

# A Method to Optimize Geometric Errors of Machine Tool based on SNR Quality Loss Function and Correlation Analysis

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**Abstract.** Instead improving the accuracy of machine tool by increasing the precision of key components level blindly in the production process, the method of combination of SNR quality loss function and machine tool geometric error correlation analysis to optimize five-axis machine tool geometric errors will be adopted. Firstly, the homogeneous transformation matrix method will be used to build five-axis machine tool geometric error modeling. Secondly, the SNR quality loss function will be used for cost modeling. And then, machine tool accuracy optimal objective function will be established based on the correlation analysis. Finally, ISIGHT combined with MATLAB will be applied to optimize each error. The results show that this method is reasonable and appropriate to relax the range of tolerance values, so as to reduce the manufacturing cost of machine tools.

## 1 Introduction

With the development of advanced manufacturing technology, the requirement of machine tool is increasing, which is not only reflected in the machining accuracy of the machine tool, but also reducing costs that can meet the demand of the market. Although the machining accuracy is an important index to measure the performance of the machine, reducing the production cost of CNC machine tools also has important significance in the premise of ensuring the processing precision.

In order to facilitate the research of the relationship between machining accuracy and production cost, the error modeling is the most important task. Domestic and foreign scholars has carried out many researches in the field of machine tool error modeling and proposed many modeling method, such as kinematics model method [1], mechanism method [2], neural network method [3-4], homogeneous transformation matrix method [5], screw theory method [6], etc. These methods can accurately establish the accuracy model of the machine tool, and it is the basis of the research of all aspects of the machine tool. In this paper, the method of homogeneous transformation matrix is used to model the error model of five axis machine tool.

After the machine tool error model is established, the research between the total cost and precision machine tool will be carried out. At first, the tolerance optimization model based on the SNR quality loss function will be established, serving as cost basis to achieve optimal allocation of each parts tolerance. At present, some domestic scholars have some improvement on the quality loss function. Feng et al. [7] proposed an

improved multivariate quality loss function, which can be used for all the quality characteristics, and fully considered the correlation between the corresponding variables. Ai et al. [8] combined multivariate quality loss function and genetic algorithm to optimize the assembly quality. Duan et al. [9] improved the multivariate quality loss function, the incremental model of multivariate quality loss function is formed, and the feasibility of the model is proved by an example. Chang et al. [10] consider the trade-off problem between quality and cost for the case when the input quality characteristic is normally distributed. According to the research of Chen et al. [11], the management can adjust the investment on prevention and appraisal costs on quality improvement that enhances process capability, reduces product defect rate and, as a result, generates remarkable financial return. Ram Ganesan et al. [12] established a model which determines the optimal levels of inventory, and the production lot-size that minimizes the sum of inventory and quality-related costs. Noel Artiles-León [13] described a methodology for optimizing several quality characteristics simultaneously. It was assumed that some type of experimentation had been carried out to determine a relationship between each quality characteristic and the experimental factors. Ramesh Kumar et al. [14] applied quality loss theory to allocate tolerance economically and precisely. Balamurugan, C. et al [15] finished the optimal allocation of geometric and process tolerances based on the present worth of quality loss. Pradeep Kumar Gupta et al [16] enhanced machine accuracy by economic analysis.

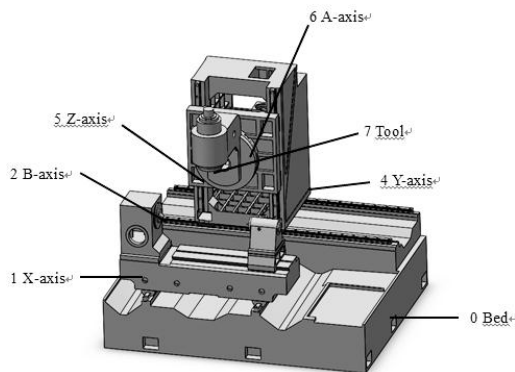
Optimization model based on the correlation analysis of the geometric error of machine tool is regarded as

another geometric error basis for the optimal allocation of each parts tolerance. There are also some researches in this respect at present, the commonly used correlation analysis method are Two step cluster [17], K - Means cluster [18], and Hierarchical cluster [19]. Hierarchical cluster analysis is a multivariate statistical analysis method, which is widely used to establish a classification. This method in the SPSS will be used to cluster and analyze the 37 geometric errors of five-axis machine tool.

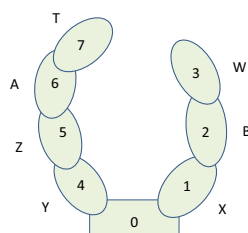
In this paper, based on the combination of the SNR quality loss function and the geometric error correlation analysis, the tolerance distribution model of the machine tool can be obtained, and the tolerance of the machine tool will be optimized. This method can ensure the machining accuracy of the machine, and appropriately increase geometric errors, so that it can reduce the cost of machine tools, which has a positive effect on the development of the machine tool market.

## 2 Error modeling of five-axis machine tool based on homogeneous transformation matrix

In this paper, the XKH800 five-axis linkage blade machining center is used to develop the geometric error model. Three-dimensional model of this XHK800 five-axis machining center is shown in Figure 1. There are 37 geometric errors including positioning errors, straightness errors, angular errors, etc., which are shown in Table 1. The topological structure of this machining center is shown in Figure 2. The homogeneous transformation matrices of the machine tool between the moving parts are shown in Table 2.



**Figure 1.** The three-dimensional model of the XHK800 five-axis machining center.



**Figure 2.** The topological structure of the XHK800 five-axis machining center.

Suppose that the tool forming point coordinate in the tool coordinate system is:

$$P_t = (p_{tx} \quad p_{ty} \quad p_{tz} \quad 1)^T \quad (1)$$

The work-piece forming point coordinate in the work-piece coordinate system is:

$$P_w = (p_{wx} \quad p_{wy} \quad p_{wz} \quad 1)^T \quad (2)$$

When the machine tool moves in ideal case, the machine tool is without error, that means the tool forming point and work-piece forming point will overlap together, which can be described as:

$$T_{07} P_t = T_{03} P_{wideal} \quad (3)$$

The ideal tool forming point in the work-piece coordinate system can be obtained according to Eq. (3):

$$P_{wideal} = (T_{01}^P T_{01}^S T_{12}^P T_{12}^S T_{23}^P T_{23}^S)^{-1} T_{04}^P T_{04}^S T_{45}^P T_{45}^S T_{56}^P T_{56}^S T_{67}^P T_{67}^S P_t \quad (4)$$

In Eq. (4), P and S mean static and dynamic respectively, so  $T_{J-1,J}^P$  refers to the ideal static homogenous transformation matrix of the adjacent body and  $T_{J-1,J}^S$  refers to the ideal dynamic homogenous transformation matrix as shown in Table 3. In the actual machining process, the position of actual tool forming point will inevitably deviate from the ideal. Therefore, the actual tool forming point in the work-piece coordinate system is:

$$P_{wactual} = ({}^E T_{03})^{-1} {}^E T_{07} P_t \quad (5)$$

And thereupon, geometric errors of five-axis machine tool caused by the gap between actual and ideal forming point can be described as:

$$E = {}^E T_{03} P_{wideal} - {}^E T_{07} P_t \quad (6)$$

The values of the expressions in the Eq. (4)-Eq. (6) can be checked from Table 2.  $E$  represents geometric errors of this machining center including  $E_x$ ,  $E_y$ , and  $E_z$ , which is:

$$E = [E_x \quad E_y \quad E_z \quad 1]^T \quad (7)$$

## 3 Tolerance optimal allocation modeling based on the total cost of the SNR quality loss function

This model consists of two parts: manufacturing cost function and SNR quality loss function. Manufacturing cost function control the precondition which is meeting the requirements of product performance and the lowest manufacturing cost, and optimize the parts tolerance. The basic purpose of quality loss function is to evaluate the loss caused by low quality in a quantitative way [7].

### 3.1 Manufacturing cost

In this article, the tolerance and accuracy of the 37 machine tool geometric errors are supposed to be  $T_i$  and  $A_i$  separately. Suppose that the half tolerance bandwidth of machine tool geometric accuracy is  $t_i$  ( $t_i = T_i / 2$ ;  $i=1, 2, \dots, 37$ ). As the implementation cost of the machine tool

geometric accuracy is different, the cost weight coefficient  $h_i$  and the cost characteristic index  $r_i$  are introduced. The cost of the machine tool can be expressed as:

$$C_{\Sigma}(t_i) = \sum_{i=1}^{37} (h_i t_i)^{-r_i} \quad (8)$$

**Table 1.** Description and representation of the five-axis machine tool geometric errors.

Identification mark	Geometrical significance	Identification mark	Geometrical significance
$\Delta x_x$	positioning error	$\Delta x_A$	X direction run-out error
$\Delta y_x$	Y direction of straightness error	$\Delta y_A$	Y direction run-out error
$\Delta z_x$	Z direction of straightness error	$\Delta z_A$	Z direction run-out error
$\Delta \alpha_x$	Roll error	$\Delta \alpha_A$	Turning error
$\Delta \beta_x$	Pitch error	$\Delta \beta_A$	Around the Y-axis turning error
$\Delta \gamma_x$	Yaw error	$\Delta \gamma_A$	Around the Z-axis turning error
$\Delta x_y$	X direction of straightness error	$\Delta x_B$	X direction run-out error
$\Delta y_y$	positioning error	$\Delta y_B$	Y direction run-out error
$\Delta z_y$	Z direction of straightness error	$\Delta z_B$	Z direction run-out error
$\Delta \alpha_y$	Pitch error	$\Delta \alpha_B$	Around the X-axis turning error
$\Delta \beta_y$	Roll error	$\Delta \beta_B$	Turning error
$\Delta \gamma_y$	Yaw error	$\Delta \gamma_B$	Around the Z-axis turning error
$\Delta x_z$	X direction of straightness error	$\Delta \gamma_{xy}$	X,Y-axis perpendicularity error
$\Delta y_z$	Y direction of straightness error	$\Delta \beta_{xz}$	X,Z-axis perpendicularity error
$\Delta z_z$	positioning error	$\Delta \alpha_{yz}$	Y,Z-axis perpendicularity error
$\Delta \alpha_z$	Pitch error	$\Delta \gamma_{xB}$	B-axis parallelism error in YZ plane
$\Delta \beta_z$	Yaw error	$\Delta \alpha_{zB}$	B-axis parallelism error in XY plane
$\Delta \gamma_z$	Roll error	$\Delta \gamma_{yA}$	A-axis parallelism error in XZ plane
		$\Delta \beta_{zA}$	A-axis parallelism error in XY plane

**Table 2.** Ideal and error homogeneous transformation matrixes of the five-axis machine tool.

Adjacent body	Body ideal static, motion HTMs ( $T_{J-1,J}^P, T_{J-1,J}^S$ )	Body ideal static, motion error HTMs ( ${}^eT_{J-1,J}^P, {}^eT_{J-1,J}^S$ )
0-1 X-axis	$T_{01}^P = I_{4 \times 4}$ $T_{01}^S = \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	${}^eT_{01}^P = I_{4 \times 4}$ ${}^eT_{01}^S = \begin{pmatrix} 1 & -\Delta \gamma_x & \Delta \beta_x & \Delta x_x \\ \Delta \gamma_x & 1 & -\Delta \alpha_x & \Delta y_x \\ -\Delta \beta_x & \Delta \alpha_x & 1 & \Delta z_x \\ 0 & 0 & 0 & 1 \end{pmatrix}$

<p>1-2 B-axis</p>	$T_{12}^P = I_{4 \times 4}$ $T_{12}^S = \begin{pmatrix} \cos B & 0 & \sin B & 0 \\ 0 & 1 & 0 & 0 \\ -\sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	${}^eT_{12}^P = \begin{pmatrix} 1 & -\Delta\gamma_{xB} & 0 & 0 \\ \Delta\gamma_{xB} & 1 & -\Delta\alpha_{zB} & 0 \\ 0 & \Delta\alpha_{zB} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^eT_{12}^S = \begin{pmatrix} 1 & -\Delta\gamma_B & \Delta\beta_B & \Delta x_B \\ \Delta\gamma_B & 1 & -\Delta\alpha_B & \Delta y_B \\ -\Delta\beta_B & \Delta\alpha_B & 1 & \Delta z_B \\ 0 & 0 & 0 & 1 \end{pmatrix}$
<p>2-3 Work piece</p>	$T_{23}^P = \begin{pmatrix} 1 & 0 & 0 & x_{wd} \\ 0 & 1 & 0 & y_{wd} \\ 0 & 0 & 1 & z_{wd} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $T_{23}^S = I_{4 \times 4}$	${}^eT_{23}^P = \begin{pmatrix} 1 & -\Delta\gamma_{wd} & \Delta\beta_{wd} & \Delta x_{wd} \\ \Delta\gamma_{wd} & 1 & -\Delta\alpha_{wd} & \Delta y_{wd} \\ -\Delta\beta_{wd} & \Delta\alpha_{wd} & 1 & \Delta z_{wd} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^eT_{23}^S = I_{4 \times 4}$
<p>0-4 Y-axis</p>	$T_{04}^P = I_{4 \times 4}$ $T_{04}^S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	${}^eT_{04}^P = \begin{pmatrix} 1 & -\Delta\gamma_{xy} & 0 & 0 \\ \Delta\gamma_{xy} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^eT_{04}^S = \begin{pmatrix} 1 & -\Delta\gamma_y & \Delta\beta_y & \Delta x_y \\ \Delta\gamma_y & 1 & -\Delta\alpha_y & \Delta y_y \\ -\Delta\beta_y & \Delta\alpha_y & 1 & \Delta z_y \\ 0 & 0 & 0 & 1 \end{pmatrix}$
<p>4-5 Z-axis</p>	$T_{45}^P = I_{4 \times 4}$ $T_{45}^S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$	${}^eT_{45}^P = \begin{pmatrix} 1 & 0 & \Delta\beta_{xz} & 0 \\ 0 & 1 & -\Delta\alpha_{yz} & 0 \\ -\Delta\beta_{xz} & \Delta\alpha_{yz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^eT_{45}^S = \begin{pmatrix} 1 & -\Delta\gamma_z & \Delta\beta_z & \Delta x_z \\ \Delta\gamma_z & 1 & -\Delta\alpha_z & \Delta y_z \\ -\Delta\beta_z & \Delta\alpha_z & 1 & \Delta z_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$
<p>5-6 A-axis</p>	$T_{56}^P = I_{4 \times 4}$ $T_{56}^S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos A & -\sin A & 0 \\ 0 & \sin A & \cos A & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	${}^eT_{56}^P = \begin{pmatrix} 1 & -\Delta\gamma_{yA} & \Delta\beta_{zA} & 0 \\ \Delta\gamma_{yA} & 1 & 0 & 0 \\ -\Delta\beta_{zA} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^eT_{56}^S = \begin{pmatrix} 1 & -\Delta\gamma_A & \Delta\beta_A & \Delta x_A \\ \Delta\gamma_A & 1 & -\Delta\alpha_A & \Delta y_A \\ -\Delta\beta_A & \Delta\alpha_A & 1 & \Delta z_A \\ 0 & 0 & 0 & 1 \end{pmatrix}$
<p>6-7 Tool</p>	$T_{67}^P = \begin{pmatrix} 1 & 0 & 0 & x_{td} \\ 0 & 1 & 0 & y_{td} \\ 0 & 0 & 1 & z_{td} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $T_{67}^S = I_{4 \times 4}$	${}^eT_{67}^P = \begin{pmatrix} 1 & -\Delta\gamma_{td} & \Delta\beta_{td} & \Delta x_{td} \\ \Delta\gamma_{td} & 1 & -\Delta\alpha_{td} & \Delta y_{td} \\ -\Delta\beta_{td} & \Delta\alpha_{td} & 1 & \Delta z_{td} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^eT_{67}^S = I_{4 \times 4}$

In order to determine the cost weight coefficient, the 37 geometric accuracies of machine tool are classified according to the categories, and it is considered that the cost weight coefficient of the same type of machine tool's geometric accuracy is the same. Linear displacement positioning accuracy is regarded as a benchmark, of which the cost weight coefficient is 1. And introducing 3 proportionality coefficients  $h_p$ ,  $h_a$ , and  $h_d$ , which severally means the ratio of the cost weight coefficient of

positional accuracy, angular displacement positioning accuracy and non-positioning accuracy to the cost weight coefficient of linear displacement positioning accuracy, and they are shown in Table 3. According to the specific structure of the machine tool chosen by this paper and the reference [20] [21], This paper set up  $h_p = 3$ ,  $h_a = 4$ ,  $h_d = 5$ ,  $r_i = 2$ .

**Table 3.** The classification and cost weight coefficient of machine tool geometric accuracy.

Category		The symbol of geometric accuracy	Cost weight coefficient
Positional accuracy		$A_{31} A_{32} A_{33} A_{34} A_{35} A_{36} A_{37}$	$h_p$
Kinematic accuracy	Linear displacement positioning accuracy	$A_1 A_8 A_{15}$	1
	Angle position precision	$A_{22} A_{29}$	$h_a$
	Linear displacement non-positioning accuracy	$A_2 A_3 A_7 A_9 A_{13} A_{14} A_{19} A_{20} A_{21} A_{25} A_{26}$ $A_{27}$	$h_d$
	Angle non-position precision	$A_4 A_5 A_6 A_{10} A_{11} A_{12} A_{16} A_{17} A_{18} A_{23} A_{24}$ $A_{28} A_{30}$	$h_a h_d$

### 3.2 Cost of quality loss based on signal-noise ratio

The smaller-the-better characteristic refers to the closer to zero the better quality characteristics. In the actual calculation, the SNR of the smaller-the-better characteristic is:

$$\eta_s = 10 \log \frac{1}{\sigma^2 + \mu^2} \tag{9}$$

Its estimated value is:

$$\eta_s = 10 \log \frac{1}{\sigma^2 + \mu^2} = -10 \log \left( \frac{1}{n} \sum_{i=1}^n y_i^2 \right) \tag{10}$$

The greater the value of  $\eta$  is, the product quality will be more stable, and the loss smaller [22]. According to this conclusion, the size of the contribution of the quality indicators to the volatility will be determined, and different weights can be assigned:

$$\lambda_i = \frac{\frac{1}{\eta_i}}{\sum_{j=1}^n \frac{1}{\eta_j}} \tag{11}$$

In this paper, 37 geometric errors can be regards as the quality characteristics of the machine tool, and they are the smaller-the-better characteristic. The multivariate quality loss function of the smaller-the-better characteristic can be expressed as:

$$L(y_1, y_2, \dots, y_n) = \sum_{i=1}^n \lambda_i L_S(y_i) = \sum_{i=1}^n \lambda_i y_i^2 \tag{12}$$

Calculating SNR  $\eta_i$  of the 37 quality indexes  $t_i$  by Eq. (10), and replacing them into Eq. (11) and Eq. (12) in turn, the quality loss function of 37 geometric errors will be obtained:

$$L_\Sigma(t_i) = \sum_{i=1}^{37} \lambda_i t_i^2 \tag{13}$$

### 3.3 Total cost optimization model

The sum of the cost of integrated manufacturing and quality lose reaches the minimum, which is the evidence of tolerance optimization allocation of various parts [23]. According to the manufacturing cost and quality loss cost mentioned above, the objective function of the machine total cost can be expressed as:

$$M1_{\min} [C_\Sigma(t_i) + L_\Sigma(t_i)] = \min \left[ \sum_{i=1}^{37} (h_i t_i)^{-\eta_i} + \sum_{i=1}^{37} \lambda_i t_i^2 \right] \tag{14}$$

Constraint conditions are:

$$\begin{aligned} \Delta x_{\max} &\leq def(\Delta x) \\ \Delta y_{\max} &\leq def(\Delta y) \\ \Delta z_{\max} &\leq def(\Delta z) \end{aligned} \tag{15}$$

In Eq. (15),  $def(\Delta x)$ ——Maximum allowable error in X direction;  $def(\Delta y)$ ——Maximum allowable error in Y direction;  $def(\Delta z)$ ——Maximum allowable error in Z direction.

## 4 Optimal distribution model based on machine tool geometric error correlation analysis

### 4.1 Machine tool geometric error correlation statistics analysis and classification

In this article hierarchical clustering method in the SPSS (Statistical Product and Service Solutions) is applied to count the relationship between each error term, and the error terms which ones have stronger relationship are divided into a category. According to the subsequent requirements of modeling, geometric error terms will be divided into four categories. The first kind contains  $\Delta\alpha_x$ ,

$$\Delta\beta_x, \Delta\gamma_x, \Delta\alpha_y, \Delta\beta_y, \Delta\gamma_y, \Delta\alpha_z, \Delta\beta_z, \Delta\gamma_z,$$

$$\Delta\alpha_A, \Delta\beta_A, \Delta\alpha_B, \Delta\beta_B, \Delta\gamma_B, \Delta\gamma_{xy}, \Delta\beta_{xz}, \Delta\alpha_{yz}$$

$$M2_{\max} = \max \sqrt{\sum_{i=1}^3 \Delta s_i^2(x) + \sum_{j=1}^3 \Delta s_j^2(y) + \sum_{k=1}^3 \Delta s_k^2(z) + \sum_{m=1}^3 \Delta s_m^2(A) + \sum_{l=1}^3 \Delta s_l^2(B) + S^2(x) + S^2(y) + S^2(z) + S^2(A) + S^2(B)} \quad (16)$$

$$= \max \sqrt{t_1^2 + t_2^2 + t_3^2 + t_4^2 + t_7^2 + t_8^2 + t_9^2 + t_{11}^2 + t_{13}^2 + t_{14}^2 + t_{15}^2 + t_{18}^2 + t_{19}^2 + t_{20}^2 + t_{21}^2 + t_{22}^2 + t_{25}^2 + t_{26}^2 + t_{27}^2 + t_{29}^2}$$

Combined with the results of the previous error terms correlation analysis and classification, the formula (16) is changed into the following form and the optimal

$$M2_{\max} = \max \left( \sqrt{t_1^2 + t_2^2 + t_3^2 + t_7^2 + t_8^2 + t_{13}^2 + t_{14}^2 + t_{15}^2} + \sqrt{t_4^2 + t_{11}^2 + t_{18}^2 + t_{22}^2 + t_{29}^2} + \sqrt{t_9^2} + \sqrt{t_{19}^2 + t_{20}^2 + t_{21}^2 + t_{25}^2 + t_{26}^2 + t_{27}^2} \right) \quad (17)$$

The constraint conditions are as follows:

$$\begin{aligned} \Delta x_{\max} &\leq \text{def}(\Delta x) \\ \Delta y_{\max} &\leq \text{def}(\Delta y) \\ \Delta z_{\max} &\leq \text{def}(\Delta z) \\ lb(\Delta s_i(u)) &\leq \Delta s_i(u) \leq ub(\Delta s_i(u)) \\ (i=1, 2, 3; u=x, y, z) \\ lb(S(u)) &\leq S(u) \leq ub(S(u)) \\ (u=x, y, z) \end{aligned} \quad (18)$$

In the constraint conditions, the last two inequalities limit the upper and lower bounds of the component accuracy parameters. Meanwhile, the implied condition is

$$M1_{\min} [C_2(t_i) + L_2(t_i)] = \min \left[ \sum_{i=1}^{37} (h_i t_i)^{-r_i} + \sum_{i=1}^{37} \lambda_i t_i^2 \right] \quad (19)$$

$$M2_{\max} = \max \left( \sqrt{t_1^2 + t_2^2 + t_3^2 + t_7^2 + t_8^2 + t_{13}^2 + t_{14}^2 + t_{15}^2} + \sqrt{t_4^2 + t_{11}^2 + t_{18}^2 + t_{22}^2 + t_{29}^2} + \sqrt{t_9^2} + \sqrt{t_{19}^2 + t_{20}^2 + t_{21}^2 + t_{25}^2 + t_{26}^2 + t_{27}^2} \right)$$

The constraint conditions are as follows:

$$\begin{aligned} \Delta x_{\max} &\leq \text{def}(\Delta x) \\ \Delta y_{\max} &\leq \text{def}(\Delta y) \\ \Delta z_{\max} &\leq \text{def}(\Delta z) \\ lb(\Delta s_i(u)) &\leq \Delta s_i(u) \leq ub(\Delta s_i(u)) \\ (i=1, 2, 3; u=x, y, z, A, B) \\ lb(S_{300P}(u)) &\leq S_{300P}(u) \leq ub(S_{300P}(u)) \\ (u=x, y, z, A, B) \end{aligned} \quad (20)$$

$\Delta\gamma_{xB}, \Delta\alpha_{zB}, \Delta\gamma_{yA}, \Delta\beta_{zA}$ . The second kind contains  $\Delta z_y$ . The third kind contains  $\Delta x_A, \Delta y_A, \Delta z_A, \Delta\gamma_A, \Delta x_B, \Delta y_B, \Delta z_B$ . And the fourth kind contains  $\Delta x_x, \Delta y_x, \Delta z_x, \Delta x_y, \Delta y_y, \Delta x_z, \Delta y_z, \Delta z_z$ .

### 4.2 The optimal distribution model based on precision grade of machine tool key components

There is a certain mutual influence between the geometric errors of machine tools and the geometric precision of components. According to the relationship between the components precision parameters and the geometric errors, the precision parameters of components are regarded as design variables, and the following objective function can be constructed.

matching model is obtained based on the correlation analysis of the machine tool geometric errors.

that the difference between the precision parameters of the same component is not too large, and the optimized variable value should be greater than the standard value.

### 4.3 Tolerance optimal distribution model of machine tool

From the foregoing, the optimization problem of this paper can be understood as the goal of minimizing the total cost and the maximization of the Euclidean norm of error terms in the constraint condition of ensuring the machining accuracy of the machine tool. The optimization model is as follows:

In Eq. (20)  $M_1$  represents total cost function;  $M_2$  represents Euclidean norm function of all the parameters variable. According to the requirement of user's processing precision, the maximum allowable error in X, Y, Z direction of the five axis machining center are:

$$\text{def}(\Delta x) = -0.015, \text{def}(\Delta y) = 0.015, \text{def}(\Delta z) = 0.01 \quad (21)$$

At the same time, in order to achieve the optimization objective, the optimized error should not be less than standard value, and accuracy parameters of the same parts should not be too much difference. Therefore, the

standard value is taken as the lower bound of the variable, and the 3 times of the standard value is taken as the upper bound of the variable.

## 5 Accuracy optimization allocation based on NSGA-II

The ISIGHT and MATLAB integration method will be applied to optimize the above multi-objective optimization problem. The above two objective functions are as the target function, and after the variables constraints range of the 37 geometric errors and requirements of machine tool precision are set, NSGA-II can be used to optimize the 37 geometric errors through combining with MATLAB. Optimization results are shown in Table 4.

**Table 4.** The optimization results of the five-axis machining center 37 geometric errors.

Error terms	Standard values (mm)	Optimal values (mm)	Error terms	Standard values (mm)	Optimal values (mm)
$\Delta x_x$	0.005	0.0150	$\Delta \beta_x$	0.003125/1000	0.009342/1000
$\Delta y_x$	0.005	0.0150	$\Delta \gamma_x$	0.003125/1000	0.003215/1000
$\Delta z_x$	0.005	0.0150	$\Delta \alpha_y$	0.003125/1000	0.009323/1000
$\Delta x_y$	0.005	0.0100	$\Delta \beta_y$	0.003125/1000	0.009331/1000
$\Delta y_y$	0.005	0.0150	$\Delta \gamma_y$	0.003125/1000	0.003126/1000
$\Delta z_y$	0.005	0.0150	$\Delta \alpha_z$	0.003125/1000	0.003126/1000
$\Delta x_z$	0.005	0.0150	$\Delta \beta_z$	0.003125/1000	0.003281/1000
$\Delta y_z$	0.005	0.00866	$\Delta \gamma_z$	0.003125/1000	0.003138/1000
$\Delta z_z$	0.005	0.00827	$\Delta \alpha_A$	0.04/1000	0.007509/1000
$\Delta x_A$	0.0075	0.0231	$\Delta \beta_A$	0.0075/1000	0.04026/1000
$\Delta y_A$	0.0075	0.0206	$\Delta \gamma_A$	0.0075/1000	0.008015/1000
$\Delta z_A$	0.0075	0.00801	$\Delta \alpha_B$	0.0075/1000	0.007531/1000
$\Delta x_B$	0.0075	0.0225	$\Delta \beta_B$	0.04/1000	0.04001/1000
$\Delta y_B$	0.0075	0.0218	$\Delta \gamma_B$	0.0075/1000	0.009955/1000
$\Delta z_B$	0.0075	0.00790	$\Delta \gamma_{xB}$	0.0125/300	0.01251/300
$\Delta \gamma_{xy}$	0.005/500	0.0108/500	$\Delta \alpha_{zB}$	0.0125/300	0.01503/300
$\Delta \beta_{xz}$	0.005/500	0.0050/500	$\Delta \gamma_{yA}$	0.0125/300	0.01263/1000
$\Delta \alpha_{yz}$	0.005/500	0.0050/500	$\Delta \beta_{zA}$	0.0125/300	0.01486/1000
$\Delta \alpha_x$	0.003125/1000	0.009374/1000			

## 6 Conclusions

Table 4 shows that all the variables optimized are less than the standard value. At the same time, the value of  $e_x$ ,  $e_y$ ,  $e_z$  are at the maximum allowable error range. It shows that the geometric errors are appropriate amplification in the premise of ensuring the processing precision of machine tool, so that it can avoid improving the precision grade of the key components blindly in the production process in order to improve the precision of the machine tool. It is found that through the optimization method in this paper, it was able to appropriately enlarge the geometric errors, which can reduce the manufacturing

cost of the machine tool, in the premise of ensuring the machining accuracy of the machine tool.

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