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Measuring coordinates of objects with adaptation expansion options

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Abstract. The parametric optimization of coordinates measurement with variations of definition and frame rate by limiting the readout speed of the signal from the image sensor problem is considered. It is shown that the optimal values of definition and frame rate are determined by the alignment of the inter-element and inter-frame difference variances estimates; switching thresholds can be decreased by the delay introducing into the mechanism of measuring these dispersions to increase the observation time with close values of the estimates of these variances.

1 Introduction

More than half a century spacecrafts (SC) have been docking in orbit with usage of radio and optical-electronic measuring systems (OES) intended to measure the time-varying spatial coordinates of cooperating spacecrafts. To ensure the implementation of the SC convergence method it is necessary to measure the coordinates of the guidance object. An important principle of the coordinate measuring of that object, especially at long range is the principle of the tracking measurement. An important feature of SC convergence and docking control that was introduced by S.P. Korolev is principle iterative coordinate measurement: changing distance between SC leads to changing parameters of measuring system. In last few years, this principle is complemented by the method of adaptation of image decomposition parameters using solid-state matrix image sensors [1-3]. This trend in the construction of measuring systems is connected on the one hand with the change in signal statistics over time (due to changes in distance between the SC), on the other hand - the restriction of signal readout speed from matrix image sensors. Task of finding spatiotemporal coordinates total estimation error minimum refers to isoperimetric problems, where partial errors estimation $\varepsilon_k = D_{\Delta k}$ act as variables, and available reading speed of sensor – as a «perimeter». To solve this type of problem Euler equation with undefined Lagrange multipliers is used. The solution of the equation is similar to the solution of error distribution in the signal spectrum problem (the calculation of the Epsilon-entropy) and leads [1-3] to the concept: for each plot (target-background environment), there is a best video from which is derived the rule of variances equality increment of signal by all the arguments (the rule of "equilibrium" increments). In the simplest case of the image

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observation in a single spectral range this rule corresponds to the requirement of variance equality inter-element (horizontally and vertically) and a frame difference of videosignal: $D_{\Delta x} = D_{\Delta y} = D_{\Delta t}$. Such equation, that supposing measurement procedure of increments dispersions for different coordinates allows minimizing the miss in spatiotemporal coordinates of SC.

2 Discussion

The presence of such a conditional decomposition parameters optimum intuitively clear, because apparent size and speed of the object are small at the long-range, and the ultimate clarity is actual and in the low range size and speed of the object are high, and more relevant parameter is high frame rate.

Thus, the system of coordinates measurement, that implements the alignment by means of definition (number of elements of decomposition) and frame rate *interchange*, includes the measurement subsystem of z statistics (relative to the current ratio of the inter-element and inter-frame variances estimates $z = D_{\Delta x}/D_{\Delta t}$). It is necessary to use traditional methods to ensure the stability of the automatic control system as well as new methods based on non-linearity and persistence in the third closed loop [1-3]. Due to the discreteness of the matrix image sensors a change in image sharpness can only be done discretely which makes its specificity in the synthesis of control systems. When measuring coordinates of the object in each *k*-frame an alternative of TV image quality arises. In the simplest case, the adaptive system to the dynamic plot can have two states:

- $z > \gamma_{high}$ transition to a state with maximum clarity;
- $z < \gamma_{low}$ transition to a state with maximum frame rate;
- $\gamma_{low} \leq z \leq \gamma_{high} \rightarrow$ preservation of the existing state.

The minimum ratio threshold $d = \gamma_{high}/\gamma_{low}$ corresponds to a change intervals relations of spatial and temporal sampling, which is equal to 4 in this case. However, using thresholds $d = \gamma_{high}/\gamma_{low} = 4$ for stability of the adaptive system is not enough, due to the discreteness of the raster, and inevitable presence of noise in the video signal. Measurements show that the monotonous variation of range in this problem gives monotonous variation of *z*-statistics. At the same time in the measurement of fluctuations relative to the expectation of the smoothed function $f(t)$ [3] are identified, that determines the necessary of incrimination of the thresholds ratio $d = \gamma_{high}/\gamma_{low}$, which ensures the stability of automatic control system. If we assume that the observed signal can be modeled by a stochastic process with a normal distribution, including uncorrelated signal and noise components, the *z* statistics becomes a modified Fischer-Snedecor statistics [2]. This distribution $p(z|f)$ for a fixed mathematical expectation f of *z*-statistics for large number of samples (corresponding to the number of elements in the image sensor array, which is $10^5 \ldots 10^7$ typically) is symmetrical. The calculation results show that the analysis in real time of each frame (without delay) for reasonable values of the probability of false alarm (untimely changes in the system state is fraught with immediate reverse change; $p_{fa} \leq 10^{-3}$) thresholds differ significantly from the minimum, which can be obtained from image discreteness and thresholds ratio $d_{1\leftrightarrow 2} = \gamma_{high}/\gamma_{low}$, in the absence of fluctuations is equal to 4, must be increased approximately to 9 [2].

The high function quality of the objects coordinate measuring OES is provided by decomposition parameters adaptation based on the maintenance (approximate, with a certain confidence interval) of video-signal increments dispersions equality by all of the arguments.

Next, the simplest case of a system with two states is considered. In real systems, expectation rate of change *f(t)* of reference *z*-statistics is small, and the deviation of *z* statistics from the mean value *f(t)* are statistically independent in the adjacent frames, whereby it becomes possible to minimize the time when the system is in the "wrong" state (when different arguments signal increment variances differ), or for the simplest embodiment of a system with two states – the possibility of increasing the residence time of the system in a state of ½ <z <2. This problem relates to the optimal filtering problems (non-linear generally), in which the evaluation of the current value of the derivative of the mathematical expectation *f(t)* of *z*-statistics in time *df(t)/dt* are formed. Function *f(t)* can be modeled by a piecewise-linear relationship (in the real docking conditions correcting engine are working, which introduce additive jumps in the variation of *f(t)*). Considering that *f(t)* is a monotonous function, it is possible to obtain an assessment using the optimal Calman filter, introducing a variable time inertia in the estimation process. This version of the problem of obtaining the optimal inertia control system methodologically akin to obtaining the linear filter's optimal inertia in the Wiener-Kolmogorov's filtering theory. This is evident from the fact that, as in the linear filtering, there are two opposite effects: the introduction of inertia reduces the noise error, but increases latency (the analog of the linear error).

Then, considering the discrete form of the measurements formation *f(n),* where $n = 0$, 1, 2 … (n corresponds to the moments of time t_n), state vector $\mathbf{f}(n) = \left[f(n) \quad \dot{f}(n) \right]^{\mathrm{T}}$ the generation algorithm of estimates vector parameters $\widehat{\mathbf{f}}(n)$, is of the form:

$$\widehat{\mathbf{f}}(n) = \mathbf{\Phi}(n)\widehat{\mathbf{f}}(n-1) + \mathbf{K}(n)\Delta f(n), \quad \widehat{\mathbf{f}}(0) = \widehat{\mathbf{f}}_0, \tag{1}$$
$$\Delta f(n) = f(n) - \mathbf{C}\Phi(n)\widehat{\mathbf{f}}(n-1).$$

where $\mathbf{\Phi}(n)$ – 2×1 extrapolation matrix; $\mathbf{K}(n)$ weight matrix; $\Delta f(n)$ – fault; $\mathbf{C} = [1\ 0]$ – 1×2 observation row matrix, showing that only one value $f(t)$ is measured.

Based on analysis and equations for Kalman's filter weight matrix K(n) transformation the new equations can be obtained:

$$\left.\begin{aligned} \mathbf{K}(n) &= \mathbf{P}_{\mathrm{e}}(n)\mathbf{C}^{\mathrm{T}}[\mathbf{C}\mathbf{P}_{\mathrm{e}}(n)\mathbf{C}^{\mathrm{T}} + R(n)]^{-1}, \mathbf{P}_{\mathrm{e}}(1) = \mathbf{P}_1; \\ \mathbf{P}(n) &= \mathbf{P}_{\mathrm{e}}(n) - \mathbf{K}(n)\mathbf{C}\mathbf{P}_{\mathrm{e}}(n); \\ \mathbf{P}_{\mathrm{e}}(n+1) &= \mathbf{K}(n)\mathbf{K}^{\mathrm{T}}(n)\overline{(\Delta f(n))^2} + \mathbf{P}(n). \end{aligned}\right\} \tag{2}$$

Here $\mathbf{P}_{\mathrm{e}}(n)$ and $\mathbf{P}(n)$ – 2×2 extrapolation and estimation errors covariance matrixes respectively; $R(n)$ – measurement error covariance matrix, which is scalar at indicated above observation matrix **C**; «T» − transposition sign. The value $\overline{(\Delta f(n))^2}$ should be determined statistically over several frames.

Estimation algorithm for parameters of vector $\mathbf{f}(n)$ by the expressions (1) и (2) defines the structure and parameters of the third control loop (image sensors' parameters of decomposition) and allows the system to minimize the statistic's $f(t)$ estimation errors on it's linear sections, as well as on "jump like" variations of $df(t)/dt$, responding to any changes in the input value $f(t)$ by changing component values of the weight matrix $\mathbf{K}(n)$.

Simplified version of this filtration can be implemented using a nonlinear accumulation, i.e. making a decision after reference *z*-statistic exceeds reduced threshold in several frames. In this simple case there is optimum amount m jointly analyzed serial frames, depending on current value of the derivative of the mathematical expectation f of z-statistics in time $df(t)/dt$. Therefore, to minimize system "wrong" state spent time an extremum of the functional should be found:

$$t_m = m + \frac{\gamma_m - \gamma_\infty}{\mathrm{arctg}\left(df(t)/dt\right)} \to \min. \quad (3)$$

Analytic solution of the problem (3) is possible when threshold dependence $\gamma_m(m)$ is clearly formalized. This dependence can't be found in analytical functions, and for the required value of optimal number of frames evaluation the function approximation $\gamma_m(m)$ can be used. Numerical analysis has shown that a good approximation of this dependence is a function $\gamma_m(m) \approx \gamma_\infty + m^{-1/4}$. Then, after taking the derivative dt_m/dm and equating it to zero, we can find the optimal number of frames m, for which solutions about z-statistics threshold γ_m excision should be multiplied:

$$m_o \approx \left[4\mathrm{arctg}(df(t)/dt)\right]^{-4/5}. \quad (4)$$

Formula (4) calculations (small values $df(t)/dt$ simplify the formula) shows that when $df(t)/dt > 10^{-1}$ optimal inertia is only 2 frames, and for average observation distance at typical SC docking problem [3] $df(t)/dt = 10^{-2}\ldots10^{-3}$ inertia is several tens of frames. For these values borders of threshold ratio $d_{1\leftrightarrow 2}$ of making a decision to change imaging mode are about 5.8…5.5, which is not 4 or 9, as we get in absence of inertia.

3 Conclusions

Thus, the solution to the problem of minimizing measurement errors for varying in time objects coordinates with limited readout speed from image sensors allows finding conventional optimum definition and framing rate for discretely switchable interchange. Achieving sustainability of the automatic control system by the measurement system's decomposition parameters requires the introduction of confidence interval when measuring statistics of the signal that exceeds minimum allowed value determined by the discrete image.

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