Two Error Models for Calibrating SCARA Robots based on the MDH Model

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Abstract. This paper describes the process of using two error models for calibrating Selective Compliance Assembly Robot Arm (SCARA) robots based on the modified Denavit-Hartenberg (MDH) model, with the aim of improving the robot’s accuracy. One of the error models is the position error model, which uses robot position errors with respect to an accurate robot base frame built before the measurement commenced. The other model is the distance error model, which uses only the robot moving distance to calculate errors. Because calibration requires the end-effector to be accurately measured, a laser tracker was used to measure the robot position and distance errors. After calibrating the robot and the end-effector locations were measured again compensating the error models’ parameters obtained from the calibration. The finding is that the robot’s accuracy improved greatly after compensating the calibrated parameters.

1 Introduction

Industrial robots usually work from one taught point to another taught point, which makes them highly repeatable although their accuracy may be poor due mainly to geometric errors.

The good news is that the accuracy can be improved by calibrating the robots. The two most important parts of robot calibration are building an accurate model and a valid method to measure the robot’s position. Although some researchers, such as Marko Švaco [1], have calibrated robots using the Denavit-Hartenberg (DH) model, it is not good enough for some robots that have parallel links. So Hayati [2] proposed a model called the MDH model, which many researchers now use to calibrate robots. Some researchers have proposed methods for third level calibrations, such as Albert Nubiola [3, 4] who introduced the temperature influence. Wang Zhen Hua [5] proposed a method to calibrate robots without building a base frame by using the distance errors of moving robots. Zhou Jian [6] proposed a pose selection algorithm that allows one to select a given number of optimal poses out of a large set of previously measured poses to avoid local convergence.

Some researchers concentrated on the measure methods, such as: laser tracker [7], coordinate measuring machine (CMM) [8, 9], telescoping ballbar [4], and the vision system [10]. There are also some researchers who use novel methods such as a probe and a ball [11] to measure the robot’s moving distance, and XiNing [12] used a laser and a PSD to calibrate robots.

In this paper, two error models are used to calibrate a SCARA robot. One is a position error model and the other is a distance error model, both of which are based on the MDH model.

The paper is organized into the following sections: section 2 proposes a kinematic model of the industrial robot called MDH model; section 3 presents the two calibration error models; section 4 presents the experiments and results; and section 5 is the conclusion.

2 MDH Model

A robot model must be built before the robot can be calibrated. The DH model is the most popular model as its four parameters can express two link positions very well. But the DH model will result in singularities with respect to two adjacent parallel links, even though a very small parallel error will cause a large deviation. So a MDH model developed by Hayati [2] was used, involving a new parameter termed β as the angle link i rotates around the y axis, written as $\text{Rot}(y, \beta)$. The MDH model can be expressed as follows:

$$
\begin{bmatrix}
    \alpha \\
    \gamma \\
    \beta \\
    \delta
\end{bmatrix} = 
\begin{bmatrix}
    \cos \theta \cos \beta & -\sin \theta & \cos \beta \sin \theta & \alpha \\
    \sin \gamma + \cos \alpha \sin \beta & \cos \alpha \cos \beta \sin \theta & -\cos \alpha \sin \beta \sin \theta & -\sin \gamma \\
    \cos \beta \sin \gamma - \cos \gamma \sin \beta & \cos \gamma \cos \beta \sin \theta & -\cos \gamma \sin \beta \sin \theta & \cos \gamma \cos \beta \\
    0 & 0 & 0 & 1
\end{bmatrix}
$$

(1)

3 Error Model

Assume that the model errors are very small and use $A_i$ to
express the nominal transformation matrix from joint \( i-1 \) to joint \( i \); \( A_i \) then expresses the actual transformation matrix as:

\[
A_i' = R_x (\alpha_i - \delta \alpha_{i-1}) \cdot D_x (a_i + \delta a_{i-1}) \\
\cdot R_y (\theta_i + \delta \theta_i) \cdot D_y (d_i + \delta d_i) \cdot R_z (\beta_i + \delta \beta_i)
\]  
(2)

Therefore the error of MDH model is:

\[
d_i' = A_i(d_i + \delta d_i)
\]

(3)

The error of one joint is equal to the sum of errors caused by every model error[5]:

\[
dA = \sum_{i=1}^{n} \frac{\partial A_i}{\partial \alpha_{i-1}} \delta \alpha_{i-1} + \frac{\partial A_i}{\partial d_i} \delta d_i + \frac{\partial A_i}{\partial \theta_i} \delta \theta_i + \frac{\partial A_i}{\partial \beta_i} \delta \beta_i
\]  
(3)

Just let,

\[
\begin{bmatrix}
    dx \\
    dy \\
    dz
\end{bmatrix} = \sum_{i=1}^{n} B_i \delta_i + B_{mod} \delta_{mod}
\]

(12)

\[
\delta_i = \begin{bmatrix}
    \delta a_{i-1} \\
    \delta d_i \\
    \delta \theta_i \\
    \delta \beta_i
\end{bmatrix}
\]

Where \( \delta_i \) is the MDH model errors of the link \( i-1 \) to link \( i \) and \( B_i \) is the coefficient matrix of \( \delta_i \).

Equation(12) can be written as:

\[
\begin{bmatrix}
    dx \\
    dy \\
    dz
\end{bmatrix} = B \delta
\]  
(13)

### 3.2 Distance error model

Unlike the difficulties involved in building accurate base frame when using the position error model to calibrate the robot, the distance error model allows distance to be measured easily and accurately without the need for a pre-determined base frame. Accordingly, the distance error model is used here to calibrate the robot.

The measured distance when the robot is moving is \( l_b \), the command distance of the robot is \( l_c \), and \( \Delta l = l_b - l_c \) is the distance error between the measured distance and the command distance. Therefore:

\[
\Delta l + l_b \approx \left( x_b(j) - x_b(i) + \Delta x (j) - \Delta x(i) \right) \\
+ \left( y_b(j) - y_b(i) + \Delta y (j) - \Delta y(i) \right) \\
+ \left( z_b(j) - z_b(i) + \Delta z (j) - \Delta z(i) \right)
\]

(14)

So the final distance error model can be written as:

\[
\Delta l' \approx \begin{bmatrix}
    x_b(j) - x_b(i) \\
    y_b(j) - y_b(i) \\
    z_b(j) - z_b(i)
\end{bmatrix}
\]

(15)

### 4 Experiment and results

#### 4.1 Experiment set-up

The first step is to build the SCARA MDH model. The
SCARA is a very simple robot with all of its links parallel and three of its four joints rotational. The process involved in building the MDH model is depicted in Figure 1 and Table 1.

### Table 1. Nominal kinematic parameters

<table>
<thead>
<tr>
<th>Joint</th>
<th>$a_{i-1}$</th>
<th>$a_{r-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
<th>$\beta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
<td>0</td>
</tr>
</tbody>
</table>

The end-effector could only be measured after the MDH model had been built. In this experiment, a FARO laser tracker with an accuracy of 10um +2.5um/m was used for measuring the errors of the SCARA robot. In order to measure robot position error, a robot base frame was built to allow the robot to move in the XY plane and was fitted so that the XY plane normal vector is the Z axis. The base frame was completed by allowing the robot to move along the X axis so that the axis could be identified as such.

![Figure 1. SCARA MDH model](image1.png)

The lease-square equation for the position error model was built according to equation (13) using the measured data:

$$b_p = C_p \Delta q_p$$

Where, $b_p$ is the position errors vector between the measured position and command position of the robot end-effector, $C_p$ is the coefficient matrix consisting of Jacobian matrix $B_i$, and $\Delta q_p$ is the MDH model errors.

### Table 2. Calibration results of position error

<table>
<thead>
<tr>
<th>i</th>
<th>$\delta a_{i-1}$</th>
<th>$\delta a_{r-1}$</th>
<th>$\delta d_i$</th>
<th>$\delta \theta_i$</th>
<th>$\delta \beta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0002</td>
<td>0</td>
<td>-0.0065</td>
<td>0.0027</td>
</tr>
<tr>
<td>2</td>
<td>-0.8340</td>
<td>0.0011</td>
<td>0</td>
<td>0.0105</td>
<td>0.0018</td>
</tr>
<tr>
<td>3</td>
<td>-0.2226</td>
<td>0.0003</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>tool</td>
<td>$\delta x = -0.8438$</td>
<td>$\delta y = -1.8113$</td>
<td>$\delta z = -18.5336$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 is the MDH model errors after equation(16) had been calibrated. The absolute error of the end-effector before calibration and after calibration of the calibrated group points is shown in Figure 3.

![Figure 3. Position error of calibrated group](image3.png)

Because the model errors were calculated with the calibrated group points only, the improvement of accuracy may only be effective for the calibrated groups. To compensate for this another 88 points were measured as a check group in different positions. Their position errors after calibration are shown in Figure 4.

![Figure 4. Position error of check group](image4.png)
Once the data was obtained, the results were analyzed and the position error expressed respectively in the X, Y and Z axes. From Table 3 it can be seen that for the calibrated group points after the calibration the mean position errors could be decreased from 1.6101mm to 0.0173mm in the X axis; 1.1801mm to 0.0181mm in the Y axis; and 0.4173mm to 0.0361mm in the Z axis. The standard deviation (SD) could be improved from 0.6438 to 0.0117 in the X axis; 0.6931 to 0.0131 in the Y axis; and 0.0943 to 0.0218 in the Z axis. For the check group points, the mean position errors are 0.0214 mm in the X axis; 0.0204 mm in the Y axis; and 0.0359 mm in the Z axis. The SD in three axes respectively is 0.0162 mm, 0.0139 mm, and 0.0258 mm. These data show that this method substantially improves robot positioning accuracy. However, the method requires that before measuring position errors it is necessary to build a robot base frame and confirm the relationship between the base frame and the measurement system. If the relationship is not accurate, it may cause other errors. To overcome this difficulty, a distance error model [5, 11] is proposed in the next section.

### 4.3 Distance error model results

When using the laser tracker as the measurement system, it is easy to confirm the base frame's accuracy because of the laser's high measurement accuracy. However, when it is not possible to get an accurate base frame, a distance error model can make it easier to calibrate the robot.

After the measurements are obtained, they can be put into equation (15), so that:

\[ \Delta \text{error}_d = D_d \Delta q_d \]  

Where, \( \Delta \text{error}_d \) is the distance error vector between the measured positions and command positions; \( D_d \) is the coefficient matrix consisting of \( B_j - B_i \), and \( \Delta q_d \) is the MDH model errors.

After the robot is calibrated with the distance error, the results are as shown in Table 4.

#### Table 4. Results of distance error model

<table>
<thead>
<tr>
<th>i</th>
<th>( \delta a_{i-1} )</th>
<th>( \delta a_{i+1} )</th>
<th>( \delta d_l )</th>
<th>( \delta \theta_l )</th>
<th>( \delta \beta_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.0022</td>
</tr>
<tr>
<td>2</td>
<td>-0.9140</td>
<td>0</td>
<td>0</td>
<td>0.0104</td>
<td>-0.0042</td>
</tr>
<tr>
<td>3</td>
<td>-0.3299</td>
<td>0.0014</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4.3.1 Distance error model results

The robot can be corrected for distance errors with the calibration parameters: the distance errors before and after calibration can be seen in Fig. 5. Just as with the position error, the calibration may only improve the accuracy of the calibrated group, so it is necessary to measure distance errors of other points that are different to the calibrated group points. Therefore, 88 points, which is 80 distance errors, were measured and are shown in Fig. 6.

#### Table 5. Distance error analysis

<table>
<thead>
<tr>
<th>Error</th>
<th>Calibrated Group</th>
<th>Check Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Calibration</td>
<td>Mean 0.2031</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>SD 0.0928</td>
<td></td>
</tr>
<tr>
<td>After Calibration</td>
<td>Mean 0.0225</td>
<td>0.0230</td>
</tr>
<tr>
<td></td>
<td>SD 0.0260</td>
<td>0.0268</td>
</tr>
</tbody>
</table>

From Figure 6 and Table 5, it can be seen that for the calibrated group points after the calibration, the mean distance errors decreased from 0.2031mm to 0.0225mm, the standard deviation (SD) improved from 0.0928 mm to 0.0260mm, and the check group points' mean distance error and SD are 0.0230 mm and 0.0268 mm respectively.

From these data it is clear that the distance error model works very well and can be used as a method when it is not
possible to construct an accurate base frame.

5 Conclusion

This paper described the building of a SCARA robot model based on the MDH model, and proposed two error models for calibrating the robot. After the robot was calibrated, the parameters were used to correct the robot, which greatly improved the robot's accuracy. When it is not possible to build an accurate robot base frame and establish the relationship of the base to the measurement system, it is suggested to use the distance error model to achieve accuracy. Both error models proved to be effective for calibrating the SCARA robot based on an accurate robot base frame.

Acknowledgment

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