Mathematical model of temperature field distribution in thin plates during polishing with a free abrasive

Alex Avilov[1,*], *Vladimir* Syrovatka[1], *Rustam* Hubiev[1], and *Olga* Baryshnikova[1]

[1]Don State Technical University, 344000 Rostov-on-Don, Russia

Abstract. The purpose of this paper is to estimate the dynamic characteristics of the heating process of thin plates during polishing with a free abrasive. A mathematical model of the temperature field distribution in space and time according to the plate thickness is based on Lagrange equation of the second kind in the thermodynamics of irreversible processes (variation principle Bio). The research results of thermo elasticity of thin plates (membranes) will allow to correct the modes of polishing with a free abrasive to receive the exact reflecting surfaces of satellites reflector, to increase temperature stability and the ability of radio signal reflection, satellite precision guidance. Calculations of temperature fields in thin plates of different thicknesses (membranes) is held in the Excel, a graphical characteristics of temperature fields in thin plates (membranes) show non-linearity of temperature distribution according to the thickness of thin plates (membranes).

1 Introduction

In the aerospace industry circular membranes are used as reflectors of optic-electronic devices for orientation and celestial navigation of spacecraft and satellites. The reflector must have sufficient accuracy of the reflecting surface, which affects the pointing accuracy, temperature stability and radio-reflection ability. Modern technology and equipment used in polishing of thin plates (membranes) allow to receive precision of surface machining, commensurate with the fractions of light wavelength. Membranes are processed by domestic polishing-lapping machine 3PD320 and also by machines OptoTech foreign production using abrasive pastes as a free abrasive [1]. But the process of thin steel plates (membranes) polishing is accompanied by a considerable friction, the uneven distribution of heat flow in the plate material, the fluctuations of the technological system elements, the occurrence of thermo elasticity effects, the substantial depth of the broken layer and other phenomena that affect the quality of thin plates (membranes) surface [2]. As a result of these reasons, about 50% of the treated plates do not meet the required parameters of the surface quality: roughness parameters, accuracy of the surface geometrical shape, deviation from flatness etc. Production reject occurs very often – plate warping. The manufacturer

*Corresponding author: av100982@ramber.ru

incurs significant material and time costs (expenses), because the polishing operation takes approximately 30 minutes and it is accompanied by a significant consumption of abrasive material. The goal of theoretical studies is the analysis of thermal processes occurring in polishing of thin plates (membranes) with a free abrasive.

2 A mathematical model of the temperature field in thin plates (membranes)

To assess the dynamic characteristics of the heating process in the plate with thickness δ - it is necessary to determine the speed of the thermal wave propagation, for which the variation principle Bio in thermodynamics of irreversible processes has the form [3]:

$$\frac{\partial U}{\partial \alpha_k} + \frac{\partial \Phi}{\partial \dot{\alpha}_k} = F_k, \tag{1}$$

where: U - heat capacity (the temperature elastic potential Bio), an analogue of the potential function in mechanics; Φ -the potential of scattering, analogue of the dissipative function in mechanics; F_k - thermal mechanical strength, the analogue of the external force in mechanics; α_k - the generalized coordinate.

The distribution of temperature field in space and in time according to the plate thickness is approximated by the curve of the second order [3]:

$$T(z,\tau) = T_1(\tau) \cdot \left(1 - \frac{z}{\delta}\right)^2, \tag{2}$$

where $T(z,\tau)$ - the distribution of temperature field in time according to the thickness of the plate 0...z...δ; $T_1(\tau)$ - the distribution of temperature field in time; τ - time non-stationary process.

Heat capacity is defined as follows [3]:

$$U = \frac{1}{2} c\rho \int_0^\delta T_1^2(z,\tau)\, dz = \frac{1}{2} c\rho \int_0^\delta T_1^2(\tau) \left(1 - \frac{z}{\delta}\right)^4 dz, \tag{3}$$

where c - is the specific volumetric heat capacity; ρ - density of the plate material

Thermal displacement H is represented by the ratio [3]:

$$\rho c T_1(z,\tau) = -\frac{\partial H}{\partial z}. \tag{4}$$

Then the expression (4) with (2) has the form

$$H = c\rho T_1(\tau) \left(\frac{z^2}{\delta} - \frac{1}{3}\frac{z^3}{\delta^2} - z + \frac{\delta}{3} \right). \tag{5}$$

To calculate the potential scattering [3] computed the derivative of the magnitude of the thermal drift:

$$\Phi = \frac{1}{2\lambda_1}\int_0^{\delta} \dot{H}^2 dz\,, \tag{6}$$

where: λ_1 - coefficient of heat conductivity of the plate material.

The expression (5) after the differentiation and integrating has the form:

$$\int_0^{\delta} \dot{H}^2 dz = c^2\rho^2 T_1^2(\tau)\dot{\delta}^2\left[\frac{z}{9}-\frac{2z^3}{9\delta^2}+\frac{z^4}{9\delta^3}+\frac{z^5}{5\delta^4}-\frac{2z^6}{9\delta^5}+\frac{4z^7}{63\delta^7}\right]_0^{\delta} = \frac{13\delta}{315}c^2\rho^2 T_1^2(\tau)\dot{\delta}^2 \tag{7}$$

By substituting the expression (7) into (6) and after differentiation finally we have:

$$\frac{\partial\Phi}{\partial\dot{\delta}} = \frac{13}{315}\frac{c^2\rho^2 T_1^2(\tau)}{\lambda_1}\dot{\delta}\cdot\delta \tag{8}$$

Generalized thermal force F is represented as a ratio of the variations:

$$F = T(\tau)\cdot\frac{\delta H}{\delta z}\Big|_{z=0}\,. \tag{9}$$

Or taking into account the expression (5)

$$F = \frac{c\rho}{3}T_1^2(\tau)\,. \tag{10}$$

The expression heat capacity (3) after the integration has the form:

$$U = \frac{1}{2}c\rho T_1^2(z,\tau)\left(z-\frac{2z^2}{\delta}+\frac{2z^3}{\delta^2}-\frac{z^4}{\delta^3}+\frac{z^5}{5\delta^4}\right), \tag{11}$$

then

$$\frac{\partial U}{\partial\delta}\bigg|_{z=0} = \frac{c\rho}{10}\cdot T_1^2(\tau)\,. \tag{12}$$

Substituting the obtained expressions (8), (10), (12) the Lagrange equation of the second kind (1), we have:

$$\frac{d\delta}{dt}\cdot\delta = \frac{7}{30}\cdot\frac{315}{13}\cdot\frac{\lambda_1}{c\rho} = 5{,}65\cdot\frac{\lambda_1}{c\rho}\,. \tag{13}$$

After integration and transformation of the expression (13) finally we have received the form: $\delta = 3{,}36\cdot\sqrt{\frac{t\cdot\lambda_1}{c\cdot\rho}}$.

From the above calculation we have the following form:

$$t = \frac{c\rho\delta^2}{11{,}29\lambda_1}\,, \tag{14}$$

where: *t*- is the time-setting process temperature.

In Figure 1 the results of the researches are presented, the graphic characteristics of the temperature field in plates of various thickness have been received. The design results of time for plate loss stability, according to the form (14), i.e. the time, after which the warping of the plate (relaxation-setting process temperature) begins, is represented in figure 2.

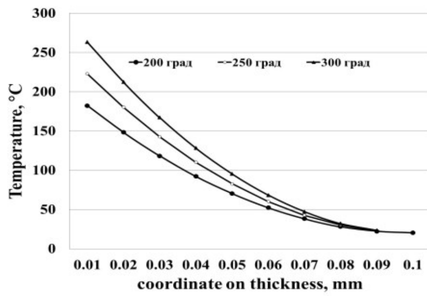

Fig. 1. Temperature field of the plate is h=0.1 mm.

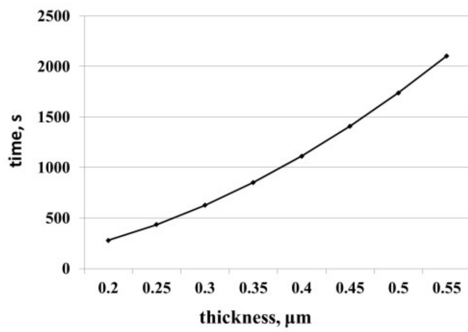

Fig. 2. The relaxation time of process of heating of the plate.

3 Conclusions

According to the results of theoretical researches we can draw the following conclusions:
1. The temperature field varies nonlinearly across the thickness of a thin plate, i.e. the heat flux in the plate is distributed unevenly, causing the appearance of the temperature elastic effects and, hence, loss stability and warping.
2. The relaxation time of heating process of a thin plate depends on the thickness of a thin plate, has a nonlinear character and increases with increasing of the plate thickness.
3. To eliminate the warping process of thin plates it is necessary to adjust the technological process of thin plates polishing, including the modes of polishing to reduce operating temperatures when polishing.

References

1. V.G. Miroshnichenko. STIN. J. **5**, 12 (2009)
2. A.M. Zenkour. Quart. J.of Mech. and Appl. Math. **57**, 205 (2004)
3. A. D. Kovalenko. Fundamental bases of temperature elastic (Scientific thought, Kiev, 1970)