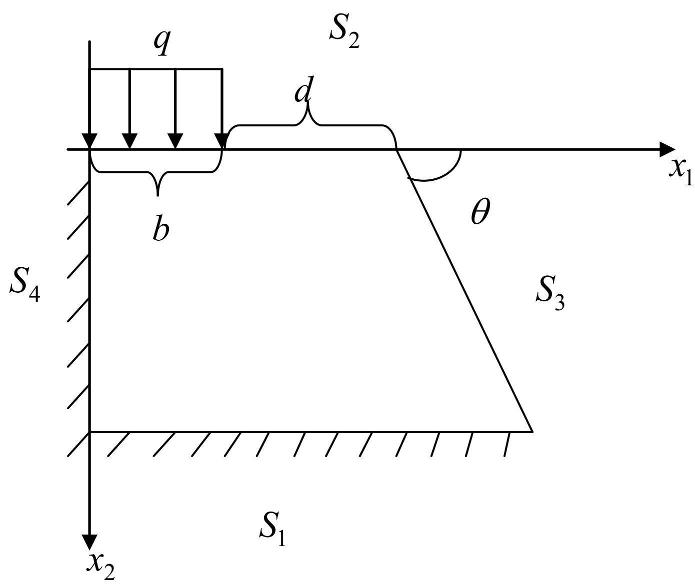

Fig. 1. The problem is axisymmetric with respect to the action of the external load q .

When solving this problem we used a kinematic model of water-saturated ground and made the following assumptions:

1. Instead of a single-phase medium with the presence of nonhomogeneities we consider a two-phase medium, in which each of the phases (solid and liquid) is a continuous homogeneous medium.
2. The movements of the solid (s) and fluid (l) phases are small. Relative modulo deformations do not exceed 0.01.
3. The six hypotheses of elasticity theory are true for the skeleton: those of perfect elasticity, isotropism and homogeneity, linear relationship between stress and strain, continuity, independence of the effect of forces, natural relaxed state of the body.
4. The hypothesis (Maltsev L.E.) of the linear relationship between the partial derivative of the stress in the liquid phase and its relative deformation is introduced for the liquid phase.
5. Another new kinematic hypothesis (Maltsev L.E.) on the interaction of the two phasesis added to the system of hypotheses. It states that during the skeleton compression it pushes a portion of the liquid out of its pores or a part of the volume released by liquid is occupied with the skeleton.

The system of the kinematic model equations describing the SSS of the two-phase half-plane includes: equilibrium Equation 1, Hooke's law with additional Equation 2, which reflect the liquid phase impact on the skeleton, physical Equations 3, for the pore water the Cauchy Equation 4, kinematic Equations 5 and the boundary conditions. The system of equations describes the stress and strain state of the porous body under load, the pores of which are completely filled with liquid, which is hydraulically continuous.

$$-\left(\frac{\partial(\sigma_{sx}-\sigma_{lx})}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}\right)=X,\quad -\left(\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial(\sigma_{sy}-\sigma_{ly})}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}\right)=Y. \tag{1}$$

$$\sigma_{sx}=2G_s\varepsilon_{sx}+\lambda_s\theta_s+\frac{E_{lx}}{\aleph_x^2}\varepsilon_{sx};\qquad \sigma_{sy}=2G_s\varepsilon_{sy}+\lambda_s\theta_s+\frac{E_{ly}}{\aleph_y^2}\varepsilon_{sy};\quad \theta_s=\varepsilon_{sx}+\varepsilon_{sy};\ \tau_{xy}=G_s\gamma_{xy}. \tag{2}$$

$$\lambda_s=\frac{\nu\cdot E_s}{(1+\nu)(1-2\nu)}\ G_S=\frac{E_S}{2(1+\nu)},\quad P_{lx}=E_{lx}\varepsilon_{lx};\quad P_{ly}=E_{ly}\varepsilon_{ly}.\quad P_{lx}=h_x\frac{\partial\sigma_{lx}}{\partial x};\quad P_{ly}=h_y\frac{\partial\sigma_{ly}}{\partial y}. \tag{3}$$

$$\varepsilon_{sx}=\frac{\partial u_s}{\partial x},\quad \varepsilon_{sy}=\frac{\partial v_s}{\partial y},\gamma_{xy}=\frac{\partial u_s}{\partial y}+\frac{\partial v_s}{\partial x},\quad \varepsilon_{lx}=\frac{\partial u_l}{\partial x},\quad \varepsilon_{ly}=\frac{\partial v_l}{\partial y}. \tag{4}$$

$$\varepsilon_{sx}=-\aleph_x\varepsilon_{lx},\qquad \varepsilon_{sy}=-\aleph_y\varepsilon_{ly}. \tag{5}$$

WhereE_{si} , E_{li} – modules of skeleton deformation and the liquid phase determined from the experiment with a biphasic sample or from toughing or field testing, $\aleph_i$, h_i – model parameters, which are also determined experimentally, ν - Poisson's ratio for the skeleton, G_s– shear modulus, λ_s – Lame’s constant for the skeleton.

Boundary conditions:

$$\boldsymbol{u}\,|_{S_1}=0\,,\ \tilde{\boldsymbol{t}}^{(\nu)}\,|_{S_2}=\mathbf{q}(x_1,x_2),$$

$$\tilde{\boldsymbol{t}}^{(\nu)}=\sum_{i,k=1}^{2}\left(\sigma_{ik}+a_i\frac{\partial u_i}{\partial x_i}\right)\cos(\nu,x_k)e_i=\sum_{i,k=1,k\neq i}^{2}\left((2G+a_i)\varepsilon_i+\lambda\varepsilon_{ik}+\sigma_{ik}\right)\cos(\nu,x_k)e_i \tag{6}$$

After transforming the equilibrium equations we obtain the following system of elliptic equations written in terms of the skeleton particles movement:

$$-\left((G_S+\lambda_S)\frac{\partial\theta_S}{\partial x}+G_S\Delta u_S+\frac{E_{lx}}{\aleph_x^2}\frac{\partial^2 u_S}{\partial x^2}+\frac{E_{lx}}{h_x\aleph_x}\frac{\partial u_S}{\partial x}\right)=X, \tag{7}$$

$$-((G_s+\lambda_s)\frac{\partial\theta_s}{\partial y}+G_s\Delta v_s+\frac{E_{ly}}{\aleph_y^2}\frac{\partial^2 v_s}{\partial y^2}+\frac{E_{ly}}{h_y\aleph_y}\frac{\partial v_s}{\partial y})=Y,$$

Which differs from the known Lame equations of elasticity theory with two additional terms in each equation. This result can be considered as a generalization of the theory of elasticity in the two-phase body.

The external load of intensity q is set on the part of the boundary S_2. The vertical and horizontal movements are absent on the boundary S_1. The position of the boundary S_3 is determined by the condition of smallness (up to 5%) of the vertical movements on the border.

3 Research description

In order to solve the problem we used an adapted version of the finite element method described in [7-10]. As finite elements we took triangular finite elements.

A program, which allows solving the set task, was written in Maple 16 Equation Editor.

The calculations were performed on a 40×40 grid. Mechanical properties of loam were taken from laboratory experiments [11]: $E_{sx}=E_{sy}=5{,}2\,МПа$, $E_{lx}=E_{ly}=3{,}27\,МПа$, $\nu=0{,}3$, $=0{,}52$. External load of intensity $q=0{,}077\,МН/м$ affects at a distance $b=7м$ .Different variants of the slope angle are considered ($\theta=\frac{\pi}{4},\frac{\pi}{6}$).Figure 2 shows the graph of horizontal movementsfor the sections X_2=0 m,X_2=2 m,X_2=4 m,X_2=6 m.

Figure3 shows graphs of vertical movements for the symmetry axis, i.e. for X_1=0 m and at a distance of 3 m and 9 m from axisX_2, respectively.

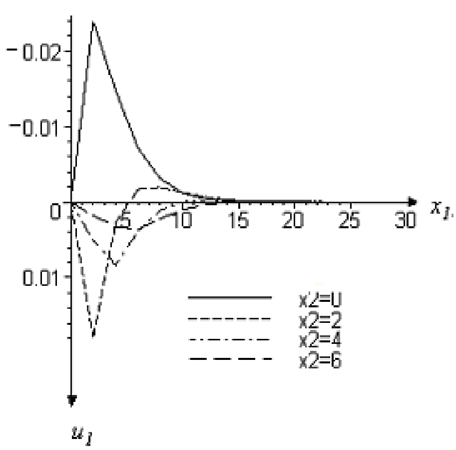

Fig. 2. Horizontal movements for various sections without taking account of the liquid phase ($\theta=\frac{\pi}{4}$).

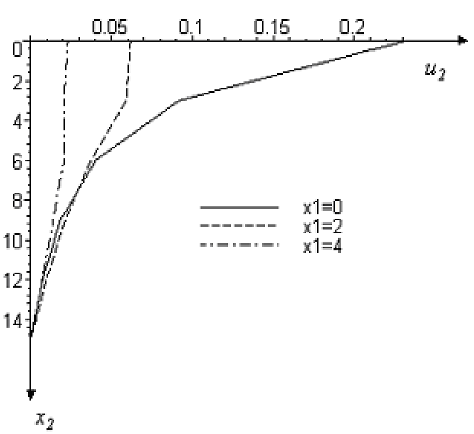

Fig. 3. Vertical movements for various sections without taking account of the liquid phase ($\theta=\frac{\pi}{4}$).

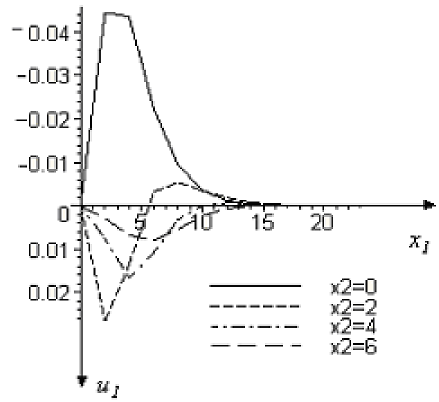

Fig. 4. Horizontal movements for various sections without taking account of the liquid phase ($\theta = \pi/6$).

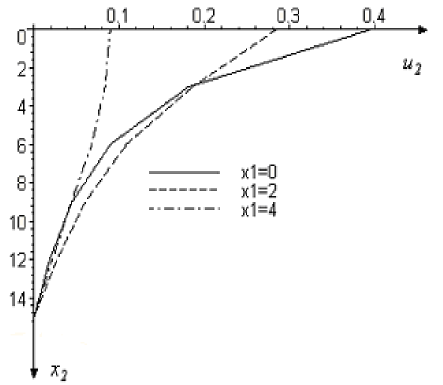

Fig. 5. Vertical movements for various sections without taking account of the liquid phase ($\theta = \pi/6$).

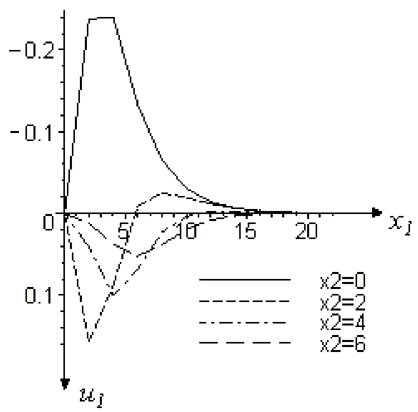

Fig.6. Horizontal movements for various sections without taking account of the liquid phase($\theta = \pi/4$).

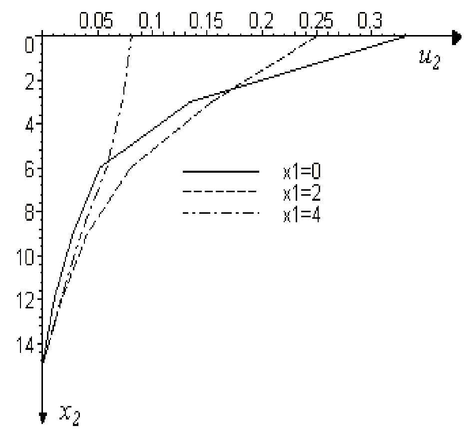

Fig. 7. Vertical movements for various sections without taking account of the liquid phase ($\theta = \pi/4$).

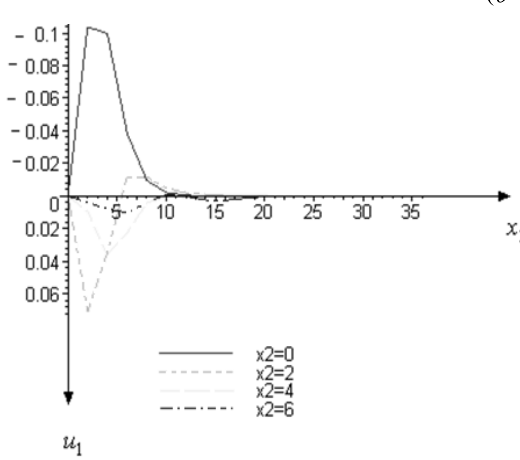

Fig. 8. Horizontal movements for various sections without taking account of the liquid phase $\theta = \pi/6$

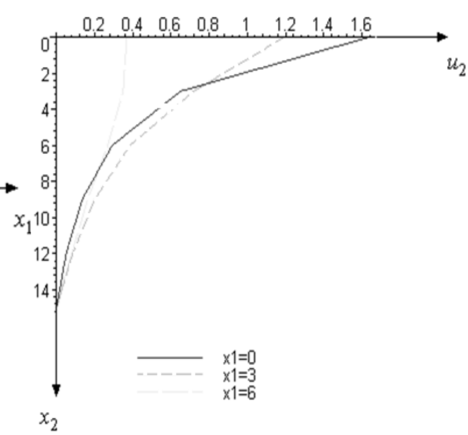

Fig. 9. Vertical movements for various sections without taking account of the liquid phase ($\theta = \pi/6$).

4 Conclusions

From the graphs (Figures 2, 4) one can see that the horizontal movements are practically null ($\theta = \pi / 4$) at a distance $d = 15\,м$ from the axis of symmetry. The distance d decreases to 12 m for $\theta = \pi / 6$.

Vertical movements starting with a depth of 3 m decrease more rapidly with a gentle slope (Figures 3, 5).

In the graphs presented in Figures 6-9, the distance from the axis of symmetry increased from 15 m to 18.5 m, ($\theta = \pi / 4$). The distance d increased from 12 m to 13.5 mfor $\theta = \pi / 6$.

It is clear from the presented research that the pore water takes up a part of the load affecting the foundation.

The results obtained can be used in the calculation of the stress and strain state of saturated grounds, as well as when designing and constructing buildings and structures in the immediate proximity of the slopes.

References

1. A.K.Bugrov, A.V.Golli, A.A.Kagan, S.N.Kuraev, I.A.Pirogov, A.G. Shashkin, Osnovaniya, fundamenty i mekhanika gruntov **1**, 2-9 (1999)
2. L.E.Mal'cev, V.F.Baj, T.V.Mal'ceva, *Kinematicheskaya model' gruntaibiomaterialov* (Strojizdat, Saint Petersburg,2002)
3. T.V.Mal'ceva, E.R.Trefilina, Mathematical modeling **16(11)**, 47 (2004)
4. E.R. Trefilina, *Research into stress-strain state of two-phase viscoelastic half-space* (Tyumen, 2004)
5. T.V. Saltanova, *Mathematical modeling of the excess residual pore pressure by means of finite element method* (Tyumen, 2008)
6. T.V. Maltseva, *Mathematical theory of water-saturated soil* ("Vector Book" Publ. H., Tyumen,2012)
7. T.Maltseva, T.Saltanova, MATEC Web Conf., **73** (2016)
8. T.V. Saltanova, TSU Newsletter**5**, 25- 33 (2007)
9. T.V. Saltanova, *Proceedings of the All-Russian seminar "Grid methods for boundary value problems and applications"* (KSU Publ. H., Kazan, 2007)
10. T.V. Saltanova, Mathematical Methods in Engineering and Technology **5**, 35-39 (2007)
11. V. F. Baj, Proceedings of the higher educational institutions **2**, 98-106 (2002)