Equilibrium contract strategies under asymmetric product substitutability

Baixun Li and Yaru Nan

School of Business Administration, Guangdong University of Finance and Economics, Guangzhou 510320, China

*Corresponding author: libaixun2002qq@163.com

Abstract. This paper investigates how asymmetric product substitutability affect the equilibrium contract choices as well as the performances of the supply chain players. We consider a market with two competing supply chains, including two competing manufacturers and two common retailers. Two frequently used contracts: wholesale-price and quantity-discount are considered. We find that under whether the wholesale-price or quantity-discount contract can be chosen by manufacturers would depend on the degree of product substitutability.

Keywords: game; product substitutability; wholesale-price contract; quantity-discount

1 Introduction

It is a common phenomenon in reality that a manufacturer sells through many competing retailers, whereas the retailers also sell the products provided by the other manufacturer competing with him. In other words, the manufacturers sell through common retailers, and the retailers carry both products. We define this phenomenon as ‘overlapping’ or ‘cross sales’ by referring to Trivedi^[1] (1998) and Wu and Mallik^[2] (2010). A typical example is the consumer goods market, where multiple retailers sell products of multiple competing brands in one area. For instance, the retailers Wal-Mart and Carrefour sell products of multiple competing brands (e.g., PHILIPS, SONY, SIEMENS, TCL, and so on) in the same area. One objective of this paper is to investigate the behaviors that all parties will behave in two competitive supply chains with multiple manufacturers and multiple retailers, and to study how the degree of product substitutability affects the contract choices and the performances of the players. Our results can be used to explain the reason why various contract types have been adopted in real supply chains and, especially, to explain why the sophisticated contracts have not completely eliminated the Pareto inferior wholesale-price contract in the real world.

The setting of this paper relates to the work on chain-to-chain competition and contract. Cachon and Kök^[3] (2010) study the problem of contract choice based on the many-to-one supply chain. Zhao and Shi^[4] (2011) consider two competing supply chains, which multiple upstream suppliers produce complementary products and sell to a single buyer. Govindan et al.^[5] (2013) gives a complete overview of the literature that addresses coordination by contracts. Zhang et al.^[6] (2014) explore the issue of supply chain coordination by considering trade credit. Chakraborty et al.^[7] (2015) study a supply chain comprising two competing manufacturers who sell their products through a common retailer. Ye et al.^[8] (2016) quantify the efficiency of decentralized competitive reverse supply chains (RSCs) with quality-dependent price-only contracts.

The remainder of this paper is organized as follows. Section 2 presents our model setting. Section 3 establishes and explores the sub-game solution for WW, QQ, WQ and QW case. Section 4 establishes the equilibrium contract structure. Finally, we summarize the paper, and suggest research extensions in Section 5.

2 Model setting

In this paper, we consider a market with two competing supply chains (SC_1 and SC_2), including two competing manufacturers (M_1 and M_2) and two common retailers (R_1 and R_2). The manufacturers act as the Stackelberg leader, and have more channel power than the retailers. In stage one, each of the two manufacturers simultaneously and independently chooses a contract type and designs their individual contract to maximize their individual profit. In stage two, after observing the contracts set by the two manufacturers, the retailers will choose their own purchase volumes simultaneously and independently.

In addition, each manufacturer will provide either *wholesale-price contract* (denoted by W) or *quantity-discount contract* (denoted by Q), representing the “simple” contract and the “sophisticated” contract, respectively. Consequently, in the two supply chains, there will be four possible contract combinations: WW, WQ, QW and QQ, where WW denotes that both manufacturers are restricted to choose wholesale price contracts; WQ means that manufacturer 1 (M_1)

is restricted to choose the wholesale-price contract while manufacturer 2 (M_2) is restricted to choose the quantity-discount contract. QW and QQ are similar.

For expositional simplicity, we elect to work with the inverse demand function described as follow.

$$p_{ij} = 1 - (d_{ij} + d_{i,3-j}) - \theta_i (d_{3-i,j} + d_{3-i,3-j}), \quad i = 1, 2; j = 1, 2. \quad (1)$$

As reflected in (1), the smaller the degree of product substitutability, the more differentiable of the two products, therefore the less potential price competition. In particular, $\theta_i = 0$ implies that the products are independent of each other, while $\theta_i = 1$ implies the perfect substitution between products

3 Model solution

In this section, we can derive closed form equilibrium solutions for various quantities such as prices and profits for different contract structures, including the WW, QQ, WQ, and QW case. In the QQ case, the retailer j 's problem is

$$\max_{d_{ij}^{QQ}, d_{i,3-j}^{QQ}} \pi_{R_j}^{QQ} = \sum_{i=1}^2 \left[(p_{ij}^{QQ} - w_i^{QQ}) d_{ij}^{QQ} + v_i^{QQ} (d_{ij}^{QQ})^2 / 2 \right] \quad (2)$$

where the superscript QQ denotes the quantity discount contracts offered by both manufacturers.

To derive the retailers' best response functions, we apply the first order conditions to their profit functions based on manufacturers' and competitors' decisions, thus getting,

$$\partial \pi_{R_j}^{QQ} / \partial d_{ij}^{QQ} = 1 + (v_i^{QQ} - 2) d_{ij}^{QQ} - d_{i,3-j}^{QQ} - (\theta_i + \theta_{3-i}) d_{3-i,j}^{QQ} - \theta_i d_{3-i,3-j}^{QQ} - w_i^{QQ} = 0 \quad (3)$$

$$\partial \pi_{R_j}^{QQ} / \partial d_{i,3-j}^{QQ} = 1 + (v_{3-i}^{QQ} - 2) d_{i,3-j}^{QQ} - d_{3-i,j}^{QQ} - (\theta_i + \theta_{3-i}) d_{i,3-j}^{QQ} - \theta_{3-i} d_{3-i,j}^{QQ} - w_{3-i}^{QQ} = 0 \quad (4)$$

then solving for the conditional Nash equilibrium values for $d_{i,j}^{QQ}$ as a function of w_i^{QQ} and v_i^{QQ} .

$$d_{ij}^{QQ}(w_i^{QQ}, v_i^{QQ}) = \frac{d_{i,3-j}^{QQ} [\theta_{3-i} + \theta (v_{3-i}^{QQ} - 1)] + d_{3-i,j}^{QQ} [\theta_{3-i} + \theta_{3-i} + v_{3-i}^{QQ} - 2] + (\theta_i + \theta_{3-i}) w_{3-i}^{QQ} + (v_{3-i}^{QQ} - 2) w_i^{QQ} + 2 - \theta_i - \theta_{3-i}}{4 - \theta_i^2 - 2\theta_i \theta_{3-i} - \theta_{3-i}^2 - 2v_{3-i}^{QQ} + v_i^{QQ} (v_{3-i}^{QQ} - 2)} \quad (5)$$

We now turn to the manufacturers' decisions. The manufacturers' reaction functions can be derived from the following first-order conditions of the manufacturers' profit maximization problems.

$$\max_{w_i^{QQ}, v_i^{QQ}} \pi_{M_i}^{QQ} = \sum_{j=1}^2 \left[w_i^{QQ} d_{ij}^{QQ} - (w_i^{QQ}, v_i^{QQ}) - v_i^{QQ} (d_{ij}^{QQ} (w_i^{QQ}, v_i^{QQ}))^2 / 2 \right] \quad (6)$$

By differentiating Equation (6) first partially with respect to w_i^{QQ} , we can determine Nash equilibrium wholesale prices.

$$w_i^{QQ}(w_{3-i}^{QQ}, v_i^{QQ}) = \frac{(2\theta_i^2 + 5\theta_i \theta_{3-i} + 2\theta_{3-i}^2 + 3v_{3-i}^{QQ} - 9) [(2\theta_i + \theta_{3-i}) (1 - w_{3-i}^{QQ}) + v_{3-i}^{QQ} - 3]}{(v_{3-i}^{QQ} - 3) [4(\theta_i^2 + \theta_{3-i}^2) + 10\theta_i \theta_{3-i} + 6v_{3-i}^{QQ} - v_i^{QQ} (v_{3-i}^{QQ} - 3) - 18]} \quad (7)$$

Substituting $w_i^{QQ}(w_{3-i}^{QQ}, v_i^{QQ})$ to Equations (6), and differentiating partially with respect to v_i^{QQ} , we get

$$\frac{\partial \pi_{M_i}^{QQ}}{\partial v_i^{QQ}} = \frac{[(2\theta_i + \theta_{3-i}) (1 - w_{3-i}^{QQ}) + v_{3-i}^{QQ} - 3]^2}{[4(\theta_i^2 + \theta_{3-i}^2) + 10\theta_i \theta_{3-i} + 6v_{3-i}^{QQ} - v_i^{QQ} (v_{3-i}^{QQ} - 3) - 18]^2} > 0 \quad (8)$$

It can be found that given the retailers' best response functions and optimal wholesale price, the manufacturer's profit function increases with quantity-discount coefficient. Therefore, each manufacturer's best response is to choose upper bound of quantity discount coefficient, and let $v_i^{QQ} = 2 - \theta_i - \theta_{3-i}$. Substituting $v_i^{QQ} = 2 - \theta_i - \theta_{3-i}$ into Equation (5) and (7) respectively will yield the equilibrium decisions of the two retailers and manufacturers. Thus, we can obtain the equilibrium values of profit for manufacturers and retailers under the QQ cases, which are only a function of the degree of product substitutability. By similar ways, we can obtain the equilibrium values of profit for manufacturers and retailers under the WW\WQ\QW cases.

4 Contract strategies under asymmetric product substitutability

In reality, if two products belong to different class brand type, the degree of product substitutability from product 1 to product 2 is the different with the latter to former, that is, the degree of product substitutability is asymmetric. In this section, we will consider the problem of contract choice game between two overlapping supply chains given that the degree of product substitutability is asymmetric. For notational convenience, we show the definitions of several functions as follows.

$$(a) \quad f_a(\theta_1, \theta_2) = \pi_{M_1}^{*WW}(\theta_1, \theta_2) - \pi_{M_1}^{*QW}(\theta_1, \theta_2), \quad (b) \quad f_b(\theta_1, \theta_2) = \pi_{M_1}^{*WQ}(\theta_1, \theta_2) - \pi_{M_1}^{*QQ}(\theta_1, \theta_2),$$

$$(c) \quad f_c(\theta_1, \theta_2) = \pi_{M_2}^{*WW}(\theta_1, \theta_2) - \pi_{M_2}^{*WQ}(\theta_1, \theta_2), \quad (d) \quad f_d(\theta_1, \theta_2) = \pi_{M_2}^{*QW}(\theta_1, \theta_2) - \pi_{M_2}^{*QQ}(\theta_1, \theta_2),$$

$$(e) \quad f_e(\theta_1, \theta_2) = \pi_{M_1}^{*WW}(\theta_1, \theta_2) - \pi_{M_1}^{*QQ}(\theta_1, \theta_2).$$

And define,

$$A_1 = \{(\theta_1, \theta_2) | f_a < 0, f_b < 0, f_c < 0, f_d < 0, f_e < 0, \text{and}, 0 < \theta_1 < 1, 0 < \theta_2 < 1\};$$

$$A_2 = \{(\theta_1, \theta_2) | f_a < 0, f_b < 0, f_c < 0, f_d < 0, f_e > 0, \text{and}, 0 < \theta_1 < 1, 0 < \theta_2 < 1\};$$

$$A_3 = \{(\theta_1, \theta_2) | f_a < 0, f_b > 0, f_c < 0, f_d > 0, f_e > 0, \text{and}, 0 < \theta_1 < 1, 0 < \theta_2 < 1\};$$

$$A_4 = \{(\theta_1, \theta_2) | f_a > 0, f_b > 0, f_c > 0, f_d > 0, f_e > 0, \text{and}, 0 < \theta_1 < 1, 0 < \theta_2 < 1\}.$$

It is obvious to have $\bigcup_{n=1}^{n=4} A_n = \{(\theta_1, \theta_2) | 0 < \theta_1 < 1, \text{and}, 0 < \theta_2 < 1\}$. Figure 1 describes graphically the

manufacturers' Nash equilibrium of contracts choice game in the (θ_1, θ_2) plane. The outcomes can be classified into four distinct regions by five critical curves. (a) $f_a=0$, (b) $f_b=0$, (c) $f_c=0$, (d) $f_d=0$, and (e) $f_e=0$.

Figure 1 Sketches of four equilibrium type regions of manufacturers

Region I. This region is defined by the set A_1 , and suggests that on the one hand, no matter the manufacturer chooses quantity-discount contract or wholesale-price contract, it is optimal for the other manufacturer to choose quantity-discount contract, therefore, quantity-discount contract is the dominant strategy for both manufacturers in this scenario. On the other hand, both manufacturers are better off with the pure quantity-discount contract than with the pure wholesale-price contract. Therefore, in this region, the pure quantity-discount contract is not only the unique equilibrium, but also the best option for both manufacturers.

Region II. This region is defined by the set A_2 , and suggests that on the one hand, similar to Region I, the pure quantity-discount contract is the unique equilibrium. However, both manufacturers are better off with the pure wholesale-price contract than with the pure quantity-discount contract. Hence, in this region, although the manufacturers would choose quantity-discount contract if they are not restricted in their contract choice, they could be better off if they have been restricted to offer only wholesale-price contract, then, the problem of choosing an optimal contract structure is a classical prisoners' dilemma game.

Region III. This region is defined by the set A_3 , and suggests that as the degree of product substitutability increases, if the manufacturer chooses quantity-discount contract, the other manufacturer can obtain much more profits by choosing wholesale-price contract, in contrast, if the manufacturer chooses wholesale-price contract, the other manufacturer can obtain much more profits by choosing quantity-discount contract. As a result, counter to the intuition, the mixed contract structure WQ/QW case is Nash equilibrium between two manufacturers.

Region IV. This region is defined by the set A_4 , and suggests that when the degree of product substitutability increases farther, especially, both θ_1 and θ_2 approach one, on the one hand, no matter the manufacturer chooses quantity-discount contract or wholesale-price contract, it is optimal for the other manufacturer to choose wholesale-price contract, that is, wholesale-price contract is the dominant strategy for both manufacturers in this scenario. On the other hand, both manufacturers are better off with the pure wholesale-price contract than with the pure quantity-discount contract. Therefore, in this region, the pure wholesale-price contract is not only the unique equilibrium, but also the best option for both manufacturers.

In summary, Figure 4 describes the general trend that in the Manufacturer Stackelberg game, as the degree of product substitutability increase, the manufacturers will gradually switch from quantity-discount contract to wholesale-price contract. Our results are consistent with the fact that when the degree of product substitutability (degree of competition) is low, especially, the manufacturers function monopolistically, the relative power (the channel bargaining power) of retailers is small, the manufacturers will extract concessions from retailers, and are better off with the pure quantity-discount contract structure (i.e., in Region I and II). On the other hand, the manufacturers may tend to drop retail prices to capture more market demand by adopting quantity-discount contract. However, as the degree of product substitutability increases, in other words, the two manufacturers compete more and more severely, the manufacturers can prevent the retail price from dropping too much by adopting the pure wholesale-price contracts, owing to the fact that in the Manufacturer Stackelberg game, the retail price is always higher under wholesale-price contract structure than within quantity-discount contract structure.

These results indicate that, to prevent fierce price competition, it is important for the manufacturer to invest product differentiation by considering important non-price factors such as the provision of product information, faster check-out, or after-sales service. That is to say, the manufacturers can benefit from the use of sophisticated contracts (e.g., quantity-discount contract) with high brand loyalty (e.g., low product substitutability) by important non-price factors.

5 Conclusions

In this paper, we consider a market with two competing supply chains, where the manufacturers produce competing products, and distribute through two common retailers. We investigate the impact of asymmetric product substitutability on the equilibrium contract structure and the supply chain players' performances. We achieve this by deriving and comparing the equilibria that arise under WW, QQ, WQ and QW case respectively. Finally, we would like to mention some potentially interesting directions for future research. First, we model competition as a single stage (static) game in this paper. It is interesting but challenging to study an infinitely repeated (dynamic) game. Second, we have assumed that information is common knowledge. Relaxing this assumption would lead to the study of games with asymmetric information, and would generate insights on the impact of information on the contract choice of the supply chain members. Yet another extension would be to consider this problem under demand uncertainty. We believe that the ideas and models presented in this paper lay the motivational ground for future research in these directions.

Acknowledgement

This research was supported in part by the National Natural Science Foundation of China under Grants nos. 71401042, 71472143, the Youth Foundation of Ministry of Education in China, Humanities and Social Science Research Projects under Grant nos. 13YJC630068. the Guangdong Natural Science Foundation under Grants nos. 2014A030313606.

References

1. M. Trivedi, *Manage Sci*, **44**(7) , 896-909(1998).
2. C. Q., Wu, S., Mallik, *Int J Prod Econ*, **126**(5), 158-167(2010).
3. G., Cachon, A., Kök, *Manage Sci*, **56**(3), 571–589(2010).
4. X., Zhao, C., Shi, *Int J Prod Econ*, **134**(2), 434-446(2011).
5. K., Govindan, M.N., Popiuc, A., Diabat, *J Cle Prod*, **47**, 319-334(2013).
6. Q.H., Zhang, M., Dong, J.W., Luo, et al. *Int J Prod Econ*, **153**, 352-360(2014).
7. T., Chakraborty, S.S., Chauhan, N., Vidyarthi, *Int J Prod Econ*, **166**, 103-118(2015).
8. S., Ye, Z.J., Ma, Y., Dai, *Transport Res E-Log*, **89**, 86-107(2016).