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Backstepping Adaptive Fuzzy Control of Servo System With LuGre Friction

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Abstract. Aiming at the control problem of servo system with friction nonlinearity, we introduced the improved LuGre friction model of system. We used adaptive fuzzy system to approximate the nonlinear part and unknown parameters in system online. The purpose is to avoid the complex calculation in deducing the adaptive law of each unknown parameter. By using backstepping approach and recursively selecting the Lyapunov function, introducing the virtual control quantity, we designed an adaptive fuzzy controller with state feedback. Then we analyzed the stability of adaptive fuzzy controller. We carried out the system response analysis and system robustness analysis. We used sinusoidal signal as simulation input signal. Two cases were considered in simulation test comparing backstepping control with conventional PID control. Simulation results show that the proposed backstepping control has better position tracking performance and robustness than conventional PID control.

1 Introduction

In electromechanical servo system, the existence of friction nonlinearity will affect the system dynamic performance and steady-state precision [1]. Hence, it is necessary to study the control strategy of weakening the effect of friction nonlinearity to system. So far, scholars at home and abroad have carried on a large amount of researches in friction modeling. The famous models of friction nonlinearity including the Dahl model [2], the LuGre model[3], the Elasto-plastic model [4], the Leuven model [5] and the GMS model [6]. Among them, the LuGre model is widely used [7] because it is simple and accurate in describing the friction characteristics at low speed. However, when system is running in high speed, the system friction are mainly coulomb friction and viscous friction, if still using the LuGre model, it will inevitably increase the complexity of the system. In addition, due to the changes of temperature, lubrication and contact force, the parameters of friction torque will also change.

Many scholars have studied the compensation control strategies of friction nonlinearity. Y. Wang [8] proposed an effevtive observer-based compensation scheme for the effects of friction. The results show that the amplitudes of limit cycles are greatly reduced with the disturbance observer-based compensation, and the stick-slip motion at low velocities is also eliminated. S. Yang [9] proposed a new repetitive adaptive compensation scheme for the dynamic friction. Computer simulations verify

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the effectiveness of the proposed scheme for the high-precision servo system. In [10], aiming at unknown dynamic friction parameters and unknown load characteristic, an adaptive friction compensation is designed for the servo system. Simulation and experiment results are shown to effectively inhibit the disadvantageous influence of friction and lay the foundation for improvement of dynamic performance of servo system. In [11], a friction torque compensation controller without distal-end feedback is designed to ensure that the output of the tendon-sheath actuation system can effectively track the expected torque trajectory, and the effectiveness of the scheme is validated by experiments. B. Feng [12] proposed a novel adaptive configuration method of friction compensation pulse characteristic parameters. The experiment results show that the proposed method has a better performance. In [13], a feedforward friction observer is proposed as formulating an explicit analytical expression for the applied observation function. The simulation results show that the proportional control combined with the observer compensates faster for frictional disturbances at suddenly changing frictional conditions than PI control. M.T.S. Aung [14] presented a new method of friction compensation that can be applied to geared actuators with high presliding stiffness. The proposed method is validated through experiments employing a harmonic drive transmission. Howerer, [8-14] have not used adaptive fuzzy control.

Adaptive fuzzy logic system proposed by Wang [15] has universal approximation property, which can effectively approximate the nonlinear function of system. In recent years, it has been widely used in the nonlinear control of various servo systems [16-20]. In this paper, an improved LuGre model [21-22] is used to describe the friction torque of system at low speed and high speed. We use an adaptive fuzzy logic system to approximate the nonlinear part of servo system with friction, design a backstepping [23-24] adaptive fuzzy controller under the condition of the unknown system parameters and load torque. Simulation and practical experiments validate the efficacy of the proposed control strategy.

2 System Modeling

If only considering the influence of LuGre friction nonlinear, the structure diagram of servo system is shown in Fig. 1.

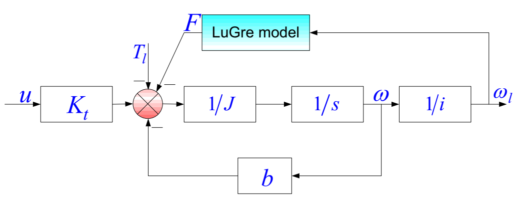


Figure 1. The structure diagram of servo system.

We can get the dynamic equation of servo system is as follows [1].

$$\begin{cases} \dfrac{d\theta}{dt} = \omega \\ J\dfrac{d\omega}{dt} = K_t u - b\omega - F - T_l \\ y = \theta \end{cases} \tag{1}$$

where θ and ω are the position and speed of motor output shaft respectively. J and b are equivalent moment of inertia and equivalent damping coefficient converted to the motor shaft respectively. K_t is motor torque constant. u is control variable. F is load friction torque. T_l is load torque converted to the motor shaft. If the LuGre model is adopted [21], then the load friction torque can be expressed as follows.

$$\frac{dz}{dt} = s(|\omega_l|)(\omega_l - \frac{|\omega_l|}{g(\omega_l)}z) \tag{2}$$

$$F = \sigma_0 s(|\omega_l|)z + \sigma_1 \dot{z} + F_c \operatorname{sgn}(\omega_l)(1 - s(|\omega_l|)) + \alpha_2 \omega_l \tag{3}$$

$$\sigma_0 g(\omega_l) = F_c + (F_s - F_c)e^{-\frac{\omega_l}{\omega_s}} = \alpha_0 + \alpha_1 e^{-\frac{\omega_l}{\omega_s}} \tag{4}$$

Where

$$s(|\omega_l|) = \begin{cases} 1 & |\omega_l| < \omega_1 \\ \dfrac{\omega_2 - |\omega_l|}{\omega_2 - \omega_1} & \omega_1 \leqslant |\omega_l| \leqslant \omega_2 \\ 0 & |\omega_l| > \omega_2 \end{cases} \tag{5}$$

where $\omega_2 > \omega_1 > 0$.

In Eq. (2)-(4), z is the average deformation amount of bristles. ω_l is load speed. σ_0 is the stiffness coefficient of bristles. σ_1 is the damping coefficient of bristles. α_2 is the viscous damping coefficient. F_c is the coulomb friction coefficient. F_s is the static friction coefficient. ω_s is the Stribeck velocity.

When the system is running at low speed, $s(|\omega|) = 1$, $F = \sigma_0 z + \sigma_1 \dot{z} + \alpha_2 \omega$, the friction model is equivalent to the LuGre model. When the system is running at high speed, $s(|\omega|) = 0$, $F = F_c \operatorname{sgn}(\omega) + \alpha_2 \omega$, the friction model is equivalent to a static friction model. In the actual LuGre model, parameters will change due to the influence of temperature, lubrication, material wear and other factors. So we use λ to reflect the influence of parameter change on the friction torque. Then the load friction torque is as follows.

$$F = \lambda(\sigma_0 s(|\omega_l|)z + \sigma_1 \dot{z} + F_c \operatorname{sgn}(\omega_l)(1 - s(|\omega_l|)) + \alpha_2 \omega_l) \tag{6}$$

So the servo system model can be expressed as follows.

$$\begin{cases} \dfrac{d\theta}{dt} = \omega \\ \dfrac{d\omega}{dt} = C_0 u - C_1 \omega - \lambda C_2 \omega - \lambda \varphi z - \lambda C_5 F_c - C_6 T_l \\ y = \theta \end{cases} \tag{7}$$

where,

$C_0 = \frac{K_t}{J}$, $C_1 = \frac{b}{J}$, $C_2 = \frac{\sigma_1 s(|\omega_l|) + \alpha_2}{J}$, $C_3 = \frac{\sigma_0 s(|\omega_l|)}{J}$, $C_4 = \frac{\sigma_0 \sigma_1 s(|\omega_l|)}{J}$, $C_5 = \frac{\operatorname{sgn}(\omega_l)(1 - s(|\omega_l|))}{J}$, $C_6 = \frac{1}{J}$, $\varphi = C_3 - \frac{C_4 |\omega|}{g(\omega_l)}$.

3 Backstepping Adaptive Fuzzy Control

3.1 Fuzzy Approximation System

In order to reduce the influence of the friction nonlinearity and the load uncertainty in servo system, we design controller employing adaptive fuzzy approximation system[19] to compensate them. The purpose is to make system can accurately track the given position signal.

We use the following fuzzy inference rules:

R^j :If x_1 is F_1^j and x_2 is F_2^j …and x_n is F_n^j, then y is G^j $j=1,2,\cdots,M$

where $x=[x_1,x_2,\cdots,x_n]^{\mathrm{T}}\in R^n$ is the input of fuzzy system, $y\in R$ is the output of fuzzy system, F_i^j and G^j are fuzzy sets.

Using the following fuzzy system:

$$y=\frac{\sum_{j=1}^{M}\bar{y}^j(\prod_{i=1}^{n}\mu_{F_i^j}(x_i))}{\sum_{j=1}^{M}\prod_{i=1}^{n}\mu_{F_i^j}(x_i)} \tag{8}$$

where, $\mu_{G^j}(\bar{y}^j)$ is fuzzy membership function, $\bar{y}^j$ is the maximum point of $\mu_{G^j}(\bar{y}^j)$.

Let

$$q_j=\frac{\prod_{i=1}^{n}\mu_{F_i^j}(x_i)}{\sum_{j=1}^{M}(\prod_{i=1}^{n}\mu_{F_i^j}(x_i))} \tag{9}$$

define $\theta=[\bar{y}^1,\bar{y}^2,\cdots,\bar{y}^M]^{\mathrm{T}}$ as the adaptive parameter vector of fuzzy system, $q(x)=[q_1(x),q_2(x),\cdots,q_M(x)]^{\mathrm{T}}$ as fuzzy basis function vector, then Eq. (8) can be expressed as follows.

$$y(x)=\theta^{\mathrm{T}}q(x) \tag{10}$$

3.2 Backstepping Design

In this paper, we designed an adaptive controller so as to make system output y asymptotically stable tracking the expected output y_d, namely $\lim_{t\to\infty}|y-y_d|=0$.

Theorem 1: For any given continuous function $p(x)$ on the compact set $U\in R^n$ and arbitrary $\varepsilon>0$, there exists a function $f(x)$ in the form of (8), such that:

$$\sup_{x\in U}\|f(x)-p(x)\|<\varepsilon \tag{11}$$

Assumption 1: System parameter K_t/J is bounded. There exist d_{m} and d_M, such that

$$0<d_{\mathrm{m}}\leqslant|K_t/J|\leqslant d_M \tag{12}$$

Assumption 2: The first and second derivative of y_d exist and are all bounded.

Step 1 Define output error e_1 as

$$e_1=y-y_d=\theta-y_d \tag{13}$$

Its derivative is

$$\dot{e}_1=\dot{\theta}-\dot{y}_d=\omega-\dot{y}_d=\omega-\omega_d \tag{14}$$

Choose the following control Lyapunov function V_1

$$V_1=\frac{1}{2}e_1^2 \tag{15}$$

Its derivative is

$$\dot{V}_1=e_1\dot{e}_1=e_1(\omega-\dot{y}_d) \tag{16}$$

Let ω be virtual control variable, define α_1 as feedback control input, the value of α_1 is:

$$\alpha_1 = -k_1 e_1 + \dot{y}_d \tag{17}$$

where, $k_1 > 0$ is a design parameter, then Eq.(16) can be expressed as follows.

$$\dot{V}_1 = e_1(\omega - \alpha_1 - k_1 e_1) = -k_1 e_1^2 + e_1(\omega - \alpha_1) \tag{18}$$

Step 2 Define the error between ω and α_1 as

$$e_2 = \omega - \alpha_1 \tag{19}$$

Choose the following augmented control Lyapunov function V_2

$$V_2 = V_1 + \frac{1}{2} e_2^2 \tag{20}$$

Its derivative is

$$\dot{V}_2 = -k_1 e_1^2 + e_2(f^*(x) + g(x)u) \tag{21}$$

where,

$$f^*(x) = e_1 + (-b\omega - F - T_l)/J - \dot{\alpha}_1 \tag{22}$$

$$\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial \alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial \alpha_1}{\partial \dot{y}_d}\ddot{y}_d \tag{23}$$

We can see that $f^*(x)$ is an unknown function. For any given accuracy requirement $\varepsilon > 0$, there exist a fuzzy logic system, such that

$$f^* = \theta^{\mathrm{T}} q(x) + \delta(x), \ |\delta(x)| < \varepsilon \tag{24}$$

where $\delta(x)$ is an approximation error function.
define

$$\beta = \frac{1}{d_m}\|\theta\|^2 \tag{25}$$

Define the estimated value of β as $\hat{\beta}$ which satisfying the following differential equation.

$$\dot{\hat{\beta}} = \frac{r}{2a^2} e_2^2 q^{\mathrm{T}} q - \sigma\hat{\beta} \tag{26}$$

where, $r > 0$, $\sigma > 0$ are design parameters.
We choose control law as follows.

$$u = -(k_2 + 0.5)e_2 - \frac{1}{2a^2} e_2 \hat{\beta} q^{\mathrm{T}} q \tag{27}$$

where, $k_2 > 0$, $a > 0$ are design parameters.
then we can get

$$e_2 f^* = e_2 \theta^{\mathrm{T}} q(x) + e_2 \delta(x) \leqslant \frac{1}{2a^2} e_2^2 q^{\mathrm{T}} q \|\theta\|^2 + \frac{a^2}{2}\frac{\theta^{\mathrm{T}}\theta}{\|\theta\|^2} + \frac{1}{2d_m}\varepsilon^2 \tag{28}$$

$$e_2 g(x) u \leqslant -(k_2 + 0.5) d_m e_2^2 - \frac{d_m}{2a^2} e_2^2 \hat{\beta} q^{\mathrm{T}} q \tag{29}$$

substitute Eqs.(28),(29) into Eqs.(21), we can get

$$\dot{V}_2 \leqslant -k_1 e_1^2 - k_2 d_m e_2^2 + \frac{1}{2}a^2 + \frac{1}{2d_m}\varepsilon^2 + \frac{d_m}{2a^2} e_2^2 q^T q \tilde{\beta} \tag{30}$$

where $\tilde{\beta} = \beta - \hat{\beta}$

3.3 Stability Analysis

We choose the augmented control Lyapunov function as follows.

$$V_3 = V_2 + \frac{d_m}{2r}\tilde{\beta}^2 \tag{31}$$

Its derivative is

$$\dot{V}_3 \leqslant -k_1 e_1^2 - k_2 d_m e_2^2 + \frac{1}{2}a^2 + \frac{1}{2d_m}\varepsilon^2 + \frac{d_m}{2a^2}e_2^2 q^{\mathrm{T}} q\tilde{\beta} - \frac{d_m}{r}\tilde{\beta}\dot{\hat{\beta}} = -k_1 e_1^2 - k_2 d_m e_2^2 + \frac{1}{2}a^2 + \frac{1}{2d_m}\varepsilon^2$$

$$+\frac{d_m}{r}\tilde{\beta}(\frac{r}{2a^2}e_2^2 q^{\mathrm{T}} q - \dot{\hat{\beta}}) = -k_1 e_1^2 - k_2 d_m e_2^2 + \frac{1}{2}a^2 + \frac{1}{2d_m}\varepsilon^2 + \frac{\sigma d_m}{r}\tilde{\beta}\hat{\beta} \tag{32}$$

notice that

$$\tilde{\beta}\hat{\beta} = \tilde{\beta}(\beta - \tilde{\beta}) \leqslant \frac{1}{2}\beta^2 - \frac{1}{2}\tilde{\beta}^2 \tag{33}$$

So Eq. (32) can be expressed as follows.

$$\dot{V}_3 \leqslant -k_1 e_1^2 - k_2 d_m e_2^2 + \frac{1}{2}a^2 + \frac{1}{2d_m}\varepsilon^2 + \frac{\sigma d_m}{2r}\beta^2 - \frac{\sigma d_m}{2r}\tilde{\beta}^2 \leqslant -m_0 V_3 + n_0 \tag{34}$$

where, $m_0 = \min\{2k_1, 2d_m k_2, \sigma\}$, $n_0 = \frac{1}{2}a^2 + \frac{1}{2d_m}\varepsilon^2 + \frac{\sigma d_m}{2r}\beta^2$

Calculating the integral on both sides of eq.(34), we have:

$$V_3(t) \leqslant e^{-m_0 t}V_3(0) + \frac{n_0}{m_0}(1 - e^{-m_0 t}) \tag{35}$$

According to eqs.(31),(35), we have:

$$e_1^2 \leqslant 2V_3(t), \lim_{t\to\infty} e_1(t) \leqslant \sqrt{\frac{2n_0}{m_0}} \tag{36}$$

Because k_1, k_2, r, σ, a are all design parameters, we can appropriately choose them to ensure that n_0/m_0 is small sufficiently, which can ensure that the error e_1 converges to a sufficient small neighborhood of the origin according to the exponential law.

4 Simulation Experiment

The following simulation graphics are the comparison charts of backstepping adaptive fuzzy control (BAFC) and the conventional PID control.

The parameters of servo system model are as follows: $J = 0.8\mathrm{kg\cdot m^2}$, $K_t = 1$, $b = 0.5\mathrm{N\cdot m\cdot s/rad}$, $i = 15$. The parameters of LuGre friction model are as follows: $\sigma_0 = 9500\mathrm{N\cdot m}$, $\sigma_1 = 25\mathrm{N\cdot m}$, $\sigma_2 = 0.3\mathrm{N\cdot m\cdot s/rad}$, $F_c = 0.5\mathrm{N\cdot m}$, $F_s = 0.45\mathrm{N\cdot m}$, $\omega_s = 0.01\mathrm{rad/s}$. The initial state is: $\theta = 0$, $\omega = 0$. The design parameters are: $k_1 = 90$, $k_2 = 80$, $a = 0.3$, $r = 6.2$, $\sigma = 0.6$. The fuzzy membership functions are as follows.

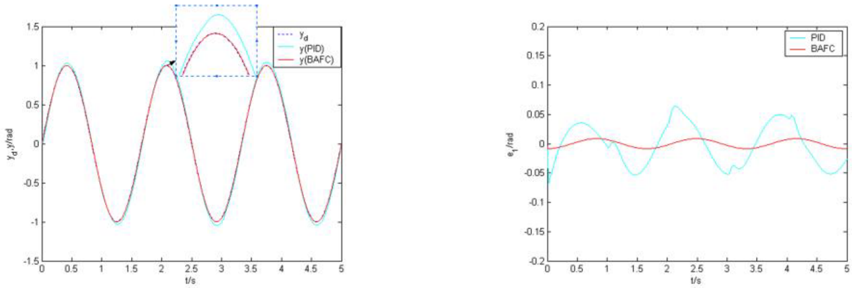
$\mu_{F_i^1}(x_i) = 1/(1+\exp(-6(x_i - 6)))$, $\mu_{F_i^2}(x_i) = \exp(-0.5(x_i - 4)^2/2)$, $\mu_{F_i^3}(x_i) = \exp(-0.5(x_i - 2)^2/2)$

$\mu_{F_i^4}(x_i) = \exp(-0.5(x_i - 0)^2/2)$, $\mu_{F_i^5}(x_i) = \exp(-0.5(x_i + 2)^2/2)$, $\mu_{F_i^6}(x_i) = \exp(-0.5(x_i + 4)^2/2)$

$\mu_{F_i^7}(x_i) = 1/(1+\exp(6(x_i + 6)))$

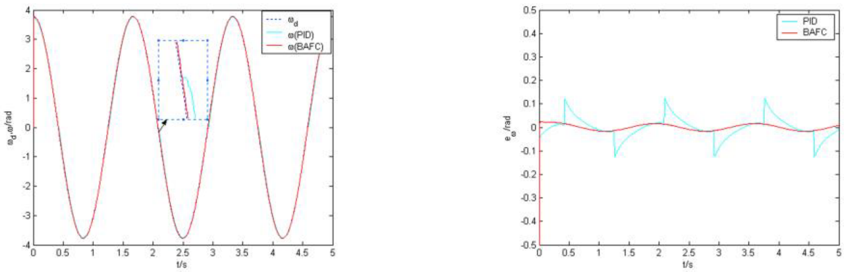
4.1 System Response Analysis

Fig. 2(a) is the system position tracking curve for sinusoidal signal input $y_d = \sin(1.2\pi t)$ by using PID control and BAFC, respectively. Fig. 2(b) is the error curve between y and y_d for sinusoidal signal input. Fig. 3(a) is the system velocity tracking curve for sinusoidal signal input $y_d = \sin(1.2\pi t)$ by using PID control and BAFC, respectively. Fig. 3(b) is the error curve between ω and ω_d for sinusoidal signal input. Fig. 4 is the controller output curve by using PID control and BAFC, respectively. From Fig. 2 and Fig. 3, we can see that position tracking exist flat top phenomenon and velocity tracking exists deadzone phenomenon when using PID control. But there are no flat top phenomenon and deadzone phenomenon exist when using BAFC. So the system has better tracking performance and smaller tracking error when using BAFC.



(a) System position tracking for sinusoidal signal (b) Position tracking error for sinusoidal signal

Figure 2. System position tracking and tracking error for sinusoidal signal input.



(a) System velocity tracking for sinusoidal signal (b) Velocity tracking error for sinusoidal signal

Figure 3. System velocity tracking and tracking error for sinusoidal signal input.

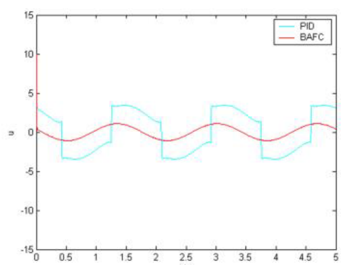
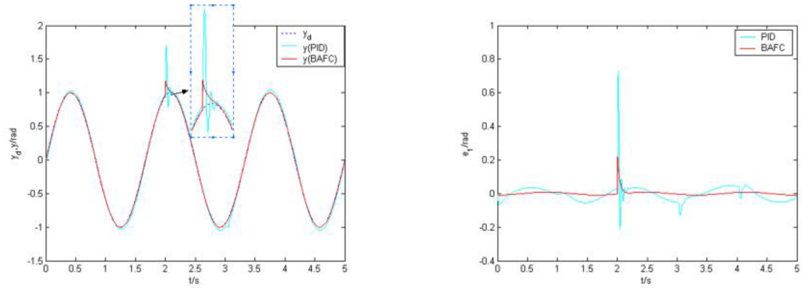


Figure 4. System controller output.

4.2 System Robustness Analysis

In Fig. 5, 10N·m disturbance signal is applied to system at 2s, the system not only has a larger fluctuation at 2s but also the tracking performance become worse after 2s when using PID control. But the system tracking performance don't become worse after 2s, only has a smaller fluctuation at 2s when using BAFC. Thus BAFC has better robustness than PID control.



(a) System sinusoidal position tracking containing disturbance

(b) Sinusoidal position tracking error containing disturbance

Figure 5. System position tracking and tracking error for sinusoidal signal when disturbance exists.

5 Conclusion

In this paper, we designed the improved LuGre friction model-based backstepping adaptive fuzzy control of servo system. The proposed control strategy has the characteristics of higher tracking accuracy, better robustness and adaptability than conventional PID control. But the system also exists backlash nonlinearity, we don't consider the coexistence of backlash and friction nonlinearlity in the system. It will be our future work considering the coexistence of them.

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The authors confirm that this article content has no conflicts of interest.

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